$x = r\cos\theta \rightarrow \frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta + r(-\sin\theta)$ $\quad y = r\sin\theta \rightarrow \frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$

AP Calculus Polar Curves and Derivatives Homework B **Name:** _____

1. Find $dr/d\theta$ of the curve $r = -1 + sin\theta$ at $\theta = 0$ and $\theta = \pi$.

$\frac{d}{d\theta}[r = -1 + \sin\theta]$

$\frac{dr}{d\theta} = \cos\theta$

$\left.\frac{dr}{d\theta}\right|_{\theta=0} = \cos 0 = 1$ $\quad \left.\frac{dr}{d\theta}\right|_{\theta=\pi} = \cos\pi = -1$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

$\frac{dy}{dx} = \frac{0}{\text{Not }0}$ → Horizontal Tangent

$= \frac{\text{Not }0}{0}$ → Vertical Tangent

$= \frac{0}{0}$ → Probably cusp

2. Find the slope of the curve $r = -1 + sin\theta$ at $\theta = 0$ and $\theta = -\frac{\pi}{2}$.

$\left.\frac{dy}{d\theta}\right|_{\theta=0} = (1)(0) + (-1)(1) = -1$

$\frac{dy}{dx} = \frac{-1}{1} = -1$

$\left.\frac{dx}{d\theta}\right|_{\theta=0} = (1)(1) + (-1)(0) = 1$

$\left.\frac{dy}{d\theta}\right|_{\theta=-\frac{\pi}{2}} = (0)(-1) + (-2)(0) = 0$

$\frac{dy}{dx} = \frac{0}{-2} = 0$

$\left.\frac{dx}{d\theta}\right|_{\theta=-\frac{\pi}{2}} = (0)(0) - (-2)(-1) = -2$

3. Find the equation of the tangent line to the curve $r = 3cos\theta$ when $r = 0$ and $0 \le \theta \le 2\pi$

$x, y, \frac{dy}{dx}$

$0 = 3\cos\theta$

$0 = \cos\theta$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{dy}{dx} = \frac{-3\sin^2\theta + 3\cos^2\theta}{-6\sin\theta\cos\theta}$ → $dy/d\theta$ / $dx/d\theta$

at $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ → $\frac{dy}{dx}$ is undefined

$x = r\cos\theta$ $\quad y = r\sin\theta$

$x = 0\cos\theta$ $\quad y = 0\sin\theta$

$x = 0$ $\quad y = 0$

Equation. $x = 0$

4. Find values of θ where the tangent lines to the curve $r = 2sin\theta$ are horizontal and vertical over $0 \le \theta \le \pi$.

$y = r\sin\theta$

$y = 2\sin^2\theta$

$\frac{dy}{d\theta} = 4\sin\theta\cos\theta$

$0 = 4\sin\theta\cos\theta$ → $\theta = 0, \frac{\pi}{2}, \pi$ (HT)

$x = r\cos\theta$

$x = 2\sin\theta\cos\theta$

$\frac{dx}{d\theta} = 2\cos\theta\cdot\cos\theta + 2\sin\theta(-\sin\theta)$

$\frac{dx}{d\theta} = 2(\cos^2\theta - \sin^2\theta)$

$0 = 2(\cos^2\theta - \sin^2\theta)$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$ → VT

$(\frac{dy}{d\theta} = 0)$ $\quad (\frac{dx}{d\theta} = 0)$ [* as long as they are not Both 0]

5. Find two equivalent polar coordinates and the Rectangular coordinates for the polar coordinate pair.

$\left(-3, \frac{2\pi}{3}\right)$

6. Find two equivalent pairs of polar coordinates for the rectangular coordinate pair.

$(-1, 1)$