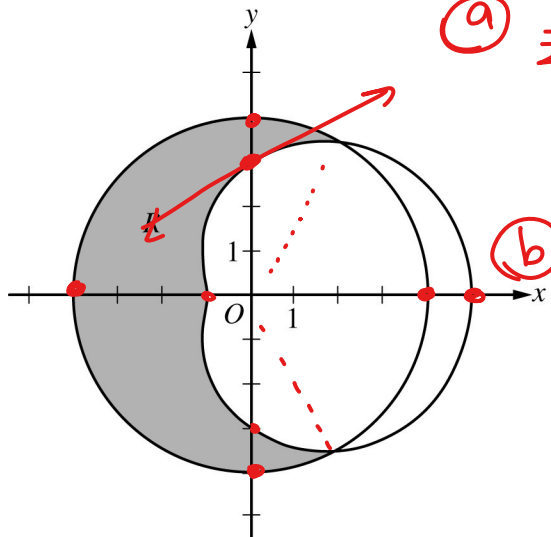


2018 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

$4 = 3 + 2\cos\theta$
 $1 = 2\cos\theta$
 $\frac{1}{2} = \cos\theta$
 $\frac{\pi}{3}, \frac{5\pi}{3}$



(a) $\frac{1}{2} \int_{\pi/3}^{5\pi/3} (4^2 - (3+2\cos\theta)^2) d\theta$

5. The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect

at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.

(a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R .

(b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$. $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-2}{-3}$

(c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per

$\frac{dr}{dt} = 3$

second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

$\frac{d}{dt} [r = 3 + 2\cos\theta]$

$\frac{dr}{dt} = -2\sin\theta \frac{d\theta}{dt}$

$3 = -2\sin\frac{\pi}{3} \frac{d\theta}{dt}$

$\frac{d\theta}{dt} = \frac{3}{-2\sin\frac{\pi}{3}}$

$y = r\sin\theta \rightarrow y = (3 + 2\cos\theta)\sin\theta$

$\frac{dy}{d\theta} = (-2\sin\theta)\sin\theta + (3 + 2\cos\theta)\cos\theta$

$\frac{dy}{d\theta} \Big|_{\theta=\pi/2} = (-2(1))(1) + (3 + 2(0))(0) = -2$

$x = r\cos\theta \rightarrow x = (3 + 2\cos\theta)\cos\theta$

$\frac{dx}{d\theta} = (-2\sin\theta)\cos\theta + (3 + 2\cos\theta)(-\sin\theta)$