

$$1.) \quad 4 = r \sin \theta$$

$$\frac{4}{\sin \theta} = r$$

$$\boxed{4 \csc \theta = r}$$

$$5.) \quad r = 2 \sin \theta$$

$$r^2 = 2r \sin \theta$$

$$x^2 + y^2 = 2y$$

$$x^2 + y^2 - 2y + 1 = 0 + 1$$

$$\boxed{x^2 + (y-1)^2 = 1}$$

$$2.) \quad 3r \cos \theta - 5r \sin \theta + 2 = 0$$

$$r(3 \cos \theta - 5 \sin \theta) = -2$$

$$\boxed{r = \frac{-2}{3 \cos \theta - 5 \sin \theta}}$$

$$3.) \quad r^2 = 25$$

$$\boxed{r = 5}$$

$$4.) \quad r = \frac{3}{\cos \theta}$$

$$r \cos \theta = 3$$

$$\boxed{x = 3}$$

$$6.) \quad \tan \theta = \frac{y}{x}$$

$$\tan \frac{5\pi}{6} = \frac{y}{x}$$

$$-\frac{\sqrt{3}}{3} = \frac{y}{x}$$

$$3y = -x\sqrt{3}$$

$$\boxed{y = \frac{-x\sqrt{3}}{3}}$$

$$7.) r = 2 + 3\sin\theta \quad \theta = \frac{3\pi}{2}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$

$$r' = 3\cos\theta \rightarrow r'(\theta) \rightarrow r'\left(\frac{3\pi}{2}\right) = 3\cos\frac{3\pi}{2} = 0$$

$$\sin\frac{3\pi}{2} = -1 \quad \cos\frac{3\pi}{2} = 0 \quad r\left(\frac{3\pi}{2}\right) = -1$$

$$\frac{dy}{dx}\bigg|_{\theta=\frac{3\pi}{2}} = \frac{0 \cdot (-1) + (-1)(0)}{0(0) - (-1)(-1)} = \frac{0}{-1} = \textcircled{0}$$

$$8.) r = 3 - 3\cos\theta$$

$$r'(\theta) = 3\sin\theta$$

$$r'\left(\frac{\pi}{2}\right) = 3 \quad r\left(\frac{\pi}{2}\right) = 3$$

$$\cos\left(\frac{\pi}{2}\right) = 0 \quad \sin\left(\frac{\pi}{2}\right) = 1$$

$$\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$

$$\frac{dy}{dx}\bigg|_{\theta=\frac{\pi}{2}} = \frac{3(1) + 3(0)}{3(0) - 3(1)} = \frac{3}{-3} = \textcircled{-1}$$

$$9.) r = 4\sin\theta$$

$$r' = 4\cos\theta$$

$$r\left(\frac{\pi}{3}\right) = 2\sqrt{3}$$

$$r'\left(\frac{\pi}{3}\right) = 2$$

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{3} = \frac{1}{2}$$

$$\frac{dy}{dx}\bigg|_{\theta=\frac{\pi}{3}} = \frac{2\left(\frac{\sqrt{3}}{2}\right) + 2\sqrt{3} \cdot \frac{1}{2}}{2\left(\frac{1}{2}\right) - 2\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{\sqrt{3} + \sqrt{3}}{1 - 3} = \frac{2\sqrt{3}}{-2} = \textcircled{-\sqrt{3}}$$

$$* y = r\sin\theta \rightarrow \frac{dy}{d\theta} = r'\sin\theta + r\cos\theta$$

$$* x = r\cos\theta \rightarrow \frac{dx}{d\theta} = r'\cos\theta + r(-\sin\theta)$$

$$10.) r = 2 \sin(3\theta) \quad \theta = \frac{\pi}{4}$$

$$r' = 2 \cos(3\theta) \cdot 3$$

$$r\left(\frac{\pi}{4}\right) = 2 \sin \frac{3\pi}{4} = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$$

$$r'\left(\frac{\pi}{4}\right) = 6 \cos \frac{3\pi}{4} = 6 \cdot \frac{-\sqrt{2}}{2} = -3\sqrt{2}$$

$$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \quad \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\frac{(-3\sqrt{2})\left(\frac{\sqrt{2}}{2}\right) + \sqrt{2}\left(\frac{\sqrt{2}}{2}\right)}{(-3\sqrt{2})\left(\frac{\sqrt{2}}{2}\right) - \sqrt{2}\frac{\sqrt{2}}{2}} = \frac{-3+1}{-3-1} = \frac{-2}{-4} = \left(\frac{1}{2}\right)$$

$$11.) r = 1 + \sin\theta \quad r' = \cos\theta$$

$$\frac{dy}{d\theta} = r' \sin\theta + r \cos\theta$$

$$0 = \cos\theta \sin\theta + \cos\theta(1 + \sin\theta)$$

$$0 = \cos\theta(1 + 2\sin\theta)$$

$$\cos\theta = 0 \quad \sin\theta = -\frac{1}{2}$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \quad \theta = -\frac{\pi}{6}, \frac{7\pi}{6}$$

$$r\left(\frac{\pi}{2}\right) = 2 \quad r\left(-\frac{\pi}{6}\right) = \frac{1}{2} \quad r\left(\frac{7\pi}{6}\right) = \frac{1}{2}$$

$$r\left(\frac{3\pi}{2}\right) = 0$$

$$\text{H.T: } \left(2, \frac{\pi}{2}\right) \quad \left(\frac{1}{2}, \frac{7\pi}{6}\right)$$

$$\left(\frac{1}{2}, -\frac{\pi}{6}\right) \quad \left(0, \frac{3\pi}{2}\right)$$

$$\frac{dx}{d\theta} = r' \cos\theta - r \sin\theta$$

$$= \cos^2\theta - \sin^2\theta - \sin\theta$$

$$= 1 - \sin^2\theta - \sin^2\theta - \sin\theta$$

$$= -2\sin^2\theta - \sin\theta + 1$$

$$0 = 2\sin^2\theta + \sin\theta - 1$$

$$0 = (2\sin\theta - 1)(\sin\theta + 1)$$

$$\sin\theta = \frac{1}{2} \quad \sin\theta = -1$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad \theta = \frac{3\pi}{2}$$

$$\text{V.T: } \left(1.5, \frac{\pi}{6}\right) \quad \left(1.5, \frac{5\pi}{6}\right)$$

* since both $\frac{dy}{d\theta} \neq \frac{dx}{d\theta} = 0$, it's neither a H.T. nor a V.T. \rightarrow cusp

16.) (B) → just pick an angle and eliminate options.

17.) (D) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

18.) $r = 2 \sec \theta$
 $r = \frac{2}{\cos \theta}$ (D)

$$r \cos \theta = 2$$

$$x = 2$$

* For 12-15, just use desmos and be able to make a chart/table to see specific values.