Polar Review

(r, θ) $x = r\cos\theta$ $y = r\sin\theta$

$x^2 + y^2 = r^2$

1.) (a)

$x = -4\cos(-\frac{\pi}{3})$

$y = -4\sin(-\frac{\pi}{3})$

$(-2, 2\sqrt{3})$

(b) $(-8, \pi)$ → this is polar → convert to rectangular

$x = -8\cos\pi$

$y = -8\sin\pi$

$(8, 0)$

2.) (a) $(4,4)$ $4^2 + 4^2 = r^2$ $r = 4\sqrt{2}$

$\theta = \tan^{-1}(\frac{4}{4})$ $\theta = \frac{\pi}{4}$

$(4\sqrt{2}, \frac{\pi}{4})$

(b) $(0, \sqrt{6})$ $0^2 + (\sqrt{6})^2 = r^2$ $r = \sqrt{6}$

$\theta = \tan^{-1}(\frac{\sqrt{6}}{0}) \rightarrow \frac{\pi}{2}$

$(\sqrt{6}, \frac{\pi}{2})$

3. (a) $y = 4$ $r\sin\theta = 4$ $r = \frac{4}{\sin\theta}$ or $r = 4\csc\theta$

(b) $3x - 5y + 2 = 0$

$3r\cos\theta - 5r\sin\theta = -2$

$r(3\cos\theta - 5\sin\theta) = -2$ $r = \frac{-2}{3\cos\theta - 5\sin\theta}$

3.) c $x^2+y^2=25$

$r^2=25$

$r=5$

4.) a $r=3\sec\theta$

$r=\dfrac{3}{\cos\theta}$ $\nearrow$ $r\cos\theta=3 \rightarrow x=3$

b $r=2\sin\theta$

$r^2=2r\sin\theta$ $\nearrow$ $x^2+y^2=2y$

$x^2+y^2-2y=0$

$x^2+(y-1)^2=1$

c $\theta=\dfrac{5\pi}{6}$ $\quad \tan\dfrac{5\pi}{6}=\dfrac{y}{x}$ $\quad -\dfrac{\sqrt{3}}{3}=\dfrac{y}{x}$

$-x\sqrt{3}=3y$ $\quad y=\dfrac{-x\sqrt{3}}{3}$

5.) a $\dfrac{dy}{dx}=\dfrac{dy/d\theta}{dx/d\theta}$ $\quad y=r\sin\theta$ $\quad \dfrac{dy}{d\theta}=r'\sin\theta+r\cos\theta$

$x=r\cos\theta$ $\quad \dfrac{dx}{d\theta}=r'\cos\theta-r\sin\theta$

$r'=-\cos\theta$ $\quad \dfrac{dy}{dx}=\dfrac{-\cos\theta\sin\theta+(1-\sin\theta)\cos\theta}{-\cos\theta\cos\theta-(1-\sin\theta)\sin\theta}$

$\left.\dfrac{dy}{dx}\right|_{\theta=0}=\dfrac{1}{-1}=-1$

5.) (b) $r = 6(1+\cos\theta)$ @ $\theta = \pi/2$

$r' = -6\sin\theta$

$$\frac{dy}{dx} = \frac{-6\sin\theta\sin\theta + 6(1+\cos\theta)\cos\theta}{-6\sin\theta\cos\theta - 6(1+\cos\theta)\sin\theta)}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{-6}{-6} = 1$$

(c) $r = 5\cos(3\theta)$ $\quad \theta = \pi/3$

$r' = -5\sin(3\theta)\cdot 3$
$= -15\sin(3\theta)$

$$\frac{dy}{dx} = \frac{(-15\sin 3\theta)\sin\theta + 5\cos(3\theta)\cos\theta}{(-15\sin(3\theta))\cos\theta - 5\cos(3\theta)\sin\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{-5\left(\frac{1}{2}\right)}{5\left(\frac{\sqrt{3}}{2}\right)} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

6.) (a) $A = \frac{1}{2}\int_0^{2\pi} (8+2\sin\theta)^2 \, d\theta \approx 207.345$

(b) $A = \frac{1}{2}\int_0^{\pi} (4\cos\theta + 9\sin\theta)^2 \, d\theta \approx 76.184$

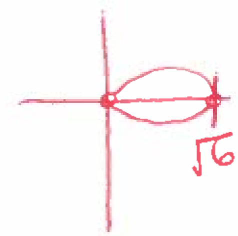
(c) $r = 0 \rightarrow 0 = 6\cos 2\theta$

$\cos 2\theta = 0$

$2\theta = \frac{\pi}{2} + \pi n$

$\theta = \frac{\pi}{4} + \frac{\pi}{2} n$

θ	r
0	$\sqrt{6}$
$\pi/4$	0
$-\pi/4$	0



$A = \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 6\cos(2\theta) \, d\theta = 3$

(d) $0 = 4 + 8\sin\theta$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$A = \frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} (4+8\sin\theta)^2 \, d\theta \approx 8.696$

(e) $A = \frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{7\pi}{6}} (4+8\sin\theta)^2 \, d\theta - 8.696 \approx 133.404$

(f) $(\sqrt{3}\sin\theta)^2 = (1+\cos\theta)^2$

$$3\sin^2\theta = 1 + 2\cos\theta + \cos^2\theta$$

$$3(1-\cos^2\theta) = 1 + 2\cos\theta + \cos^2\theta$$

$$3 - 3\cos^2\theta = 1 + 2\cos\theta + \cos^2\theta$$

$$0 = 4\cos^2\theta + 2\cos\theta - 2$$

$$0 = 2(2\cos^2\theta + \cos\theta - 1)$$

$$0 = 2(2\cos\theta - 1)(\cos\theta + 1)$$

$\cos\theta = \frac{1}{2}$ $\quad$ $\cos\theta = -1$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ extraneous $\quad$ $\theta = \pi$

$$A = \frac{1}{2}\int_{\frac{\pi}{3}}^{\pi}\left((\sqrt{3}\sin\theta)^2 - (1+\cos\theta)^2\right)d\theta$$

$$\approx 1.299$$

g) $-6\cos\theta = 3$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$$A = \frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \left((-6\cos\theta)^2 - 3^2\right) d\theta \approx 17.219$$

h) $1 + \cos\theta = 3\cos\theta$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, -\frac{\pi}{3}$

$$A = \frac{1}{2}\int_0^{\pi} (3\cos\theta)^2 d\theta - \frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left((3\cos\theta)^2 - (1+\cos\theta)^2\right) d\theta$$

$$\approx 3.927$$

7) a) $L = \int_0^{\pi/2} \sqrt{(2\sin\theta + 2\cos\theta)^2 + \frac{dr}{d\theta}^2}\, d\theta$; $\frac{dr}{d\theta} = 2\cos\theta - 2\sin\theta$

≈ 4.443

b) $L = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{(1+\cos 2\theta) + \left(\frac{dr}{d\theta}\right)^2}\; d\theta$ $\rightarrow$ $\frac{dr}{d\theta} = \frac{1}{2}(1+\cos 2\theta)^{-\frac{1}{2}} \cdot -\sin 2\theta \cdot 2$

≈ 4.443

8.) (c) $3 = 4 - 2\sin\theta$

$-1 = -2\sin\theta$

$\sin\theta = \frac{1}{2} \qquad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$A = \frac{1}{2}\int_0^{2\pi} 3^2 \, d\theta - \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(3^2 - (4-2\sin\theta)^2\right) d\theta$$

≈ 24.709

(b) $r = 4 - 2\sin\theta \qquad \theta = t^2 \qquad 1 \le t \le 2, \; x = -1$

$x = r\cos\theta \qquad , \; x = (4-2\sin\theta)\cos\theta$

$-1 = (4-2\sin\theta)\cos\theta$

(Solve w/ calculator in Func mode)

$\theta = 2.039125 \rightarrow t^2 = 2.039125$

$\rightarrow t = 1.428$ seconds

(c) $y = r\sin\theta \; , \; y = (4-2\sin\theta)\sin\theta$

Position vector: $\langle (4-2\sin\theta)\cos\theta, (4-2\sin\theta)\sin\theta \rangle$

i.t.o t $\quad \langle (4-2\sin t^2)\cos t^2, (4-2\sin t^2)\sin t^2 \rangle$

$v'(1.5) = \langle x'(1.5), y'(1.5) \rangle \approx \langle -8.072, -1.673 \rangle$

9) a

$2 = 3 + 2\cos\theta$

$\cos\theta = -\frac{1}{2}$ $\quad \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$$A = \frac{1}{2}\int_0^{2\pi} 2^2 \, d\theta - \frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \left(2^2 - (3+2\cos\theta)^2\right) d\theta$$

$$\approx 10.370$$

b $\frac{dr}{d\theta} = -2\sin\theta$

$$\left.\frac{dr}{d\theta}\right|_{\theta=\frac{\pi}{3}} = -2\sin\frac{\pi}{3} = \frac{-2\cdot\sqrt{3}}{2} = -\sqrt{3}$$

therefore $-\sqrt{3} = \frac{dr}{dt}$. The particle is moving closer to the origin when $\theta = \frac{\pi}{3}$ since $r > 0$ and $\frac{dr}{dt} < 0$

c $y = r\sin\theta$ $\quad \frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$

$$\left.\frac{dy}{d\theta}\right|_{\theta=\frac{\pi}{3}} = (-\sqrt{3})\left(\frac{\sqrt{3}}{2}\right) + \left(3 + 2\left(\tfrac{1}{2}\right)\right)\left(\tfrac{1}{2}\right)$$

$$= -\frac{3}{2} + (4)\left(\tfrac{1}{2}\right) = -\frac{3}{2} + 2 = \frac{1}{2}$$

so $\frac{dy}{dt} = \frac{dy}{d\theta}$ and the particle is moving away from the x-axis when $\theta = \frac{\pi}{3}$ since $\frac{dy}{dt} > 0$ & $y > 0$