

Systems of Conic Sections Notes

State if the point given is a solution to the system of equations.

$$1) \begin{cases} 3y^2 + 13x + 6y - 137 = 0 \\ 15x^2 - 3y^2 - 28x - 6y - 163 = 0 \end{cases}$$

Point: (7, -4)

$$3(-4)^2 + 13(7) + 6(-4) - 137$$

$$48 + 91 - 24 - 137 \neq 0$$

No

$$2) \begin{cases} x^2 - 3y^2 + 30x + 2y - 192 = 0 \\ x + y - 2 = 0 \end{cases}$$

Point: (10, -8)

$$(10)^2 - 3(-8)^2 + 30(10) + 2(-8) - 192$$

$$100 - 3(64) + 300 - 16 - 192$$

$$100 - 192 + 300 - 16 - 192$$

$$-92 + 300 - 208$$

$$-300 + 300 = 0$$

yes

Solve each system of equations.

$$3) \begin{cases} 2y^2 - 27x - 3y + 180 = 0 \\ x + y = 4 \end{cases}$$

$x = 4 - y$

$$2y^2 - 27(4 - y) - 3y + 180 = 0$$

$$2y^2 - 108 + 27y - 3y + 180 = 0$$

$$2y^2 + 24y + 72 = 0$$

$$2(y^2 + 12y + 36) = 0$$

$$2(y + 6)^2 = 0$$

$$y = -6$$

$$x = 4 - (-6)$$

$$x = 10$$

$$(10, -6)$$

$$4) \begin{cases} 2x^2 - 3y^2 + 76x + 4y + 4 = 0 \\ 3x + y = 2 \end{cases} \quad (0, 2) \quad (4, -10)$$

$$y = 2 - 3x$$

$$2x^2 - 3(2 - 3x)^2 + 76x + 4(2 - 3x) + 4 = 0$$

$$2x^2 - 3(4 - 12x + 9x^2) + 76x + 8 - 12x + 4 = 0$$

$$2x^2 - 12 + 36x - 27x^2 + 76x + 8 - 12x + 4 = 0$$

$$-25x^2 + 100x = 0$$

$$-25x(x - 4) = 0$$

$$x = 0 \text{ and } 4$$

$$\begin{aligned} y &= 2 - 3(0) = 2 \\ y &= 2 - 3(4) = -10 \end{aligned}$$

$$5) \begin{cases} 2x^2 - y^2 + 4x + 10y - 32 = 0 \\ -(14x^2 - y^2 + 28x + 10y - 128 = 0) \end{cases}$$

$$-12x^2 - 24x + 96 = 0$$

$$-12(x^2 + 2x - 8) = 0$$

$$-12(x + 4)(x - 2) = 0$$

$$x = -4 \quad x = 2$$

$$2(-4)^2 - y^2 + 4(-4) + 10y - 32 = 0$$

$$2(16) - y^2 - 16 + 10y - 32 = 0$$

$$32 - y^2 - 16 + 10y - 32 = 0$$

$$-y^2 + 10y - 16 = 0$$

$$-(y^2 - 10y + 16) = 0$$

$$-(y - 8)(y - 2) = 0$$

$$y = 8 \quad y = 2$$

$$2(2)^2 - y^2 + 4(2) + 10y - 32 = 0$$

$$2(4) - y^2 + 8 + 10y - 32 = 0$$

$$8 - y^2 + 8 + 10y - 32 = 0$$

$$-y^2 + 10y - 16 = 0$$

$$y = 8, y = 2 \quad (-4, 8) \quad (-4, 2) \quad (2, 8) \quad (2, 2)$$

$$6) \begin{cases} 4x^2 - 2y^2 - 55x + 20y + 138 = 0 \\ + \\ -9x^2 + 2y^2 + 53x - 20y - 151 = 0 \end{cases}$$

$$-5x^2 - 2x - 13 = 0$$

$$5x^2 + 2x + 13 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4(5)(13)}}{2(5)}$$

No real solution

Write the equation of each conic section in standard form.

7) $4x^2 + 25y^2 + 8x - 250y + 529 = 0$

$$4x^2 + 8x + 4 + 25y^2 - 250y + 625 = -529$$

+4
+625

$$\frac{4(x+1)^2}{100} + \frac{25(y-5)^2}{100} = \frac{100}{100}$$

$$\boxed{\frac{(x+1)^2}{25} + \frac{(y-5)^2}{4} = 1}$$

8) $6y^2 + x + 36y + 57 = 0$

$$x + 57 = -6y^2 - 36y - 54$$

$$-54 = -6(y^2 + 6y + 9)$$

$$\boxed{x + 3 = -6(y + 3)^2}$$

$$\text{or } x = -6(y + 3)^2 - 3$$

9) $2y^2 - 12x = -12y - 2x^2 - 28$

$$2x^2 - 12x + 18 + 2y^2 + 12y + 18 = -28$$

+18
+18

$$2(x^2 - 6x + 9) + 2(y^2 + 6y + 9)$$

$$2(x-3)^2 + 2(y+3)^2 = 8$$

$$\boxed{(x-3)^2 + (y+3)^2 = 4}$$

10) $4x - 6 = y^2 - x^2 - 2y$

$$x^2 + 4x + 4 - y^2 + 2y - 1 = 6$$

-1
+4

$$-(y^2 - 2y + 1)$$

$$(x+2)^2 - (y-1)^2 = 9$$

$$\boxed{\frac{(x+2)^2}{9} - \frac{(y-1)^2}{9} = 1}$$