Pre Calculus Name____

Ellipses Notes

Date________________ Period____

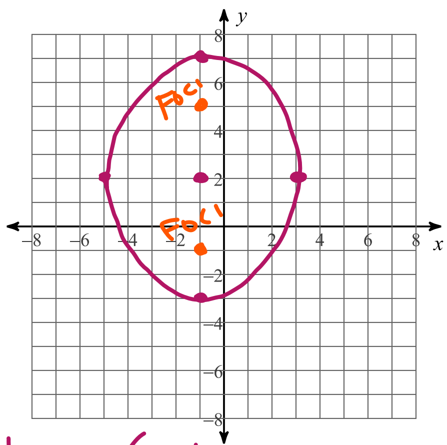
Identify the center, vertices, co-vertices, foci, length of the major axis, and length of the minor axis of each. Then sketch the graph.

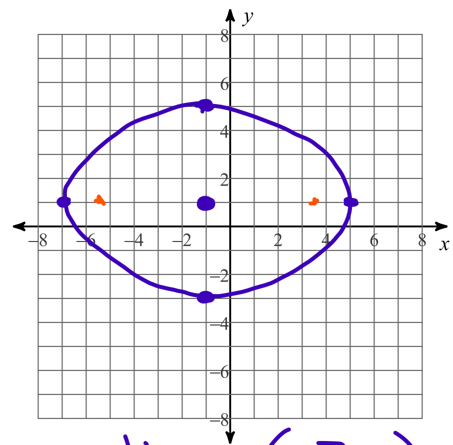
1) $\frac{(x+1)^2}{16} + \frac{(y-2)^2}{25} = 1$ | * $1 = \left(\frac{x+1}{4}\right)^2 + \left(\frac{y-2}{5}\right)^2$

Finetoo

2) $\frac{(x+1)^2}{36} + \frac{(y-1)^2}{16} = 1$

center $(-1,1)$





center $(-1,2)$

vertices $(-1,7)$ $(-1,-3)$

co-vertices $(-5,2)$ $(3,2)$

length major axis 10

Length minor axis 8

$c^2 = a^2 - b^2$ "c" is focal length (apply to the center)

a → semi-major axis length

b → semi-minor axis length

$c^2 = 25 - 16$

$= 9$

$c = 3$

Foci

$(-1,5)$ $(-1,-1)$

vertices $(-7,1)$ $(5,1)$

co-vertices $(-1,-3)$ $(-1,5)$

length major 12

Length minor 8

$c^2 = a^2 - b^2$

$= 36 - 16$

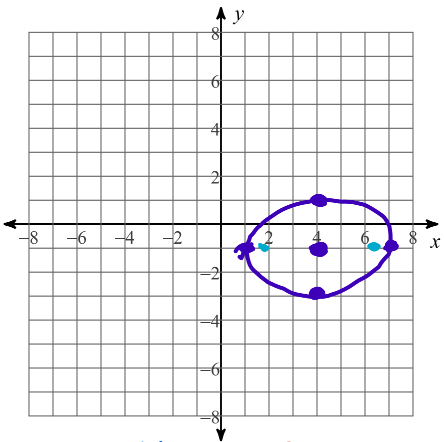
$c^2 = 20$

$c = \pm 2\sqrt{5}$

Foci $(-1 \pm 2\sqrt{5}, 1)$

$L_{maj} = 6 \quad L_{min} = 4$

3) $4x^2 + 9y^2 - 32x + 18y + 37 = 0$



$4x^2 - 32x \underline{+64} + 9y^2 + 18y \underline{+9} = -37 + 64 + 9$

$4(x^2 - 8x \underline{+16}) + 9(y^2 + 2y \underline{+1})$

$\frac{4(x-4)^2}{36} + \frac{9(y+1)^2}{36} = \frac{36}{36}$

$$\boxed{\frac{(x-4)^2}{9} + \frac{(y+1)^2}{4} = 1}$$

c $(4, -1)$

vertices $(1, -1)$ $(7, -1)$

co-vertices $(4, 1)$ $(4, -3)$

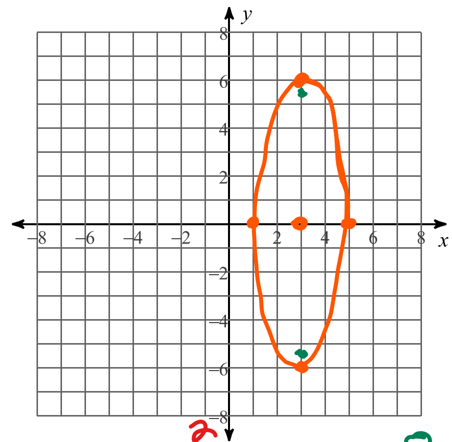
$c^2 = a^2 - b^2$

$= 9 - 4$

$c^2 = 5$ $\quad c = \sqrt{5}$

Foci. $(4 \pm \sqrt{5}, -1)$

4) $9x^2 + y^2 - 54x + 45 = 0$



$9x^2 - 54x \underline{+81} + y^2 = -45 + 81$

$9(x^2 - 6x \underline{+9}) + y^2$

$\frac{9(x-3)^2}{36} + \frac{y^2}{36} = \frac{36}{36}$

$$\boxed{\frac{(x-3)^2}{4} + \frac{y^2}{36} = 1}$$

$c^2 = 36 - 4$

$c^2 = 32$

$c = \sqrt{32} = 4\sqrt{2}$

F $(3, 0 \pm 4\sqrt{2})$

Use the information provided to write the standard form equation of each ellipse.

$\sqrt{13} = c$

$13 = c^2$

5) Vertices: $(-3, 3), (-3, -19)$
Co-vertices: $(7, -8), (-13, -8)$

center: $(-3, -8)$

$$\frac{(x+3)^2}{100} + \frac{(y+8)^2}{121} = 1$$

6) Vertices: $(-10, 16), (-10, 2)$
Foci: $(-10, 9 + \sqrt{13}), (-10, 9 - \sqrt{13})$

center: $(-10, 9)$

$$\boxed{\frac{(x+10)^2}{36} + \frac{(y-9)^2}{49} = 1}$$

$c^2 = a^2 - b^2$

$13 = 49 - b^2$

$b^2 = 36$

$-2 \leftrightarrow -12 = 10 \quad \frac{10}{2} = 5 \quad 5^2 = 25$

7) Foci: $(-7 + 2\sqrt{6}, -7), (-7 - 2\sqrt{6}, -7)$
Endpoints of minor axis: $(-7, -2), (-7, -12)$

c $(-7, -7)$

$$\boxed{\frac{(x+7)^2}{49} + \frac{(y+7)^2}{25} = 1}$$

$c = 2\sqrt{6}$

$c^2 = 4 \cdot 6 = 24$

$c^2 = a^2 - b^2$

$24 = a^2 - 25$

$a^2 = 49$

8) Center: $(-9, 3)$
Vertex: $(-19, 3)$
$c^2 = 84$

$$\boxed{\frac{(x+9)^2}{100} + \frac{(y-3)^2}{16} = 1}$$

$a^2 - b^2 = c^2$

$100 - b^2 = 84$

$c = 5 \rightarrow c^2 = 25$

9) Foci: $\left(\frac{17}{2}, -4\right), \left(-\frac{3}{2}, -4\right)$

Co-vertices: $\left(\frac{7}{2}, -4 + \sqrt{55}\right), \left(\frac{7}{2}, -4 - \sqrt{55}\right)$

Center: $\left(\frac{7}{2}, -4\right)$

$$\boxed{\frac{\left(x - \frac{7}{2}\right)^2}{80} + \frac{(y+4)^2}{55} = 1}$$

$\sqrt{55} = b \rightarrow b^2 = 55$

$c^2 = a^2 - b^2$

$25 = a^2 - 55$

$a^2 = 80$

$c = 3\sqrt{7} \rightarrow c^2 = 9 \cdot 7 = 63$

10) Foci: $\left(-10, 2 + 3\sqrt{7}\right), \left(-10, 2 - 3\sqrt{7}\right)$

Point on the ellipse: $\left(-7, 2 + 8\sqrt{2}\right)$

$$\frac{(x+10)^2}{a^2 - 63} + \frac{(y-2)^2}{a^2} = 1$$

$a^2 - b^2 = 63$

$$\frac{(-7+10)^2}{a^2 - 63} + \frac{(2 + 8\sqrt{2} - 2)^2}{a^2} = 1$$

$$\frac{a^2}{a^2} \cdot \frac{9}{a^2 - 63} + \frac{128}{a^2} \cdot \frac{a^2 - 63}{a^2 - 63} = 1 \rightarrow \frac{a^2(a^2 - 63)}{a^2(a^2 - 63)}$$

$$9a^2 + 128(a^2 - 63) = a^2(a^2 - 63)$$

$$9a^2 + 128a^2 - 8064 = a^4 - 63a^2$$

$$0 = a^4 - 200a^2 + 8064$$

$$0 = \cancel{(a^2 - 56)}(a^2 - 144)$$

$$a^2 = 144$$

$$\boxed{\frac{(x+10)^2}{81} + \frac{(y-2)^2}{144} = 1}$$