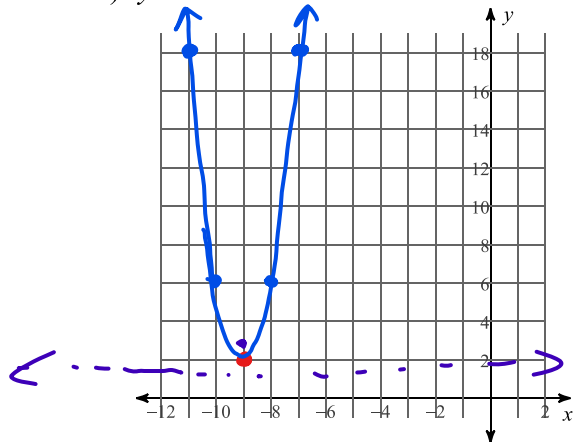


$$y = a(x-h)^2 + k$$

Parabolas and Circles Notes

Use the information provided to write the vertex form equation of each parabola. Identify the vertex, focus, directrix, y-intercept, and x-intercepts of each. Then sketch the graph.

1) $y = 4x^2 + 72x + 326$



$$y - 326 = 4(x^2 + 18x + 81) + 481$$

$$y - 2 = 4(x + 9)^2$$

$$\text{or } y = 4(x + 9)^2 + 2$$

$$0 - 2 = 4(x + 9)^2$$

$- \neq +$ No x-int

Vertex: $(-9, 2)$

y-int (x=0) $(0, 326)$

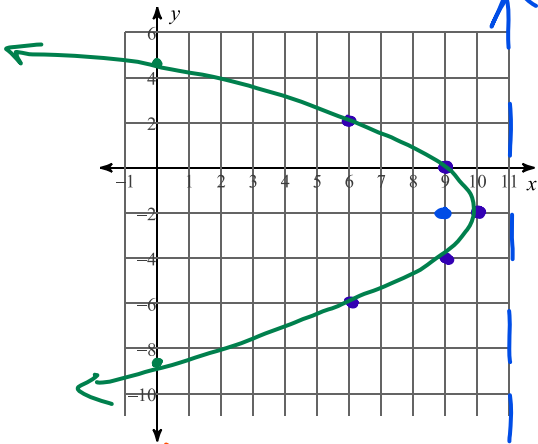
Find focal length $p = \frac{1}{4a}$ → the distance from the vertex x to the focus

$$p = \frac{1}{4(4)} = \frac{1}{16}$$

Focus $(-9, 2\frac{1}{16})$

Directrix $y = 1\frac{15}{16}$

$$2) x = -\frac{1}{4}y^2 - y + 9$$



y-int (x=0)

$$0 - 10 = -\frac{1}{4}(y+2)^2$$

$$\sqrt{40} = \sqrt{(y+2)^2}$$

$$\pm 2\sqrt{10} = y+2$$

y-int: $(0, -2 \pm 2\sqrt{10})$

$$x - 9 = -\frac{1}{4}(y^2 + 4y + 4)$$

$$-\frac{1}{4}(4)$$

$$x - 10 = -\frac{1}{4}(y+2)^2$$

$$\text{or } x = -\frac{1}{4}(y+2)^2 + 10$$

vertex $(10, -2)$

x-int $(9, 0)$

Focal length

$$p = \frac{1}{4a} = \frac{1}{4(-\frac{1}{4})} = -1$$

Focus $(9, -2)$

Directrix $x = 11$

axis of symmetry $x = \frac{-b}{2a}$

Use the information provided to write the vertex form equation of each parabola.

$$3) y = -\frac{1}{4}x^2 - \frac{5}{2}x - \frac{61}{4}$$

$$x = \frac{\frac{5}{2}}{2(-\frac{1}{4})}$$

$$x = \frac{5}{2} \cdot \frac{-4}{2}$$

$$x = -5$$

$$y(-5) = -\frac{1}{4}(25) - \frac{5}{2}(-5) - \frac{61}{4}$$

$$= -\frac{25}{4} + \frac{50}{4} - \frac{61}{4}$$

$$= \frac{-36}{4} = -9$$

$$y + 9 = -\frac{1}{4}(x + 5)^2$$

$$4) x = 2y^2 + 32y + 133$$

$$y = \frac{-32}{2(2)} = \frac{-32}{4} = -8$$

$$x(-8) = 2(-8)^2 + 32(-8) + 133$$

$$2(64) - 256 + 133$$

$$128 - 256 + 133$$

$$-128 + 133$$

5

$$x - 5 = 2(y + 8)^2$$

Completing the square

Use the information provided to write the vertex form equation of each parabola. Identify the vertex, focus, directrix, y-intercept, and x-intercepts of each.

5) $y = \frac{1}{3}x^2 - 3x + \frac{14}{3}$

$\frac{1}{4} \cdot \frac{-14}{3} + \frac{27}{4} \cdot \frac{3}{3}$

$= \frac{-56 + 81}{12} = \frac{25}{12}$

* $y - \frac{14}{3} = \frac{1}{3} \left(x^2 - 9x + \frac{81}{4} \right)$

$+ \frac{1}{3} \cdot \frac{81}{4}$

$$y + \frac{25}{12} = \frac{1}{3} \left(x - \frac{9}{2} \right)^2$$

V $\left(\frac{9}{2}, -\frac{25}{12} \right)$

y-int. $\left(0, \frac{14}{3} \right)$

$$3 \left[0 + \frac{25}{12} = \frac{1}{3} \left(x - \frac{9}{2} \right)^2 \right]$$

$$\sqrt{\frac{25}{4}} = \sqrt{\left(x - \frac{9}{2} \right)^2}$$

$$\pm \frac{5}{2} = x - \frac{9}{2}$$

$$x = \frac{9}{2} \pm \frac{5}{2}$$

x-int $(7, 0)$ $(2, 0)$

$$p = \frac{1}{4 \left(\frac{1}{3} \right)} = \frac{3}{4}$$

Focus: $\left(\frac{9}{2}, -\frac{25}{12} + \frac{3}{4} \right)$

Directrix: $y = -\frac{25}{12} - \frac{3}{4}$

6) $x = -2y^2 + 22y - 60$

* $\frac{120}{2} - \frac{121}{2} = -\frac{1}{2}$

* $x + 60 = -2 \left(y^2 - 11y + \frac{121}{4} \right)$

$-2 \left(\frac{121}{4} \right)$

$$x - \frac{1}{2} = -2 \left(y - \frac{11}{2} \right)^2$$

V $\left(\frac{1}{2}, \frac{11}{2} \right)$

x-int $(-60, 0)$

$$0 - \frac{1}{2} = -2 \left(y - \frac{11}{2} \right)^2$$

$$\sqrt{\frac{1}{4}} = \sqrt{\left(y - \frac{11}{2} \right)^2}$$

$$\pm \frac{1}{2} = y - \frac{11}{2} \rightarrow y = \frac{11}{2} \pm \frac{1}{2}$$

y-int. $(0, 5)$ $(0, 6)$

$$p = \frac{1}{4(-2)} = -\frac{1}{8}$$

Focus $\left(\frac{1}{2} - \frac{1}{8}, \frac{11}{2} \right)$

Directrix $x = \frac{1}{2} + \frac{1}{8}$

Use the information provided to write the standard form equation of each circle.

7) Three points on the circle:

$(-13, -10)$, $(-3, -10)$, and $(-15, 2)$

$$(-13-h)^2 + (-10-k)^2 = r^2$$

$$(-3-h)^2 + (-10-k)^2 = r^2$$

$$(-15-h)^2 + (2-k)^2 = r^2$$

$$(-15+8)^2 + 4 - 4k + k^2 = r^2$$

$$49 + 4 - 4k + k^2 = r^2$$

$$(-13+8)^2 + 100 + 20k + k^2 = r^2$$

$$25 + 100 + 20k + k^2 = r^2$$

so $(-13-h)^2 = (-3-h)^2$

$$169 + 26h + h^2 = 9 + 6h + h^2$$

$$169 + 26h = 9 + 6h \quad r^2 = 53 - 4(-3) + (-3)^2$$

$$20h = -160$$

$$h = -8$$

$$= 53 + 12 + 9$$

$$r^2 = 74$$

$$53 - 4k + k^2 = r^2$$

$$-(125 + 20k + k^2 = r^2)$$

$$-72 - 24k = 0$$

$$-24k = 72$$

$$k = -3$$

$$(x+8)^2 + (y+3)^2 = 74$$

Use the information provided to write the vertex form equation of each parabola.

8) Vertex: $(-3, 2)$, Directrix: $y = \frac{47}{24}$

$$y = 1 \frac{23}{24}$$

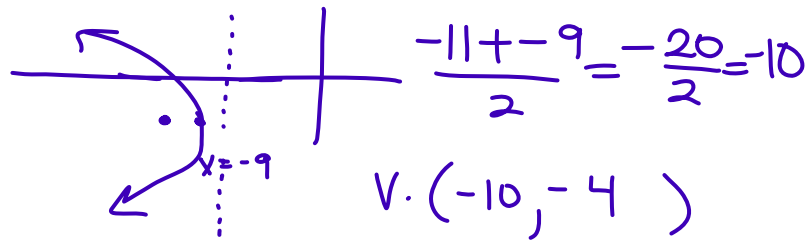
$$p = \frac{1}{24} = \frac{1}{4a}$$

$$a = 6$$

$$y - 2 = 6(x + 3)^2$$

$$y = 6(x + 3)^2 + 2$$

9) Focus: $(-11, -4)$, Directrix: $x = -9$



$$\frac{-11 + -9}{2} = \frac{-20}{2} = -10$$

$$V. (-10, -4)$$

$$p = \frac{1}{4a}$$

$$-1 = \frac{1}{4a}$$

$$a = -\frac{1}{4}$$

$$x + 10 = -\frac{1}{4}(y + 4)^2$$