Pre Calculus Name_____

Ellipses Notes

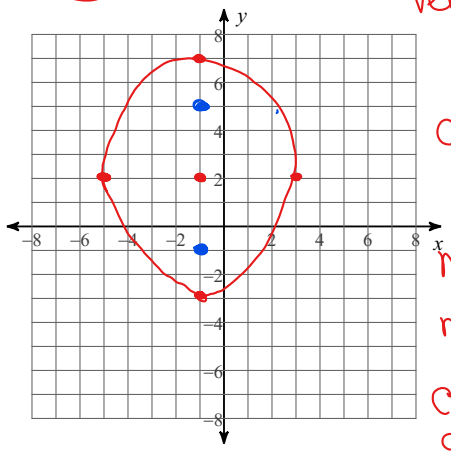
Date_____ Period____

Identify the center, vertices, co-vertices, foci, length of the major axis, and length of the minor axis of each. Then sketch the graph.

1) $\frac{(x+1)^2}{16} + \frac{(y-2)^2}{25} = 1$

$b^2 = 16$, $b = 4$ ← $a^2 = 25$, $a = 5$

C $(-1, 2)$



vertices $(-1, 7)$ $(-1, -3)$

co-vertices $(3, 2)$ $(-5, 2)$

Foci $(-1, 5)$ $(-1, -1)$

Major 10

minor 8

$c^2 = a^2 - b^2$

$c^2 = 25 - 16$

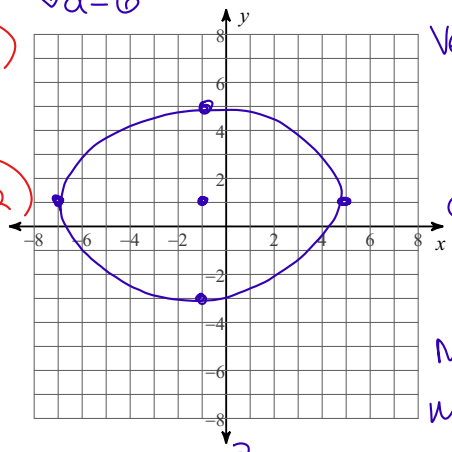
$c = 9$

$c = 3$ → focal length

2) $\frac{(x+1)^2}{36} + \frac{(y-1)^2}{16} = 1$

$a = 6$ ↓ $b = 4$

C $(-1, 1)$



vertices $(5, 1)$ $(-7, 1)$

co-vertices $(-1, 5)$ $(-1, -3)$

Major 12

minor 8

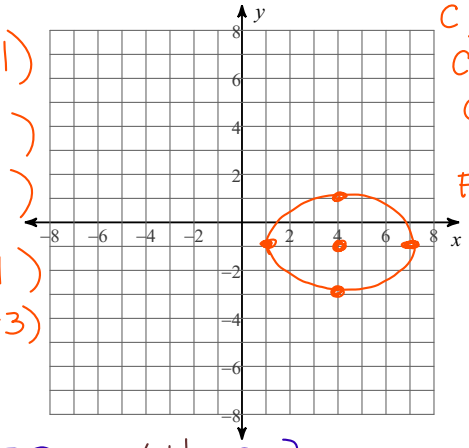
$c^2 = 36 - 16$

$c^2 = 20$

$c = \sqrt{20} = 2\sqrt{5}$

Foci $(-1 \pm 2\sqrt{5}, 1)$

3) $4x^2 + 9y^2 - 32x + 18y + 37 = 0$



C (4, -1)

V (7, -1) (1, -1)

CV (4, 1) (4, -3)

Major: 6

Minor: 4

$c^2 = 9 - 4$

$c^2 = 5$

$c = \sqrt{5}$

Foci $(4 \pm \sqrt{5}, -1)$

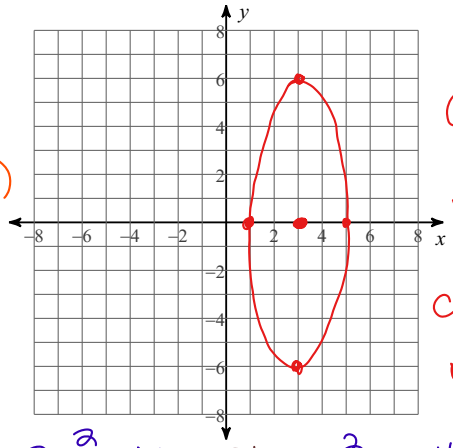
$4x^2 - 32x + 64 \quad 9y^2 + 18y + 9 = -37 + 64 + 9$

$4(x^2 - 8x + 16) + 9(y^2 + 2y + 1) = 36$

$\frac{4(x-4)^2}{36} + \frac{9(y+1)^2}{36} = \frac{36}{36}$

$$\frac{(x-4)^2}{9} + \frac{(y+1)^2}{4} = 1$$

4) $9x^2 + y^2 - 54x + 45 = 0$



C (3, 0)

V (3, 6) (3, -6)

CV (1, 0) (5, 0)

major 12

minor 4

$9x^2 - 54x + 81 \quad + y^2 = -45 + 81$

$9(x^2 - 6x + 9) + y^2$

$\frac{9(x-3)^2}{36} + \frac{y^2}{36} = \frac{36}{36}$

$$\frac{(x-3)^2}{4} + \frac{y^2}{36} = 1$$

$c^2 = 36 - 4$

$c^2 = 32$

$c = \sqrt{32}$

$c = 4\sqrt{2}$

Foci $(3, \pm 4\sqrt{2})$

Use the information provided to write the standard form equation of each ellipse.

5) Vertices: $(-3, 3)$, $(-3, -19)$
Co-vertices: $(7, -8)$, $(-13, -8)$

center $(-3, -8)$

$a = 11$ (y)

$b = 10$ (x)

$$\frac{(x+3)^2}{100} + \frac{(y+8)^2}{121} = 1$$

6) Vertices: $(-10, 16)$, $(-10, 2)$
Foci: $(-10, 9 + \sqrt{13})$, $(-10, 9 - \sqrt{13})$

C $(-10, 9)$

X (semi-minor) $= 6$

Y (semi-major) $= 7$

$c^2 = a^2 - b^2$

$(\sqrt{13})^2 = 49 - b^2$

$13 = 49 - 36$

$$\frac{(x+10)^2}{36} + \frac{(y-9)^2}{49} = 1$$

7) Foci: $(-7 + 2\sqrt{6}, -7)$, $(-7 - 2\sqrt{6}, -7)$
Endpoints of minor axis: $(-7, -2)$, $(-7, -12)$

(co-vertices)

$b = 5$

center $(-7, -7)$

$(c)^2 (2\sqrt{6})^2$

$c^2 = 4 \cdot 6 = 24$

$b^2 = 25$

$c^2 = a^2 - b^2$

$24 = a^2 - 25$

$a^2 = 49$

$$\frac{(x+7)^2}{49} + \frac{(y+7)^2}{25} = 1$$

8) Center: $(-9, 3)$
Vertex: $(-19, 3)$
$c^2 = 84$

$$\frac{(x+9)^2}{100} + \frac{(y-3)^2}{16} = 1$$

$c^2 = a^2 - b^2$

$84 = 100 - b^2$

$b^2 = 16$

9) Foci: $\left(\frac{17}{2}, -4\right), \left(-\frac{3}{2}, -4\right)$

Co-vertices: $\left(\frac{7}{2}, -4+\sqrt{55}\right), \left(\frac{7}{2}, -4-\sqrt{55}\right)$

center: $\left(\frac{7}{2}, -4\right)$

$c = 5 \rightarrow c^2 = 25$

$b = \sqrt{55} \rightarrow b^2 = 55$

$25 = a^2 - 55$

$a^2 = 80$

$$\frac{\left(x-\frac{7}{2}\right)^2}{80} + \frac{(y+4)^2}{55} = 1$$

10) Foci: $(-10, 2+3\sqrt{7}), (-10, 2-3\sqrt{7})$

Point on the ellipse: $(-7, 2+8\sqrt{2})$

center. $(-10, 2)$

$$\frac{(x+10)^2}{b^2} + \frac{(y-2)^2}{a^2} = 1$$

$c\ 3\sqrt{7} \rightarrow c^2 = 9 \cdot 7 = 63$

$c^2 = a^2 - b^2 \rightarrow 63 = a^2 - b^2$

$a^2 = 63 + b^2$

$$\frac{(-7+10)^2}{b^2} + \frac{(2+8\sqrt{2}-2)^2}{63+b^2} = 1$$

$$\frac{(63+b^2)}{(63+b^2)}\frac{9}{b^2} + \frac{128(b^2)}{63+b^2(b^2)} \mid \frac{(b^2)(63+b^2)}{(b^2)(63+b^2)}$$

$9(63+b^2) + 128b^2 = b^2(63+b^2)$

$567 + 9b^2 + 128b^2 = 63b^2 + b^4$

$0 = b^4 - 74b^2 - 567$

$0 = (b^2 - 81)(b^2 + 7)$

$b^2 - 81 = 0 \rightarrow b^2 = 81$

$63 - 9 = 54$

$128 + -54 = 74$

$a^2 = 63 + 81$

$= 144$

$$\frac{(x+10)^2}{81} + \frac{(y-2)^2}{144} = 1$$

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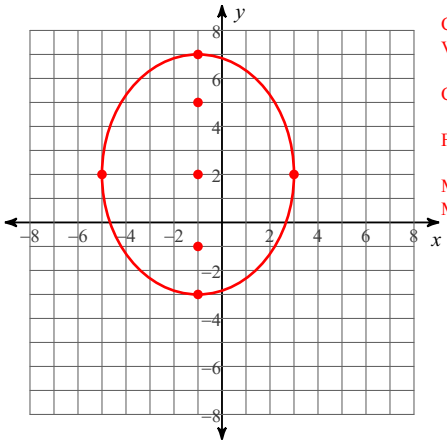
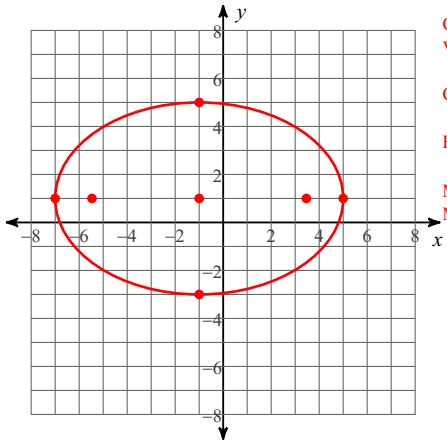
1) $\frac{(x+1)^2}{16} + \frac{(y-2)^2}{25} = 1$



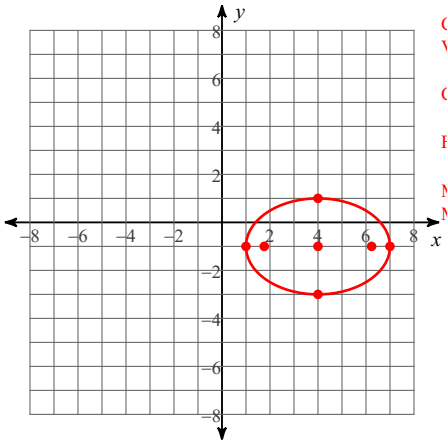
Center: $(-1, 2)$
Vertices: $(-1, 7)$
$(-1, -3)$
Co-vertices: $(3, 2)$
$(-5, 2)$
Foci: $(-1, 5)$
$(-1, -1)$
Major Axis: 10 units
Minor Axis: 8 units

2) $\frac{(x+1)^2}{36} + \frac{(y-1)^2}{16} = 1$



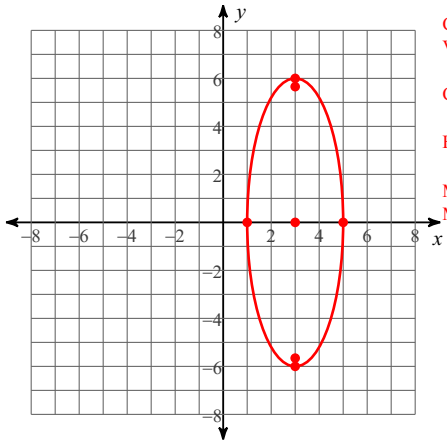
Center: $(-1, 1)$
Vertices: $(5, 1)$
$(-7, 1)$
Co-vertices: $(-1, 5)$
$(-1, -3)$
Foci: $(-1 + 2\sqrt{5}, 1)$
$(-1 - 2\sqrt{5}, 1)$
Major Axis: 12 units
Minor Axis: 8 units

3) $4x^2 + 9y^2 - 32x + 18y + 37 = 0$



Center: $(4, -1)$
Vertices: $(7, -1)$
$(1, -1)$
Co-vertices: $(4, 1)$
$(4, -3)$
Foci: $(4 + \sqrt{5}, -1)$
$(4 - \sqrt{5}, -1)$
Major Axis: 6 units
Minor Axis: 4 units

4) $9x^2 + y^2 - 54x + 45 = 0$



Center: $(3, 0)$
Vertices: $(3, 6)$
$(3, -6)$
Co-vertices: $(5, 0)$
$(1, 0)$
Foci: $(3, 4\sqrt{2})$
$(3, -4\sqrt{2})$
Major Axis: 12 units
Minor Axis: 4 units

Use the information provided to write the standard form equation of each ellipse.

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8) Center: $(-9, 3)$
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$c^2 = 84$

$$\frac{(x+9)^2}{100} + \frac{(y-3)^2}{16} = 1$$

9) Foci: $\left(\frac{17}{2}, -4\right), \left(-\frac{3}{2}, -4\right)$

Co-vertices: $\left(\frac{7}{2}, -4 + \sqrt{55}\right), \left(\frac{7}{2}, -4 - \sqrt{55}\right)$

$$\frac{\left(x - \frac{7}{2}\right)^2}{80} + \frac{(y+4)^2}{55} = 1$$

10) Foci: $\left(-10, 2 + 3\sqrt{7}\right), \left(-10, 2 - 3\sqrt{7}\right)$

Point on the ellipse: $\left(-7, 2 + 8\sqrt{2}\right)$

$$\frac{(x+10)^2}{81} + \frac{(y-2)^2}{144} = 1$$