Probability Classwork *[288 marks]*

1a. *[6 marks]*

Markscheme

(i) number of ways of getting $X = 6$ is 5 ***A1***

$\mathrm{P}(X = 6) = \frac{5}{36}$ ***A1*** ***N2***

(ii) number of ways of getting $X > 6$ is 21 ***A1***

$\mathrm{P}(X > 6) = \frac{21}{36}\left(= \frac{7}{12}\right)$ ***A1*** ***N2***

(iii) $\mathrm{P}(X = 7 | X > 6) = \frac{6}{21}\left(= \frac{2}{7}\right)$ ***A2*** ***N2***

[6 marks]

1b. *[8 marks]*

Markscheme

attempt to find $\mathrm{P}(X < 6)$ ***M1***

e.g. $1 - \frac{5}{36} - \frac{21}{36}$

$\mathrm{P}(X < 6) = \frac{10}{36}$ ***A1***

fair game if $\mathrm{E}(W) = 0$ (may be seen anywhere) ***R1***

attempt to substitute into $\mathrm{E}(X)$ formula ***M1***

e.g. $3\left(\frac{5}{36}\right) + 1\left(\frac{21}{36}\right) - k\left(\frac{10}{36}\right)$

correct substitution into $\mathrm{E}(W) = 0$ ***A1***

e.g. $3\left(\frac{5}{36}\right) + 1\left(\frac{21}{36}\right) - k\left(\frac{10}{36}\right) = 0$

work towards solving ***M1***

e.g. $15 + 21 - 10k = 0$

$36 = 10k$ ***A1***

$k = \frac{36}{10}(= 3.6)$ ***A1*** ***N4***

[8 marks]

2a. *[4 marks]*

Markscheme

(i) correct calculation ***(A1)***

e.g. $\frac{9}{20}+\frac{5}{20}-\frac{2}{20}$, $\frac{4+2+3+3}{20}$

$\mathrm{P}(\text{male or tennis})=\frac{12}{20}$ ***A1*** ***N2***

(ii) correct calculation ***(A1)***

e.g. $\frac{6}{20}\div\frac{11}{20}$, $\frac{3+3}{11}$

$\mathrm{P}(\text{not football}|\text{female})=\frac{6}{11}$ ***A1*** ***N2***

[4 marks]

2b. *[3 marks]*

Markscheme

$\mathrm{P}(\text{first not football})=\frac{11}{20}$, $\mathrm{P}(\text{second not football})=\frac{10}{19}$ ***A1***

$\mathrm{P}(\text{neither football})=\frac{11}{20}\times\frac{10}{19}$ ***A1***

$\mathrm{P}(\text{neither football})=\frac{110}{380}$ ***A1*** ***N1***

[3 marks]

3a. *[1 mark]*

Markscheme

$\mathrm{P}(A\cap B)=\mathrm{P}(A)\times\mathrm{P}(B)(=0.6x)$ ***A1*** ***N1***

[1 mark]

3b. *[4 marks]*

Markscheme

(i) evidence of using $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A)\mathrm{P}(B)$ ***(M1)***

correct substitution ***A1***

e.g. $0.8 = 0.6 + x - 0.6x$, $0.2 = 0.4x$

$x = 0.5$ ***A1*** ***N2***

(ii) $\mathrm{P}(A \cap B) = 0.3$ ***A1*** ***N1***

[4 marks]

3c. *[1 mark]*

Markscheme

valid reason, with reference to $\mathrm{P}(A \cap B)$ ***R1*** ***N1***

e.g. $\mathrm{P}(A \cap B) \neq 0$

[1 mark]

4a. *[1 mark]*

Markscheme

$\mathrm{P}(A) = \frac{1}{11}$ ***A1*** ***N1***

[1 mark]

4b. *[2 marks]*

Markscheme

$\mathrm{P}(B|A) = \frac{2}{10}$ ***A2*** ***N2***

[2 marks]

4c. *[3 marks]*

Markscheme

recognising that $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B|A)$ ***(M1)***

correct values ***(A1)***

e.g. $\mathrm{P}(A \cap B) = \frac{1}{11} \times \frac{2}{10}$

$\mathrm{P}(A \cap B) = \frac{2}{110}$ ***A1*** ***N3***

[3 marks]

5a. *[3 marks]*

Markscheme

(i) evidence of substituting into $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ ***(M1)***

e.g. $75 + 55 - 100$, Venn diagram

30 ***A1*** ***N2***

(ii) 45 ***A1*** ***N1***

[3 marks]

5b. *[4 marks]*

Markscheme

(i) **METHOD 1**

evidence of using complement, Venn diagram ***(M1)***

e.g. $1 - p$, $100 - 30$

$\frac{70}{100} \left(= \frac{7}{10}\right)$ ***A1*** ***N2***

METHOD 2

attempt to find P(only one sport) , Venn diagram ***(M1)***

e.g. $\frac{25}{100} + \frac{45}{100}$

$\frac{70}{100} \left(= \frac{7}{10}\right)$ ***A1*** ***N2***

(ii) $\frac{45}{70} \left(= \frac{9}{14}\right)$ ***A2*** ***N2***

[4 marks]

5c. *[2 marks]*

Markscheme

valid reason in words or symbols ***(R1)***

e.g. $\mathrm{P}(A \cap B) = 0$ if mutually exclusive, $\mathrm{P}(A \cap B) \neq 0$ if not mutually exclusive

correct statement in words or symbols ***A1*** ***N2***

e.g. $\mathrm{P}(A \cap B) = 0.3$, $\mathrm{P}(A \cup B) \neq \mathrm{P}(A) + \mathrm{P}(B)$, $\mathrm{P}(A) + \mathrm{P}(B) > 1$, some students play both sports, sets intersect

[2 marks]

5d. *[3 marks]*

Markscheme

valid reason for independence ***(R1)***

e.g. $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$, $\mathrm{P}(B|A) = \mathrm{P}(B)$

correct substitution ***A1A1*** ***N3***

e.g. $\frac{30}{100} \neq \frac{75}{100} \times \frac{55}{100}$, $\frac{30}{55} \neq \frac{75}{100}$

[3 marks]

6a. *[1 mark]*

Markscheme

$p = \frac{4}{5}$ ***A1*** ***N1***

[1 mark]

6b. *[3 marks]*

Markscheme

multiplying along the branches ***(M1)***

e.g. $\frac{1}{5} \times \frac{1}{4}$, $\frac{12}{40}$

adding products of probabilities of two mutually exclusive paths ***(M1)***

e.g. $\frac{1}{5} \times \frac{1}{4} + \frac{4}{5} \times \frac{3}{8}$, $\frac{1}{20} + \frac{12}{40}$

$\mathrm{P}(B) = \frac{14}{40} \left(= \frac{7}{20}\right)$ ***A1*** ***N2***

[3 marks]

6c. *[3 marks]*

Markscheme

appropriate approach which must include A' (may be seen on diagram) ***(M1)***

e.g. $\frac{\mathrm{P}(A' \cap B)}{\mathrm{P}(B)}$ (do not accept $\frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$)

$\mathrm{P}(A'|B) = \frac{\frac{4}{5} \times \frac{3}{8}}{\frac{7}{20}}$ ***(A1)***

$\mathrm{P}(A'|B) = \frac{12}{14} \left(= \frac{6}{7}\right)$ ***A1*** ***N2***

[3 marks]

7a. *[5 marks]*

Markscheme

(i) $s = 1$ ***A1*** ***N1***

(ii) evidence of appropriate approach ***(M1)***

e.g. $21 - 16$, $12 + 8 - q = 15$

$q = 5$ ***A1*** ***N2***

(iii) $p = 7$, $r = 3$ ***A1A1*** ***N2***

[5 marks]

7b. *[4 marks]*

Markscheme

(i) $\mathrm{P}(\text{art}|\text{music}) = \frac{5}{8}$ ***A2*** ***N2***

(ii) **METHOD 1**

$\mathrm{P}(\text{art}) = \frac{12}{16}\ \left(= \frac{3}{4}\right)$ ***A1***

evidence of correct reasoning ***R1***

e.g. $\frac{3}{4} \neq \frac{5}{8}$

the events are not independent ***AG*** ***N0***

METHOD 2

$\mathrm{P}(\text{art}) \times \mathrm{P}(\text{music}) = \frac{96}{256}\ \left(= \frac{3}{8}\right)$ ***A1***

evidence of correct reasoning ***R1***

e.g. $\frac{12}{16} \times \frac{8}{16} \neq \frac{5}{16}$

the events are not independent ***AG*** ***N0***

[4 marks]

7c. *[4 marks]*

Markscheme

$\mathrm{P}(\text{first takes only music}) = \frac{3}{16}$ (seen anywhere) ***A1***

$\mathrm{P}(\text{second takes only art}) = \frac{7}{15}$ (seen anywhere) ***A1***

evidence of valid approach ***(M1)***

e.g. $\frac{3}{16} \times \frac{7}{15}$

$\mathrm{P}(\text{music and art}) = \frac{21}{240}\ \left(= \frac{7}{80}\right)$ ***A1*** ***N2***

[4 marks]

8a. *[4 marks]*

Markscheme

(i) $n = 0.1$ ***A1*** ***N1***

(ii) $m = 0.2$, $p = 0.3$, $q = 0.4$ ***A1A1A1*** ***N3***

[4 marks]

8b. *[2 marks]*

Markscheme

appropriate approach

e.g. $\mathrm{P}(B') = 1 - P(B)$, $m + q$, $1 - (n + p)$ ***(M1)***

$\mathrm{P}(B') = 0.6$ ***A1*** ***N2***

[2 marks]

9a. *[4 marks]*

Markscheme

METHOD 1

evidence of discriminant ***(M1)***

e.g. $b^2 - 4ac$, discriminant = 0

correct substitution into discriminant ***A1***

e.g. $k^2 - 4 \times \frac{1}{2} \times 8$, $k^2 - 16 = 0$

$k = \pm 4$ ***A1A1*** ***N3***

METHOD 2

recognizing that equal roots means perfect square ***(R1)***

e.g. attempt to complete the square, $\frac{1}{2}(x^2 + 2kx + 16)$

correct working

e.g. $\frac{1}{2}(x + k)^2$, $\frac{1}{2}k^2 = 8$ ***A1***

$k = \pm 4$ ***A1A1*** ***N3***

[4 marks]

9b. *[4 marks]*

Markscheme

evidence of appropriate approach ***(M1)***

e.g. $b^2 - 4ac < 0$

correct working for k ***A1***

e.g. $-4 < k < 4$, $k^2 < 16$, list all correct values of k

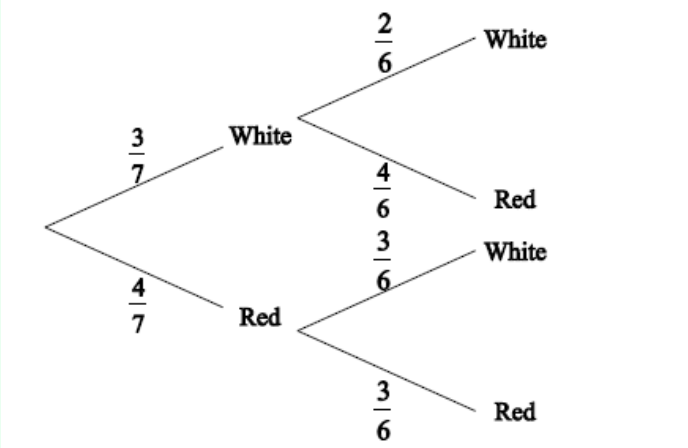
$p = \frac{7}{11}$ ***A2*** ***N3***

[4 marks]

10a. *[5 marks]*

Markscheme

(i)



$\frac{4}{6}, \frac{3}{6}$ and $\frac{3}{6}\left(\frac{2}{3}, \frac{1}{2}\text{and}\frac{1}{2}\right)$ ***A1A1A1*** ***N3***

(ii) multiplying along the correct branches (may be seen on diagram) ***(A1)***

e.g. $\frac{3}{7} \times \frac{2}{6}$

$\frac{6}{42}\left(= \frac{1}{7}\right)$ ***A1*** ***N2***

[5 marks]

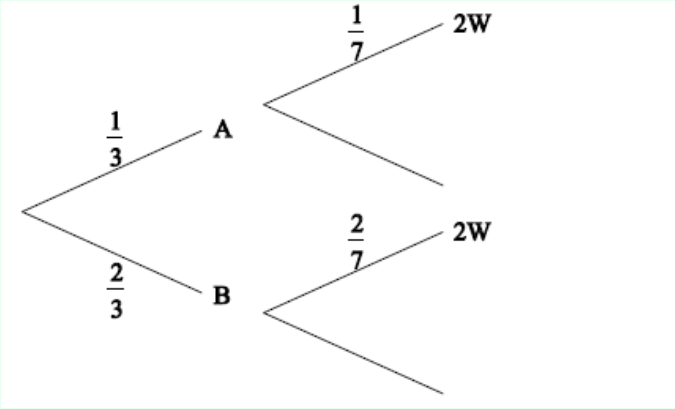
10b. *[5 marks]*

Markscheme

$\mathrm{P}(\mathrm{bagA}) = \frac{2}{6}\left(= \frac{1}{3}\right)$, $\mathrm{P}(\mathrm{bagB}) = \frac{4}{6}\left(= \frac{2}{3}\right)$ (seen anywhere) ***(A1)(A1)***

appropriate approach ***(M1)***

e.g. $\mathrm{P}(WW \cap A) + \mathrm{P}(WW \cap B)$



correct calculation ***A1***

e.g. $\frac{1}{3} \times \frac{1}{7} + \frac{2}{3} \times \frac{2}{7}$, $\frac{2}{42} + \frac{8}{42}$

$\mathrm{P}(2W) = \frac{60}{252}\left(= \frac{5}{21}\right)$ ***A1*** ***N3***

[5 marks]

10c. *[4 marks]*

Markscheme

recognizing conditional probability ***(M1)***

e.g. $\frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$, $\mathrm{P}(A|WW) = \frac{\mathrm{P}(WW \cap A)}{\mathrm{P}(WW)}$

correct numerator ***(A1)***

e.g. $\mathrm{P}(A \cap WW) = \frac{6}{42} \times \frac{2}{6}, \frac{1}{21}$

correct denominator ***(A1)***

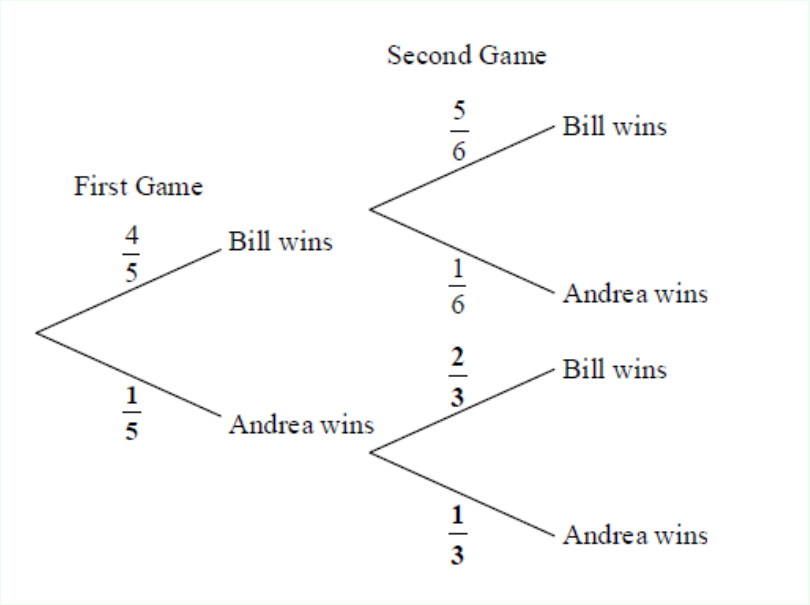
e.g. $\frac{6}{252}, \frac{5}{21}$

probability $\frac{84}{420}\left(= \frac{1}{5}\right)$ ***A1*** ***N3***

[4 marks]

11a. *[3 marks]*

Markscheme



A1A1A1 ***N3***

Note: Award ***A1*** for each correct **bold** probability.

[3 marks]

11b. *[2 marks]*

Markscheme

multiplying along the branches (may be seen on diagram) ***(M1)***

eg $\frac{4}{5} \times \frac{1}{6}$

$\frac{4}{30}\left(\frac{2}{15}\right)$ ***A1*** ***N2***

[2 marks]

11c. *[4 marks]*

Markscheme

METHOD 1

multiplying along the branches (may be seen on diagram) ***(M1)***

eg $\frac{4}{5} \times \frac{5}{6}, \frac{4}{5} \times \frac{1}{6}, \frac{1}{5} \times \frac{2}{3}$

adding their probabilities of three mutually exclusive paths ***(M1)***

eg $\frac{4}{5} \times \frac{5}{6} + \frac{4}{5} \times \frac{1}{6} + \frac{1}{5} \times \frac{2}{3}, \frac{4}{5} + \frac{1}{5} \times \frac{2}{3}$

correct simplification ***(A1)***

eg $\frac{20}{30} + \frac{4}{30} + \frac{2}{15}, \frac{2}{3} + \frac{2}{15} + \frac{2}{15}$

$\frac{28}{30} \left(= \frac{14}{15}\right)$ ***A1*** ***N3***

METHOD 2

recognizing "Bill wins at least one" is complement of "Andrea wins 2" ***(R1)***

eg finding P (Andrea wins 2)

P (Andrea wins both) $= \frac{1}{5} \times \frac{1}{3}$ ***(A1)***

evidence of complement ***(M1)***

eg $1 - p, 1 - \frac{1}{15}$

$\frac{14}{15}$ ***A1*** ***N3***

[4 marks]

11d. *[5 marks]*

Markscheme

P (*B* wins both) $\frac{4}{5} \times \frac{5}{6} \left(= \frac{2}{3}\right)$ ***A1***

evidence of recognizing conditional probability ***(R1)***

eg $\mathrm{P}(A \mid B)$, P (Bill wins both | Bill wins at least one), tree diagram

correct substitution ***(A2)***

eg $\frac{\frac{4}{5} \times \frac{5}{6}}{\frac{14}{15}}$

$\frac{20}{28} \left(= \frac{5}{7}\right)$ ***A1*** ***N3***

[5 marks]

12a. *[4 marks]*

Markscheme

appropriate approach ***(M1)***

e.g. tree diagram or a table

$\mathrm{P}(\text{win}) = \mathrm{P}(H \cap W) + \mathrm{P}(A \cap W)$ ***(M1)***

$= (0.65)(0.83) + (0.35)(0.26)$ ***A1***

$= 0.6305$ (or 0.631) ***A1*** ***N2***

[4 marks]

12b. *[4 marks]*

Markscheme

evidence of using complement ***(M1)***

e.g. $1 - p$, 0.3695

choosing a formula for conditional probability ***(M1)***

e.g. $\mathrm{P}(H|W') = \frac{\mathrm{P}(W' \cap H)}{\mathrm{P}(W')}$

correct substitution

e.g. $\frac{(0.65)(0.17)}{0.3695}$ $\left(= \frac{0.1105}{0.3695}\right)$ ***A1***

P(home) = 0.299 ***A1*** ***N3***

[4 marks]

13. *[7 marks]*

Markscheme

METHOD 1

for independence $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ ***(R1)***

expression for $\mathrm{P}(A \cap B)$, indicating $\mathrm{P}(B) = 2\mathrm{P}(A)$ ***(A1)***

e.g. $\mathrm{P}(A) \times 2\mathrm{P}(A)$, $x \times 2x$

substituting into $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***(M1)***

correct substitution ***A1***

e.g. $0.52 = x + 2x - 2x^2$, $0.52 = \mathrm{P}(A) + 2\mathrm{P}(A) - 2\mathrm{P}(A)\mathrm{P}(A)$

correct solutions to the equation ***(A2)***

e.g. $0.2, 1.3$ (accept the single answer 0.2)

$\mathrm{P}(B) = 0.4$ ***A1*** ***N6***

[7 marks]

METHOD 2

for independence $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ ***(R1)***

expression for $\mathrm{P}(A \cap B)$, indicating $\mathrm{P}(A) = \frac{1}{2}\mathrm{P}(B)$ ***(A1)***

e.g. $\mathrm{P}(B) \times \frac{1}{2}\mathrm{P}(B)$, $x \times \frac{1}{2}x$

substituting into $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***(M1)***

correct substitution ***A1***

e.g. $0.52 = 0.5x + x - 0.5x^2$, $0.52 = 0.5\mathrm{P}(B) + \mathrm{P}(B) - 0.5\mathrm{P}(B)\mathrm{P}(B)$

correct solutions to the equation ***(A2)***

e.g. 0.4, 2.6 (accept the single answer 0.4)

$\mathrm{P}(B) = 0.4$ (accept $x = 0.4$ if x set up as $\mathrm{P}(B)$) ***A1*** ***N6***

[7 marks]

14a. *[3 marks]*

Markscheme

36 outcomes (seen anywhere, even in denominator) ***(A1)***

valid approach of listing ways to get sum of 5, showing at least two pairs ***(M1)***

e.g. (1, 4)(2, 3), (1, 4)(4, 1), (1, 4)(4, 1), (2, 3)(3, 2) , lattice diagram

$\mathrm{P}(\text{prize}) = \frac{4}{36}\ \left(= \frac{1}{9}\right)$ ***A1*** ***N3***

[3 marks]

14b. *[2 marks]*

Markscheme

recognizing binomial probability ***(M1)***

e.g. $\mathrm{B}\left(8, \frac{1}{9}\right)$, binomial pdf, $\binom{8}{3}\left(\frac{1}{9}\right)^3\left(\frac{8}{9}\right)^5$

$\mathrm{P}(3 \text{ prizes}) = 0.0426$ ***A1*** ***N2***

[2 marks]

15a. *[3 marks]*

Markscheme

three correct pairs ***A1A1A1*** ***N3***

e.g. (2, 4), (3, 3), (4, 2) , *R*2 *G*4, *R*3 *G*3, *R*4 *G*2

[3 marks]

15b. *[3 marks]*

Markscheme

$p = \frac{1}{16}$, $q = \frac{2}{16}$, $r = \frac{2}{16}$ ***A1A1A1*** ***N3***

[3 marks]

15c. *[6 marks]*

Markscheme

let X be the number of times the sum of the dice is 5

evidence of valid approach ***(M1)***

e.g. $X \sim \mathrm{B}(n, p)$, tree diagram, 5 sets of outcomes produce a win

one correct parameter ***(A1)***

e.g. $n = 4$, $p = 0.25$, $q = 0.75$

Fred wins prize is $\mathrm{P}(X \geq 3)$ ***(A1)***

appropriate approach to find probability ***M1***

e.g. complement, summing probabilities, using a CDF function

correct substitution ***(A1)***

e.g. $1 - 0.949\ldots$, $1 - \frac{243}{256}$, $0.046875 + 0.00390625$, $\frac{12}{256} + \frac{1}{256}$

$\text{probability of winning} = 0.0508\ \left(\frac{13}{256}\right)$ ***A1*** ***N3***

[6 marks]

16a. *[2 marks]*

Markscheme

valid approach ***(M1)***

e.g. Venn diagram with intersection, union formula,

$\mathrm{P}(S \cap F) = 0.75 + 0.40 - 1$

15 (accept 15%) ***A1*** ***N2***

[2 marks]

16b. *[2 marks]*

Markscheme

valid approach involving subtraction ***(M1)***

e.g. Venn diagram, $75 - 15$

60 (accept 60%) ***A1*** ***N2***

[2 marks]

16c. *[5 marks]*

Markscheme

(i) valid approach ***(M1)***

e.g. tree diagram, multiplying probabilities, $\mathrm{P}(S|G) \times \mathrm{P}(G)$

correct calculation ***(A1)***

e.g. 0.52×0.85

$\mathrm{P}(G \cap S) = 0.442$ (exact) ***A1*** ***N3***

(ii) valid reasoning, with words, symbols or numbers (seen anywhere) ***R1***

e.g. $\mathrm{P}(G) \times \mathrm{P}(S) \neq \mathrm{P}(G \cap S)$, $\mathrm{P}(S|G) \neq \mathrm{P}(S)$, not equal,

one correct value ***A1***

e.g. $\mathrm{P}(G) \times \mathrm{P}(S) = 0.39$, $\mathrm{P}(S|G) = 0.85$, $0.39 \neq 0.442$

G and *S* are not independent ***AG*** ***N0***

[5 marks]

16d. *[6 marks]*

Markscheme

METHOD 1

48% are boys (seen anywhere) ***A1***

e.g. $\mathrm{P}(B) = 0.48$

appropriate approach ***(M1)***

e.g. $\mathrm{P}(\text{girl and Spanish}) + \mathrm{P}(\text{boy and Spanish}) = \mathrm{P}(\text{Spanish})$

correct approach to find P(boy and Spanish) ***(A1)***

e.g. $\mathrm{P}(B \cap S) = \mathrm{P}(S) - \mathrm{P}(G \cap S)$, $\mathrm{P}(B \cap S) = \mathrm{P}(S|B) \times \mathrm{P}(B)$, 0.308

correct substitution ***(A1)***

e.g. $0.442 + 0.48x = 0.75$, $0.48x = 0.308$

correct manipulation ***(A1)***

e.g. $\mathrm{P}(S|B) = \frac{0.308}{0.48}$

$\mathrm{P}(\text{Spanish}|\text{boy}) = 0.641666\ldots, 0.641\bar{6}$

$\mathrm{P}(\text{Spanish}|\text{boy}) = 0.642\ [0.641, 0.642]$ ***A1*** ***N3***

[6 marks]

METHOD 2

48% are boys (seen anywhere) ***A1***

e.g. 0.48 used in tree diagram

appropriate approach ***(M1)***

e.g. tree diagram

correctly labelled branches on tree diagram ***(A1)***

e.g. first branches are boy/girl, second branches are Spanish/not Spanish

correct substitution ***(A1)***

e.g. $0.442 + 0.48x = 0.75$

correct manipulation ***(A1)***

e.g. $0.48x = 0.308$, $\mathrm{P}(S|B) = \frac{0.308}{0.48}$

$\mathrm{P}(\text{Spanish}|\text{boy}) = 0.641666\ldots, 0.641\bar{6}$

$\mathrm{P}(\text{Spanish}|\text{boy}) = 0.642\ [0.641, 0.642]$

[6 marks]

17a. *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $0.5 = 0.2 + \mathrm{P}(B),\ \mathrm{P}(A \cap B) = 0$

$\mathrm{P}(B) = 0.3$ ***A1*** ***N2***

[2 marks]

17b. *[4 marks]*

Markscheme

Correct expression for $\mathrm{P}(A \cap B)$ (seen anywhere) ***A1***

eg $\mathrm{P}(A \cap B) = 0.2\mathrm{P}(B),\ 0.2x$

attempt to substitute into correct formula for $\mathrm{P}(A \cup B)$ ***(M1)***

eg $\mathrm{P}(A \cup B) = 0.2 + \mathrm{P}(B) - \mathrm{P}(A \cap B),\ \mathrm{P}(A \cup B) = 0.2 + x - 0.2x$

correct working ***(A1)***

eg $0.5 = 0.2 + \mathrm{P}(B) - 0.2\mathrm{P}(B),\ 0.8x = 0.3$

$\mathrm{P}(B) = \frac{3}{8}\ (= 0.375,\ \text{exact})$ ***A1*** ***N3***

[4 marks]

18a. *[2 marks]*

Markscheme

$\mathrm{P}(C \cap D) = 2k \times 3k^2$ ***(A1)***

$\mathrm{P}(C \cap D) = 6k^3$ ***A1*** ***N2***

[2 marks]

18b. *[2 marks]*

Markscheme

their correct equation ***(A1)***

eg $2k \times 3k^2 = 0.162,\ 6k^3 = 0.162$

$k = 0.3$ ***A1*** ***N2***

18c. *[3 marks]*

Markscheme

METHOD 1

finding **their** $\mathrm{P}(C' \cap D)$ (seen anywhere) ***(A1)***

eg $0.4 \times 0.27, 0.27 - 0.162, 0.108$

correct substitution into conditional probability formula ***(A1)***

eg $\mathrm{P}(C'|D) = \frac{\mathrm{P}(C' \cap D)}{0.27},\ \frac{(1-2k)(3k^2)}{3k^2}$

$\mathrm{P}(C'|D) = 0.4$ ***A1*** ***N2***

METHOD 2

recognizing $\mathrm{P}(C'|D) = \mathrm{P}(C')$ ***A1***

finding **their** $\mathrm{P}(C') = 1 - \mathrm{P}(C)$ (only if first line seen) ***(A1)***

eg $1 - 2k,\ 1 - 0.6$

$\mathrm{P}(C'|D) = 0.4$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

19a. *[2 marks]*

Markscheme

(i) new mean is $20 + 10 = 30$ ***A1*** ***N1***

(ii) new sd is 6 ***A1*** ***N1***

[2 marks]

19b. *[3 marks]*

Markscheme

(i) new mean is $20 \times 10 = 200$ ***A1*** ***N1***

(ii) **METHOD 1**

variance is 36 ***A1***

new variance is $36 \times 100 = 3600$ ***A1*** ***N2***

METHOD 2

new sd is 60 ***A1***

new variance is $60^2 = 3600$ ***A1*** ***N2***

[3 marks]

20a. *[2 marks]*

Markscheme

evidence of using $\sum f_i = 100$ ***(M1)***

$k = 4$ ***A1*** ***N2***

[2 marks]

20b. *[5 marks]*

Markscheme

(i) evidence of median position ***(M1)***

e.g. 50th item, $26 + 10 + 20 = 56$

$\text{median} = 3$ ***A1*** ***N2***

(ii) $Q_1 = 1$ and $Q_3 = 5$ ***(A1)(A1)***

$\text{interquartile range} = 4$ (accept 1 to 5 or $5 - 1$, etc.) ***A1*** ***N3***

[5 marks]

21a. *[2 marks]*

Markscheme

evidence of valid approach ***(M1)***

e.g. $92 + 52$, line on graph at $x = 31$

$p = 144$ ***A1*** ***N2***

[2 marks]

21b. *[5 marks]*

Markscheme

(i) evidence of valid approach ***(M1)***

e.g. line on graph, 0.8×160 , using complement

$= 29.5$ ***A1*** ***N2***

(ii) $Q_1 = 23$; $Q_3 = 29$ ***(A1)(A1)***

$\text{IQR} = 6$ (accept any notation that suggests an interval) ***A1*** ***N3***

[5 marks]

22a. *[2 marks]*

Markscheme

evidence of median position ***(M1)***

e.g. 50, line on sketch

median is 56 ***A1*** ***N2***

[2 marks]

22b. *[3 marks]*

Markscheme

lower quartile $= 40$, upper quartile $= 70$ ***(A1)(A1)***

interquartile range $= 30$ ***A1*** ***N3***

[3 marks]

23a. *[4 marks]*

Markscheme

(i) median weekly wage $= 400$ (dollars) ***A1*** ***N1***

(ii) lower quartile $= 330$, upper quartile $= 470$ ***(A1)(A1)***

$\text{IQR} = 140$ (dollars) (accept any notation suggesting interval 330 to 470)
A1 ***N3***

Note: Exception to the ***FT*** rule. Award ***A1(FT)*** for an incorrect IQR **only** if both quartiles are explicitly noted.

[4 marks]

23b. *[3 marks]*

Markscheme

(i) 330 (dollars) ***A1*** ***N1***

(ii) 400 (dollars) ***A1*** ***N1***

(iii) 700 (dollars) ***A1*** ***N1***

[3 marks]

23c. *[3 marks]*

Markscheme

valid approach ***(M1)***

e.g. $\text{hours} = \frac{\text{wages}}{\text{rate}}$

correct substitution ***(A1)***

e.g. $\frac{400}{20}$

median hours per week $= 20$ ***A1*** ***N2***

[3 marks]

23d. *[5 marks]*

Markscheme

attempt to find wages for 25 hours per week ***(M1)***

e.g. $\text{wages} = \text{hours} \times \text{rate}$

correct substitution ***(A1)***

e.g. 25×20

finding wages $= 500$ ***(A1)***

65 people (earn 500$\leqslant$) ***(A1)***

15 people (work more than 25 hours) ***A1*** ***N3***

[5 marks]

24a. *[4 marks]*

Markscheme

attempt to find p ***(M1)***

eg $120 - 70$, $50 + 20 + x = 120$

$p = 50$ ***A1*** ***N2***

attempt to find q ***(M1)***

eg $180 - 20$, $200 - 20 - 20$

$q = 160$ ***A1*** ***N2***

[4 marks]

24b. *[3 marks]*

Markscheme

(i) $\frac{70}{200}\ \left(=\frac{7}{20}\right)$ ***A1*** ***N1***

(ii) valid approach ***(M1)***

eg $20+20$, $200-160$

$\frac{40}{200}\ \left(=\frac{1}{5}\right)$ ***A1*** ***N2***

[3 marks]

24c. *[4 marks]*

Markscheme

(i) attempt to find number of girls ***(M1)***

eg 0.4, $\frac{40}{100}\times 200$

80 are not selected ***A1*** ***N2***

(ii) 120 are selected ***(A1)***

$x=20$ ***A1*** ***N2***

[4 marks]

24d. *[4 marks]*

Markscheme

(i) 30 given second chance ***A1*** ***N1***

(ii) 20 took less than 20 minutes ***(A1)***

attempt to find **their** selected total (may be seen in $\%$ calculation) ***(M1)***

eg $120 + 20\ (= 140)$, $120+$ **their** answer from (i)

$70\ (\%)$ ***A1*** ***N3***

[4 marks]

25. *[4 marks]*

Markscheme

(i) valid approach ***(M1)***

eg $\max - \min = \text{range}, c = 40 + 47$

$c = 87$ ***A1*** ***N2***

(ii) valid approach ***(M1)***

eg $Q3 - Q1 = IQR,\ 74 - 22$

$d = 52$ ***A1*** ***N2***

26a. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg between 10th and 11th, $\frac{8+8}{2}$

median $= 38$ ***A1*** ***N2***

[2 marks]

26b. *[3 marks]*

Markscheme

(i) $a = 20$ ***A1*** ***N1***

(ii) valid approach ***(M1)***

eg $Q_3 - Q_1,\ Q_1 + 14,\ b - 30 = 14$

$b = 44$ ***A1*** ***N2***

[3 marks]

26c. *[3 marks]*

Markscheme

valid approach ***(M1)***

*eg*x $40 \times 20,\ \frac{x+745}{20},\ 40 - \frac{745}{20}$

correct working ***(A1)***

eg $800 - 745,\ 20 \times 2.75$

55 (more cans) ***A1*** ***N2***

[3 marks]

26d. *[5 marks]*

Markscheme

(i) most cans in Sam's class $= 50$ ***(A1)***

5 ($) ***A1*** ***N2***

(ii) correct value of 64 or 16 ***A1***

valid approach ***(M1)***

eg $\frac{64}{80},\ 80\%,\ 80 - 64,\ \frac{16}{80}$

20% ***A1*** ***N2***

[5 marks]

26e. *[2 marks]*

Markscheme

(i) 41.4 (exact) ***A1 N1***

(ii) 18.5 ***A1*** ***N1***

[2 marks]

27a. *[3 marks]*

Markscheme

(i) $p = 17$, $q = 11$ ***A1A1*** ***N2***

(ii) $75 \leq T < 85$ ***A1*** ***N1***

[3 marks]

27b. *[2 marks]*

Markscheme

evidence of valid approach ***(M1)***

e.g. adding frequencies

$\frac{76}{93} = 0.8172043\ldots$

$\mathrm{P}(T < 95) = \frac{76}{93} = 0.817$ ***A1*** ***N2***

[2 marks]

27c. *[2 marks]*

Markscheme

(i) 10 ***A1*** ***N1***

(ii) 50 ***A1*** ***N1***

[2 marks]

27d. *[4 marks]*

Markscheme

(i) evidence of approach using mid-interval values (may be seen in part (ii)) ***(M1)***

79.1397849

$\overline{x} = 79.1$ ***A2*** ***N3***

(ii) 16.4386061

$\sigma = 16.4$ ***A1*** ***N1***

[4 marks]

27e. *[2 marks]*

Markscheme

e.g. standardizing, $z = 0.9648\ldots$

0.8326812

$\mathrm{P}(T < 95) = 0.833$ ***A1*** ***N2***

[2 marks]

28a. *[2 marks]*

Markscheme

recognizing that the median is at half the total frequency ***(M1)***

eg $\frac{2000}{2}$

$m = 2500\ (\text{dollars})$ ***A1*** ***N2***

[2 marks]

28b. *[4 marks]*

Markscheme

(i) 500 families have a monthly income less than 2000 ***A1*** ***N1***

(ii) correct cumulative frequency, 1850 ***(A1)***

subtracting **their** cumulative frequency from 2000 ***(M1)***

eg $2000 - 1850$

150 families have a monthly income of more than 4000 dollars ***A1*** ***N2***

Note: If working shown, award ***M1A1A1*** for $128 + 22 = 150$, using the table.

[4 marks]

28c. *[2 marks]*

Markscheme

correct calculation ***(A1)***

eg $2000 - (436 + 64 + 765 + 28 + 122),\ 1850 - 500 - 765$ ***(A1)***

$p = 585$ ***A1*** ***N2***

[2 marks]

28d. *[2 marks]*

Markscheme

(i) correct working ***(A1)***

eg $436 + 765 + 28$

0.6145 (exact) ***A1*** ***N2***

$\frac{1229}{2000},\ 0.615\ [0.614,\ 0.615]$

(ii) correct working/probability for number of families ***(A1)***

eg $122 + 28,\ \frac{150}{2000},\ 0.075$

0.186666

$\frac{28}{150}\ \left(= \frac{14}{75}\right),\ 0.187\ [0.186,\ 0.187]$ ***A1*** ***N2***

[4 marks]

28e. *[2 marks]*

Markscheme

evidence of using correct mid-interval values $(1500, 3000, 4500)$ ***(A1)***

attempt to substitute into $\frac{\sum fx}{\sum f}$ ***(M1)***

eg $\frac{1500\times64+3000\times p+4500\times122}{64+585+122}$

3112.84

$3110\ [3110,\ 3120]\ (\text{dollars})$ ***A1*** ***N2***

[3 marks]

Total [15 marks]

29a. *[2 marks]*

Markscheme

attempt to substitute into formula for mean ***(M1)***

eg $\frac{\Sigma x}{10}$, $\frac{252}{n}$, $\frac{252}{10}$

mean $= 25.2\ (\text{hours})$ ***A1*** ***N2***

[2 marks]

29b. *[2 marks]*

Markscheme

(i) mean $= 30.2\ (\text{hours})$ ***A1 N1***

(ii) $\sigma = 5\ (\text{hours})$ ***A1*** ***N1***

[2 marks]

29c. *[6 marks]*

Markscheme

(i) valid approach ***(M1)***

eg 95%, 5% of 27

correct working ***(A1)***

eg $0.95 \times 27,\ 27 - (5\% \text{ of } 27)$

median $= 25.65\ (\text{exact}), 25.7\ (\text{hours})$ ***A1*** ***N2***

(ii) **METHOD 1**

variance $= (\text{standard deviation})^2$ (seen anywhere) ***(A1)***

valid attempt to find new standard deviation ***(M1)***

eg $\sigma_{new} = 0.95 \times 5,\ 4.75$

variance $= 22.5625\ (\text{exact}),\ 22.6$ ***A1*** ***N2***

METHOD 2

variance $= (\text{standard deviation})^2$ (seen anywhere) ***(A1)***

valid attempt to find new variance ***(M1)***

eg $0.95^2\ ,\ 0.9025 \times \sigma^2$

new variance $= 22.5625\ (\text{exact}),\ 22.6$ ***A1*** ***N2***

[6 marks]

29d. *[6 marks]*

Markscheme

(i) both correct frequencies ***(A1)***

eg 80, 150

subtracting **their** frequencies in either order ***(M1)***

eg $150 - 80,\ 80 - 150$

70 (students) ***A1*** ***N2***

(ii) evidence of a valid approach ***(M1)***

eg 10% of 200, 90%

correct working ***(A1)***

eg $0.90 \times 200,\ 200 - 20$, 180 students

$k = 35$ ***A1*** ***N3***

[6 marks]

30a. *[1 mark]*

Markscheme

$\text{mode} = 10$ ***A1*** ***N1***

[1 mark]

30b. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $x_{\max} - x_{\min}$, interval 2 to 11

$\text{range} = 9$ ***A1*** ***N2***

[2 marks]

30c. *[2 marks]*

Markscheme

7.14666

$\text{mean} = 7.15$ ***A2*** ***N2***

[2 marks]

30d. *[2 marks]*

Markscheme

recognizing that variance is $(\text{sd})^2$ ***(M1)***

eg $\text{var} = \sigma^2,\ 2.90605^2,\ 2.92562^2$

$\sigma^2 = 8.44515$

$\sigma^2 = 8.45$ ***A1*** ***N2***

[2 marks]