AB Calculus Rates of Change and Derivatives Review Name: Key

1. Find the average rate of change of the function $f(x) = 3 + \sin x$ over the interval $[-\pi,\ \pi]$.

   $f(\pi) = 3 + \sin\pi = 3$ $f(-\pi) = 3 + \sin -\pi = 3$ $\frac{3-3}{\pi - -\pi} = \frac{0}{2\pi} = 0$

2. Find the slope of the line tangent to the curve at the given value of x.

   a) $f(x) = -3x^2 + 6x$ at $x = 6$.

   $f'(x) = -6x + 6 \rightarrow -6(6) + 6 = -30$

   b) $f(x) = \frac{-1}{x+6}$ at $x = -4$.

   $f'(x) = \frac{1}{(x+6)^2} \rightarrow \frac{1}{4}$

   c) $f(x) = 4 - 15x$ at $x = 3$.

   $f'(x) = -15$ -15

   d) $f(x) = \begin{cases} 8 + x & x \le 4 \\ -x - 6 & x > 4 \end{cases}$ at $x = 5$.

   $f'(x) = -1$ -1

3. Find the equation of the tangent line to the curve $f(x) = \frac{7}{x} - 2$ at $\left(3,\ \frac{1}{3}\right)$.

   $f'(x) = \frac{-7}{x^2} \rightarrow \frac{-7}{9}$ $y - \frac{1}{3} = -\frac{7}{9}(x-3)$

4. Find the equation of the normal line to the function $y = 4x^2$ at $(3, 36)$.

   $f'(x) = 8x$ $f'(3) = 24$ $y - 36 = -\frac{1}{24}(x-3)$

5. Find the x-value(s) where the graph of the function $f(x) = 6x^2 + 4x - 4$ has horizontal tangents.

   $f'(x) = 12x + 4 = 0$ $x = -1/3$

6. Use the limit definition of derivative to find the derivative of the function $f(x) = x^3 + 7$.

   $f'(x) = 3x^2$

7. Use the alternative definition of derivative to find the derivative of each function at the indicated point.

   a) $f(x) = \sqrt{x}$ at $x = 25$. $f'(x) = \frac{1}{2\sqrt{x}} \rightarrow \frac{1}{10}$

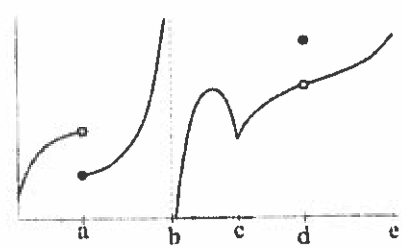
   b) $f(x) = -2x^2 + 10x$ at $x = 10$.

   $f'(x) = -4x + 10 \rightarrow -30$

8. Consider the graph of f given to the right.

   a) On what interval is f continuous?

   $(-\infty, a) \cup [a, b) \cup (b, d) \cup (d, e)$ or (d, ∞)

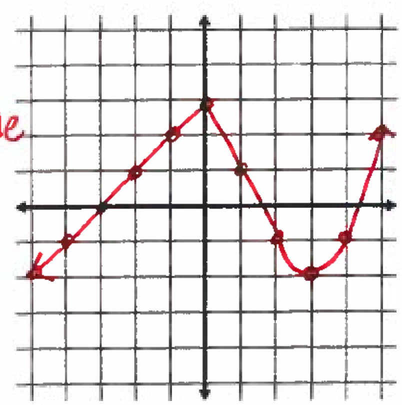


   b) On what interval is f differentiable?

   $(a, b) \cup (b, c) \cup (c, d) \cup (d, e)$

9. Graph the given function, then answer the following questions.

$$f(x) = \begin{cases} 3 + x, & x \le 0 \\ -2x + 3, & 0 < x \le 2 \\ x^2 - 6x + 7, & x > 2 \end{cases}$$



   a) Compare the right-hand and left-hand derivatives at $x = 0$ to prove whether or not the function is differentiable at $x = 0$. Explain your answer.

   $\lim_{x \to 0^-} f'(x) = 1$ $\lim_{x \to 0^+} f'(x) = -2$ Not differentiable

   b) Compare the right-hand and left-hand derivatives at $x = 2$ to prove whether or not the function is differentiable at $x = 2$. Explain your answer.

   $\lim_{x \to 2^-} f'(x) = -2$ $\lim_{x \to 2^+} f'(x) = -2$

   $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2) \rightarrow$ continuous

   Yes and $f'(x) =$

10. For each of the following functions, find the interval for which the function is differentiable.

a) $f(x) = \dfrac{1}{x^2 - 81}$ $\quad \dfrac{1}{(x-9)(x+9)}$ $\quad (-\infty,-9)\cup(-9,9)\cup(9,\infty)$

b) $f(x) = -7x + 5$ $\quad (-\infty,\infty)$

c) $f(x) = \sqrt{16 - x^2}$ $\quad (-4,4)$

11. Graph the derivative of the function below on the grid to the right.