AB Calculus Rates of Change and Tangent Lines Homework Name: Key

1. Find the average rate of change of $y = e^x$ over each interval. Leave your answer in terms of e.

a) $[-2,0]$ $\quad \dfrac{e^0 - e^{-2}}{0 - -2} = \dfrac{1 - \frac{1}{e^2}}{+2}$

$= \dfrac{1}{2} - \dfrac{1}{2e^2}$

b) $[1,3]$ $\quad \dfrac{e^3 - e^1}{3-1} = \dfrac{e^3 - e}{2}$

2. Find the average rate of change of the function $y = \cot x$ over each interval.

$\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ $\quad \dfrac{\cot\frac{3\pi}{4} - \cot\frac{\pi}{4}}{\frac{3\pi}{4} - \frac{\pi}{4}}$

$\dfrac{-1-1}{2\pi/4} = \dfrac{-2}{\pi/2} = \dfrac{-4}{\pi}$

$\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ $\quad \dfrac{\cot\frac{\pi}{2} - \cot\frac{\pi}{6}}{\frac{\pi}{2} - \frac{\pi}{6}}$

$\dfrac{0 - \sqrt{3}}{\frac{\pi}{3}} = \dfrac{-3\sqrt{3}}{\pi}$

3. For each of the following, find the slope of the curve at the indicated point.

a) $y = x^2 + 2x$ at $x = 2$

$y = 2^2 + 2(2) = 8$

$\lim\limits_{h\to 0} \dfrac{(x+h)^2 + 2(x+h) - x^2 - 2x}{h}$

$\lim\limits_{h\to 0} \dfrac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$

$\lim\limits_{h\to 0} \dfrac{2xh + h^2 + 2h}{h} \rightarrow \lim\limits_{h\to 0} 2x + h + 2 = 2x + 2$ @ $x = 2$

$2(2) + 2 = 6$

b) $y = \dfrac{1}{x-1}$ at $x = 2$

$\lim\limits_{h\to 0} \dfrac{\frac{1}{(x+h-1)}\cdot\frac{(x-1)}{(x-1)} - \frac{1}{(x-1)}\cdot\frac{(x+h-1)}{(x+h-1)}}{h} \rightarrow \dfrac{x - 1 - x - h + 1}{h(x-1)(x+h-1)}$

$\lim\limits_{h\to 0} \rightarrow \dfrac{-h}{h(x-1)(x+h-1)} \rightarrow \dfrac{-1}{(x-1)(x+h-1)} = \dfrac{-1}{(x-1)^2} = \dfrac{-1}{(2-1)^2} = \dfrac{-1}{1^2} = -1$

nd the equation of the tangent lin… the normal line to the following functions at the ind… d poi…

a) $f(x) = 3x - 4$ at $x = 1$ $\rightarrow y = -1$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $\quad \frac{3x + 3h - 4 - 3x + 4}{h}$

$\rightarrow \frac{3(x+h) - 4 - 3x + 4}{h} \nearrow \rightarrow \frac{3h}{h} = 3$

$y + 1 = 3(x - 1)$ T

$y + 1 = -\frac{1}{3}(x - 1)$ N

b) $f(x) = \sqrt{x}$ at $x = 4 \rightarrow y = 2$

$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$

$\frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}} \rightarrow \frac{1}{2\sqrt{x}} \rightarrow \frac{1}{2 \cdot \sqrt{4}} = \frac{1}{4}$

$y - 2 = \frac{1}{4}(x - 4)$ T

$y - 2 = -4(x - 4)$ N

5. An object is dropped from the top of a 100-m tower. Its height above the ground after t seconds is given by the function $s(t) = 100 - 4.9t^2$ where $s(t)$ is measured in meters. How fast is the object falling 2 seconds after it has been released?

$s'(t) = \lim_{h \to 0} \frac{100 - 4.9(x+h)^2 - 100 + 4.9x^2}{h}$

$= \frac{100 - 4.9x^2 - 9.8xh - 4.9h^2 - 100 + 4.9x^2}{h}$

x's should be t's oops :)

$= -9.8x - 4.9h = -9.8(2) = -19.6$ m/s

6. The equation for free fall on the surface of Mars is $s = 1.86t^2$ meters with t measured in seconds. Assume a rock is dropped from the top of a 200-m cliff. Find the speed of the rock at t = 1 second.

$s'(t) = \lim_{h \to 0} \frac{1.86(t+h)^2 - 1.86t^2}{h}$

$\frac{1.86t^2 + 3.72th + 1.86h^2 - 1.86t^2}{h}$

$\lim_{h \to 0} \rightarrow 3.72t + 1.86h \rightarrow 3.72(1) = 3.72$ m/s

7. At what point is the tangent line to $f(x) = x^2 - 6x + 1$ horizontal?

$\frac{(x+h)^2 - 6(x+h) + 1 - x^2 + 6x - 1}{h}$ $\hookrightarrow$ slope = 0

$\frac{x^2 + 2xh + h^2 - 6x - 6h + 1 - x^2 + 6x - 1}{h}$

$\frac{2xh - 6h + h^2}{h} \rightarrow 2x - 6 + h \rightarrow 2x - 6 = 0$

$2x = 6$

$x = 3$

$f(3) = 9 - 18 + 1 = -8$ $\quad (3, -8)$