AB Calculus Rates of Change and Tangent Lines Practice Name: Key

1. What is the difference quotient? $\frac{f(a+h)-f(a)}{h}$

2. How do you find the slope of a curve at a point (a.k.a. slope of the tangent line to a curve) at $x = a$?

$$\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

3. What is a normal line?

perpendicular

4. What is the difference between average rate of change and instantaneous rate of change?

Slope between 2 pts → secant line → ; tangent → slope @ 1 pt.

5. Find the average rate of change of each function over the indicated interval.

a) $h(x) = 2 + \sin x$ over $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\frac{h(\frac{\pi}{2})-h(-\frac{\pi}{2})}{\frac{\pi}{2}-(-\frac{\pi}{2})} = \frac{(2+1)-(2+-1)}{\pi} = \frac{3-1}{\pi} = \frac{2}{\pi}$$

b) $f(x) = x^2 - x$ over $[2,5]$

$$\frac{f(5)-f(2)}{5-2} = \frac{(5^2-5)-(2^2-2)}{3} = \frac{20-2}{3} = \frac{18}{3} = 6$$

6. Let $f(x) = x^3$

a) Write and simplify an expression for $f(a+h)$. → Pascal's Δ

$$f(a+h) = (a+h)^3 = a^3 + 3a^2h + 3ah^2 + h^3$$

b) Find the slope of $f(x)$ at $x = a$.

$$\lim_{h \to 0} \frac{f(a+h)-f(a)}{h} \Rightarrow \lim_{h \to 0} \frac{a^3 + 3a^2h + 3ah^2 + h^3 - a^3}{h} \Rightarrow \lim_{h \to 0} \frac{3a^2h + 3ah^2 + h^3}{h}$$

$$\to \lim_{h \to 0} 3a^2 + 3ah + h^2 \to 3a^2 + 3a(0) + 0^2 \to 3a^2$$

c) When does the slope equal 12?

$12 = 3a^2$

$4 = a^2$ $a = \pm 2$, so when $x = \pm 2$

d) Write the equation of the tangent line to the curve at $x = 4$.

$(4, 4^3) \to (4, 64)$

Slope $= 3(4)^2 = 3 \cdot 16 = 48$

$y - 64 = 48(x-4)$

e) Write the equation of the normal line to the curve at $x = 4$.

$y - 64 = -\frac{1}{48}(x-4)$

7. Let $f(x) = \sqrt{x}$

a) Find the average rate of change from $x = 4$ to $x = 9$.

$$\frac{\sqrt{9}-\sqrt{4}}{9-4} = \frac{3-2}{5} = \frac{1}{5}$$

b) Find the instantaneous rate of change at $x = 9$.

$$f(a+h) = \sqrt{a+h}$$

$$\lim_{h\to 0} \frac{\sqrt{a+h}-\sqrt{a}}{h} \cdot \frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}}$$

$$\lim_{h\to 0} \frac{a+h-a}{h(\sqrt{a+h}+\sqrt{a})} \to \frac{h}{h(\sqrt{a+h}+\sqrt{a})} \to \lim_{h\to 0} \frac{1}{\sqrt{a+h}+\sqrt{a}} \to \frac{1}{\sqrt{a+0}+\sqrt{a}} \to \frac{1}{\sqrt{a}+\sqrt{a}} \to \frac{1}{2\sqrt{a}}$$

if $x=9$, then $\frac{1}{2\sqrt{9}} = \frac{1}{2\cdot 3} = \frac{1}{6}$

c) Write the equation of the tangent line at $x = 9$.

$$(9,\sqrt{9}) \Rightarrow (9,3)$$

$$y-3 = \frac{1}{6}(x-9)$$

d) Write the equation of the normal line at $x = 9$.

$$y-3 = -6(x-9)$$

8. An object is dropped from the top of a 150-meter tower. It's height above the ground after t seconds is given by the function $s(t) = 150 - 4.9t^2$. How fast is the object falling 2 seconds after it was dropped?

$$s(a+h) = 150 - 4.9(a+h)^2$$

$$= 150 - 4.9a^2 - 9.8ah - 4.9h^2$$

$$\lim_{h\to 0} \frac{150-4.9a^2-9.8ah-4.9h^2-150+4.9a^2}{h}$$

$$\lim_{h\to 0} \frac{-9.8ah-4.9h^2}{h}$$

$$\lim_{h\to 0} -9.8a-4.9h \to -9.8a = -9.8(2) = -19.6$$

so falling @ 19.6 m/s

implies "–" sign