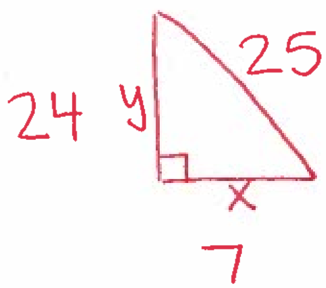
AP Calculus Related Rates Day 1 Homework (calculator) Name: Key

1. The top of a 25-foot ladder, leaning against a vertical wall, is slipping down the wall at a rate of 1 ft/min. How fast is the bottom of the ladder slipping along the ground when the bottom of the ladder is 7 ft. away from the base of the wall?

$\frac{dx}{dt}$?



$x = \sqrt{25^2 - y^2}$

$\frac{dx}{dt} = \frac{1}{2\sqrt{25^2-y^2}}(-2y)\left(\frac{dy}{dt}\right)$

$\frac{dx}{dt} = \frac{1}{2\sqrt{25^2-24^2}}(-2(24))(-1)$

$= \frac{24}{7}$ ft/min

2. A spherical snowball $\left(V = \frac{4}{3}\pi r^3\right)$ is melting at a rate of 4π cubic centimeters per hour. How fast is the diameter changing when the diameter is 20 cm?

$d = 20$

$\frac{dV}{dt} = 4\pi$

$r = \frac{d}{2}$ $V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$

$= \frac{4}{3}\pi\frac{d^3}{8}$

$= \frac{\pi d^3}{6}$

$\frac{dV}{dt} = \frac{1}{2}\pi d^2 \frac{dd}{dt}$ $-4\pi = \frac{1}{2}\pi(20)^2\frac{dd}{dt}$

$\frac{dd}{dt} = \frac{-1}{50}$ cm/hr

3. The radius of a circle is decreasing at a rate of 2.6 ft/sec. Find the rate of change of the circumference at the instant the radius has a length of 8 ft.

$\frac{dr}{dt} = -2.6$

$C = 2\pi r$

$\frac{dC}{dt} = 2\pi\frac{dr}{dt}$

$= 2\pi(-2.6) = -5.2\pi$ ft/sec

4. If $f(x) = 2^x$, find the linearization $L(x)$ of $f(x)$ centered at $x = 3$.

$f(3) = 2^3 = 8$

$f'(x) = 2^x \cdot \ln 2$

$f'(3) = 2^3 \ln 2$

$y - y_1 = m(x - x_1)$

$L(x) - 8 = 8\ln 2(x-3) \rightarrow L(x) = 8\ln 2(x-3) + 8$

5. If $f(x) = \cos x$, find the linearization $L(x)$ of $f(x)$ centered at $x = \frac{\pi}{3}$.

$f\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

$f'(x) = -\sin x$

$f'\left(\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = \frac{-\sqrt{3}}{2}$

$L(x) - \frac{1}{2} = \frac{-\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right)$

$L(x) = \frac{-\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right) + \frac{1}{2}$

6. Evaluate the following limits.

a) $\lim_{x\to 0} \frac{x\sin x}{1-\cos x}$ $\to \frac{0}{0}$ $\lim_{x\to 0} \frac{x\cos x+\sin x}{\sin x} \to \frac{0}{0}$ $\lim_{x\to 0} \frac{-x\sin x+\cos x+\cos x}{\cos x}$

$\to \frac{-x\sin x+2\cos x}{\cos x}$

$\to \frac{0+2}{1} = 2$

b) $\lim_{x\to 0^+} \frac{\ln(x^2+2x)}{\ln x}$ $\frac{\infty}{\infty} \to \lim_{x\to 0^+} \frac{\frac{1}{x^2+2x}(2x+2)}{\frac{1}{x}} \to \frac{2x+2}{x+2} = 1$

c) $\lim_{x\to\infty} x^{\frac{4}{x}}$ ∞^0

$\frac{4}{x}\ln x \to \frac{4\ln x}{x} \to \frac{\infty}{\infty}$ $\lim_{x\to\infty} \frac{\frac{4}{x}}{1} \to \frac{4}{\infty} \to 0 \to e^0 = 1$

7. If $f(x) = 4x^3 + 21x^2 + 36x - 20$, find the following.

a) Intervals on which $f(x)$ is increasing. Justify your answer.

$f'(x) = 12x^2+42x+36$

$= 6(2x^2+7x+6) \to 6(2x+3)(x+2)$

$+ \quad - \quad +$; $-2 \quad -\frac{3}{2}$

$f'(x) > 0$ Inc: $(-\infty,-2) \cup (-\frac{3}{2},\infty)$

$f'(x) < 0$ Dec: $(-2, -\frac{3}{2})$

b) Intervals on which $f(x)$ is concave down. Justify your answer.

$f''(x) = 24x+42$

$= 6(4x+7)$

$- \quad +$; $-7/4$

$f''(x) > 0$ ccu: $(-\frac{7}{4},\infty)$

$f''(x) < 0$ ccd $(-\infty, -7/4)$

c) x-values where $f(x)$ has a relative maximum and/or minimum. Justify your answer.

rel max @ $x = -2$ $f'(x)$ changes $+$ to $-$

rel min @ $x = -3/2$ $f'(x)$ changes $-$ to $+$

8. Determine if $f(x) = \ln(x^2 + 2x + 4)$ satisfies MVT over the interval $[-4, 3]$. If it does, find the c value guaranteed by the theorem. If it does not, explain why. $2^2-4(1)(4) \to 4-16 = - \to$ no zeros, always $+$

$\ln(x^2+2x+4)$ is continuous on $[-4,3]$

$\frac{d}{dx}\ln(x^2+2x+4) = \frac{1}{x^2+2x+4}(2x+2) \to \frac{2x+2}{x^2+2x+4} \to$ diff. over $(-4,3)$

$\ln(9+6+4) = \ln 19$

$\ln(16-8+4) = \ln 12$

$\frac{2x+2}{x^2+2x+4} = \frac{\ln 19 - \ln 12}{7} \to$ graph, find intersection

$x \approx .901$