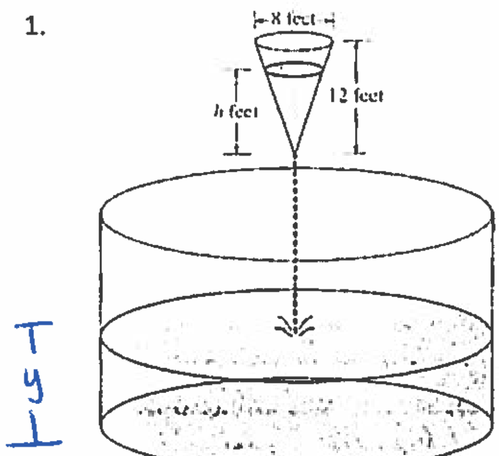
AB Calculus Related Rates Day 3 Homework Name: Key

1.



As shown in the figure to the left, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h - 12)$ feet per minute.
(The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$

$400\pi = \pi r^2$ $\quad r_{tank} = 20$ (not cone)

$\frac{dh}{dt} = h - 12$

4, 12, r, h $\quad \frac{12}{4} = \frac{h}{r}$

a) Write an expression for the volume of water in the conical tank as a function of h. $r = \frac{1}{3}h$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h \Rightarrow \boxed{V = \frac{1}{27}\pi h^3}$

b) At what rate is the volume of water in the conical tank changing when $h = 3$? Indicate units of measure.

$\frac{dV}{dt} = \frac{3}{27}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{9}\pi(3)^2(3-12) \Rightarrow \boxed{-9\pi \text{ ft}^3/\text{min}}$

c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure.

$V_{cylinder} = \pi r^2 y \rightarrow \pi(20)^2 y \rightarrow 400\pi y$

$\frac{dV}{dt} = 400\pi \frac{dy}{dt} \rightarrow 9\pi = 400\pi \frac{dy}{dt} \nearrow \frac{dy}{dt} = \frac{9\pi}{400\pi}$ $\boxed{= \frac{9}{400} \text{ ft/min}}$

2. Gas is escaping from a spherical balloon at the rate of $2\ ft^3/min$. How fast is the radius changing when the radius is 12 ft.

$V_{sphere} = \frac{4}{3}\pi r^3$

$-2 = 4\pi(12)^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\boxed{\frac{-1}{288\pi} = \frac{dr}{dt} \hookrightarrow \text{ft/min}}$

$\frac{dy}{dt} = -2$ $\quad\quad$ $\frac{dx}{dt} = ?$

3. A 15 ft. ladder is sliding down a building at a constant rate of 2 feet/min. How fast is the base of the ladder moving away from the building when the base of the ladder is 9 ft. from the building?

$x^2 + y^2 = 225$ $\quad\quad$ $9^2 + y^2 = 225 \rightarrow y = 12$

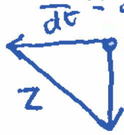
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(9)\frac{dx}{dt} + 2(12)(-2) = 0 \rightarrow 18\frac{dx}{dt} = 48$

$\boxed{\frac{dx}{dt} = \frac{48}{18} \rightarrow \frac{8}{3}\text{ ft/min}}$

4. Two cars start moving from the same point. One travels south at 60 mph and the other travels west at 25 mph. At what rate is the distance between the cars increasing two hours later?

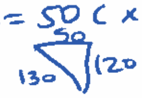
$\frac{dx}{dt} = 25$



$\frac{dy}{dt} = 60$

$x^2 + y^2 = z^2$

$60 \times 2 = 120$ (y)
$25 \times 2 = 50$ (x)



$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(50)(25) + 2(60)(120) = 2(130)\frac{dz}{dt}$

$\boxed{\frac{dz}{dt} = 65\text{ mph}}$

5. Find the linearization of the function $f(x) = 3xe^{2x-10}$ at $x = 5$. Use it to approximate $f(5.1)$. Use a calculator to determine the accuracy of your approximation.

$f'(x) = 3x \cdot e^{2x-10} \cdot 2 + 3e^{2x-10}$

$f'(5) = 3(5) \cdot e^{10-10} \cdot 2 + 3e^{10-10}$

$= 15 \cdot 1 \cdot 2 \quad + 3$

$= 30 \quad +3 = 33$

$f(5) = 3(5)e^{10-10}$
$= 15$

$y - y_1 = m(x - x_1)$

$\boxed{y - 15 = 33(x-5)}$

$y = 33(5.1-5) + 15 \rightarrow \boxed{18.3}$

$f(5.1) - 18.3$
$\approx .387$
Error < 1

6. Find dy of $y = \frac{2}{x^2}$ and evaluate dy if $x = -5$ and $dx = \frac{5}{2}$.

$\hookrightarrow 2x^{-2}$

$\frac{dy}{dx} = \cancel{(2)(2)}x^{-3}$

$\hookrightarrow \frac{dy}{dx} = -4x^{-3}$

$dy = -4(-5)^{-3}\left(\frac{5}{2}\right)$

$= \frac{-4}{-125} \cdot \frac{5}{2} \rightarrow \boxed{\frac{2}{25} = dy}$

7. Evaluate each of the following limits.

a) $\lim_{x\to\infty} 5x^2e^{-x}$

$\lim_{x\to\infty} \frac{5x^2}{e^x} \rightarrow \frac{\infty}{\infty}$ L'Hospital's

$\lim_{x\to\infty} \frac{10x}{e^x} \rightarrow \frac{\infty}{\infty}$ L'Hospitals

$\lim_{x\to\infty} \frac{10}{e^x} \rightarrow \frac{10}{\infty} \rightarrow 0$

b) $\lim_{x\to-1} \frac{2(x^2-1)}{\ln x^2}$

$\lim_{x\to-1} \frac{2((-1)^2-1)}{\ln(-1)^2} \rightarrow \frac{0}{0}$ L'Hospital's

$\lim_{x\to-1} \frac{4x}{\frac{1}{x^2}(2x)} \rightarrow \lim_{x\to-1} \frac{4x}{\frac{2x}{x^2}} \rightarrow \frac{4x}{\frac{2}{x}} \rightarrow 2x^2$

$\lim_{x\to-1} 2x^2 \rightarrow 2(-1)^2 = 2$