AB Calculus Rules for Differentiation Day 2 Practice Name: Key

1. What is the product rule? $f(x)\cdot g(x)$ $f'(x)g(x)+f(x)g'(x)$

2. What is the quotient rule? $\frac{f(x)}{g(x)}$ $\frac{g(x)f'(x)-f(x)g'(x)}{g^2(x)}$

3. Let $f(x)=(3x^3+4x^2)(2x^4-5x)$

   a) Find $f'(x)$ without using product rule $6x^7-15x^4+8x^6-20x^3 \to 42x^6+48x^5-60x^3-60x^2$

   b) Find $f'(x)$ using product rule $(3x^3+4x^2)(8x^3-5)+(9x^2+8x)(2x^4-5x)$

4. Let $f(x)=\frac{x^2+4}{x}$ $x+4x^{-1}$

   a) Find $f'(x)$ without using quotient rule $1-\frac{4}{x^2}$

   b) Find $f'(x)$ using quotient rule $\frac{x(2x)-(x^2+4)(1)}{x^2}=\frac{2x^2-x^2-4}{x^2}$

5. For each of the following, find $\frac{dy}{dx}$

   a) $y=\frac{2x-5}{3x+2}$ $\frac{(3x+2)(2)-(2x-5)(3)}{(3x+2)^2}\to\frac{6x+4-6x+15}{(3x+2)^2}\to\frac{19}{(3x+2)^2}$

   b) $y=(3-x)(2+x^2)^{-1}$ $\to\frac{3-x}{(2+x^2)}$ $\frac{(2+x^2)(-1)-(3-x)(2x)}{(2+x^2)^2}\to\frac{-2-x^2+6x+2x^2}{(2+x^2)^2}\to\frac{x^2+6x-2}{(x^2+2)^2}$

   c) $y=\frac{x^3}{8-x^2}$ $\frac{(8-x^2)(3x^2)-(x^3)(-2x)}{(8-x^2)^2}\to\frac{24x^2-3x^4+2x^4}{(8-x^2)^2}\to\frac{-x^4+24x^2}{(8-x^2)^2}\to\frac{-x^2(x^2-24)}{(8-x^2)^2}$

6. For each of the following, write an expression for $f'(x)$ and find $f'(2)$ given the information below.

   $g(2)=3$ $g'(2)=-2$

   $h(2)=-1$ $h'(2)=4$

   a) $f(x)=2g(x)+h(x)$ $f'(x)=2g'(x)+h'(x)=2(-2)+4=0$

   b) $f(x)=4-h(x)$ $f'(x)=-h'(x)=-4$

   c) $f(x)=g(x)h(x)$ $f'(x)=g'(x)h(x)+g(x)h'(x)=(-2)(-1)+(3)(4)=14$

   d) $f(x)=\frac{g(x)}{h(x)}$ $\frac{h(x)g'(x)-g(x)h'(x)}{h^2(x)}$ $\frac{(-1)(-2)-(3)(4)}{(-1)^2}=\frac{2-12}{1}=-10$

7. Suppose u and v are differentiable functions at $x=3$ and $u(3)=4$, $\left.\frac{du}{dx}\right|_{x=3}=-3$, $v(3)=2$ and $\left.\frac{dv}{dx}\right|_{x=3}=3$. Find the following at $x=3$.

   a) $\frac{d}{dx}\left[\frac{u}{v}\right]$ $\frac{v\,du-u\,dv}{v^2}=\frac{2(-3)-4(3)}{4}=\frac{-6-12}{4}=\frac{-18}{4}=-\frac{9}{2}$

   b) $\frac{d}{dx}[uv]$ $u\,dv+v\,du$ $4(3)+2(-3)$ $12-6=6$

   c) $\frac{d}{dx}[5u-2v+4uv]$ $5du-2dv+4u\,dv+4v\,du$

   d) $\frac{d}{dx}\left[\frac{v}{u}\right]$ $\frac{u\,dv-v\,du}{u^2}$ $\frac{4(3)-2(-3)}{16}=\frac{12+6}{16}=\frac{18}{16}=\frac{9}{8}$

8. Find the values of a and b so that $g(x)$ is both continuous and differentiable at $x=1$.

$$g(x)=\begin{cases}x^2+2, & x\le 1\\ a\left(x-\frac{1}{x}\right)+b, & x>1\end{cases}$$

$1+2=3=a(1-1)+b$ $3=b$

$g'(x)=2x$ $g'(x)=a+\frac{a}{x^2}$ $2=a+\frac{a}{1}$ $2=2a$ $a=1$

9. At what point on the graph of $y=\frac{1}{4}x^2$ is the tangent line parallel to the line $2x-4y=3$?

$5(-3)-2(3)+4(4)(3)+4(-3)(2)=-15-6+48-24=-21+24=3$

$y'=x$ $y'=\frac{1}{2}$ if $x=\frac{1}{2}$ $\left(\frac{1}{2},\frac{1}{8}\right)$

$-4y=-2x+3$ $y=\frac{-2}{-4}x-\frac{3}{4}\to y=\frac{1}{2}x-\frac{3}{4}$ ↓ slope