ABCALC Rules for Differentiation Day 2 Homework

Name: Key

1. Find dy/dx

a) $f(x) = -2x^3 - x^2 + 4x - 7$

$f'(x) = -6x^2 - 2x + 4$

b) $y = x^3 + \frac{1}{x} - \sqrt[3]{x} + \frac{1}{\sqrt[4]{x^5}}$ $\to x^{-1}$ $\to -x^{1/3}$ $\to x^{-5/4}$

$\frac{dy}{dx} = 3x^2 - \frac{1}{x^2} - \frac{1}{3}x^{-2/3} - \frac{5}{4}x^{-9/4}$

c) $f(x) = (6x + 5)(x^3 - 3)$

$f'(x) = 6(x^3-3) + (6x+5)(3x^2)$

$= 6x^3 - 18 + 18x^3 + 15x^2$

$= 24x^3 + 15x^2 - 18$

d) $f(x) = (\sqrt{x} + 2)(4x^3 + 3x)$

$f'(x) = \frac{1}{2\sqrt{x}}(4x^3 + 3x) + (\sqrt{x}+2)(12x^2+3)$

e) $y = \frac{2x + 5}{3x - 2}$

$\frac{dy}{dx} = \frac{(3x-2)(2) - (2x+5)(3)}{(3x-2)^2} \to \frac{-19}{(3x-2)^2}$

Num: $6x - 4 - 6x - 15$

f) $y = \frac{x^2}{1 - x^3}$

$\frac{dy}{dx} = \frac{(1-x^3)(2x) - x^2(-3x^2)}{(1-x^3)^2}$

g) $y = \frac{(x - 1)(x^2 + x + 1)}{x^3}$

$\frac{x^3\left((x-1)(2x+1) + (1)(x^2+x+1)\right) - (x-1)(x^2+x+1)(3x^2)}{x^6}$

L D Hi - H DL

L^2 → that's a lot of work....

maybe multiply 1st $y = \frac{x^3 - 1}{x^3} \to 1 - x^{-3} \to \frac{dy}{dx} \to \frac{3}{x^4}$ same answer

h) $y = \frac{(x + 1)(x + 2)}{(x - 1)(x - 2)} \to \frac{x^2+3x+2}{x^2-3x+2}$

$\frac{(x^2-3x+2)(2x+3) - (x^2+3x+2)(2x-3)}{(x^2-3x+2)^2}$

cutoff (2x-3)

2. Let $y = (x + 1)(x^2 + 1)$. Find dy/dx (a) by applying the Product Rule and (b) by multiplying the factors first and then differentiating.

$\frac{dy}{dx} = (x+1)(2x) + (1)(x^2+1) = 2x^2 + 2x + x^2 + 1 = 3x^2 + 2x + 1$

$y = x^3 + x + x^2 + 1 \to \frac{dy}{dx} = 3x^2 + 1 + 2x$

3. Use the Product Rule to show that

$$\frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx} f(x)$$

for any constant c.

$$\underbrace{\frac{dc}{dx}}_{\to 0} \cdot f(x) + c\frac{d}{dx}f(x)$$

$$= c\frac{d}{dx}f(x)$$

4. Suppose v and w are functions of x that are differentiable at $x = 0$, and that $v(0) = 5$, $v'(0) = -3$, $w(0) = -1$, $w'(0) = 2$. Find the values of the following derivatives at x = 0.

a) $\frac{d}{dx}(vw)$ $\quad \frac{dv}{dx}w + \frac{dw}{dx}v$

$(-3)(-1) + (2)(5) = 13$

b) $\frac{d}{dx}\left(\frac{v}{w}\right)$ $\quad \dfrac{w\frac{dv}{dx} - v\frac{dw}{dx}}{w^2} = \dfrac{(-1)(-3)-(5)(2)}{(-1)^2}$

$= -7$

c) $\frac{d}{dx}\left(\frac{w}{v}\right)$ $\quad \dfrac{v\frac{dw}{dx} - w\frac{dv}{dx}}{v^2} = \dfrac{5(2)-(-1)(-3)}{5^2}$

$= \frac{7}{25}$

d) $\frac{d}{dx}(7w - 2v)$

$7\frac{dw}{dx} - 2\frac{dv}{dx} = 7(2) - 2(-3) = 20$

5. Find the tangents to Newton's Serpentine (graph and function below) at the origin and the point (1, 2).

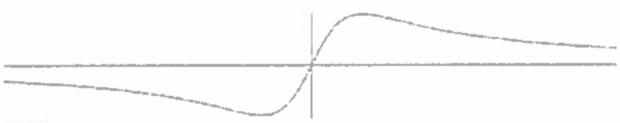

Figure 1: Newton's Serpentine

$$y = \frac{4x}{x^2+1}$$

$$\frac{dy}{dx} = \frac{(x^2+1)(4) - 4x(2x)}{(x^2+1)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{(2)(4) - 4\cdot 2}{2^2} = 0 \qquad y - 2 = 0(x-1) \to y = 2$$

$$\left.\frac{dy}{dx}\right|_{x=0} = 4$$

$$y - 0 = 4(x-0)$$

$$y = 4x$$

6. Cylindrical Pressure If gas in a cylinder is maintained at a constant temperature T, the pressure P is related to the volume V by a formula of the form

constant as well

$$P = \frac{nRT}{V - nb} - \frac{an^2}{V^2}$$

$$nRT(V-nb)^{-1} - an^2V^{-2}$$

in which a, b, n. and R are constants. Find dP/dV.

$$\frac{dP}{dV} = \frac{-nRT}{(V-nb)^2}(1) + \frac{2an^2}{V^3}$$

7. Find the values of a and b so that $f(x)$ is both differentiable and continuous.

$$f(x) = \begin{cases} ax^3, & x \le 2 \\ x^2 + b, & x > 2 \end{cases}$$

$$3ax^2 = 2x$$

$$3a(2)^2 = 2(2)$$

$$a = 1/3$$

$$a(2)^3 = 2^2 + b$$

$$8a = 4 + b$$

$$8\left(\tfrac{1}{3}\right) = 4 + b$$

$$b = \frac{8}{3} - \frac{12}{3}$$

$$b = -4/3$$