11a. *[3 marks]*

The first three terms of a geometric sequence are $\ln x^{16}, \ln x^8, \ln x^4$, for $x > 0$.

Find the common ratio.

$$r = \frac{\ln x^8}{\ln x^{16}} = \frac{8 \ln x}{16 \ln x} = \frac{1}{2}$$

11b. *[5 marks]*

$$S_\infty \rightarrow \frac{a_1}{1-r}$$

Solve $\sum_{k=1}^{\infty} 2^{5-k} \ln x = 64$.

$$\frac{a_1}{1-r} = 64$$

$$\frac{16 \ln x}{1 - 1/2} = 64$$

$$\frac{16 \ln x}{1/2} = 64$$

$$32 \ln x = 64$$

$$\ln x = 2$$

$$x = e^2$$

12. *[6 marks]*

The sum of an infinite geometric sequence is 33.25. The second term of the sequence is 7.98. Find the possible values of r.

$$33\,25 = \frac{u_1}{1-r}$$

$$33\,25(1-r) = u_1$$

$$y_1 = 33\,25(1-x)$$

$$y_2 = \frac{7.98}{x}$$

$$\frac{7\,98}{u_1} = r$$

$$\frac{7\,98}{r} = \cancel{r}\, u_1$$

$$33\,25r - 33\,25r^2 = 7.98$$

$$r = 4, \quad r = 6$$

13a. *[2 marks]*

The first term of an infinite geometric sequence is 4. The sum of the infinite sequence is 200.

Find the common ratio.

$u_1 = 4$ $\quad 200 = \frac{4}{1-r}$

$200(1-r) = 4$ $\quad r = 1 - \frac{4}{200}$

$1 - r = \frac{4}{200} \rightarrow$ $\quad = 1 - \frac{1}{50} = \frac{49}{50}$ or $.98$

13b. *[2 marks]*

Find the sum of the first 8 terms. → partial sum

$$S_8 = \frac{u_1(1-r^8)}{1-r}$$

$$= \frac{4(1-.98^8)}{1-.98} = 29.8$$

13c. *[3 marks]*

Find the least value of n for which $S_n > 163$.

$$163 = \frac{4(1-.98^n)}{1-.98}$$

$n = 83.5$

so

$\boxed{n \text{ is } 84}$

$$+.185 = +.98^n$$

$$\frac{\ln(.185)}{\ln(.98)} = \frac{n\ln(.98)}{\ln(.98)}$$

14. *[6 marks]*

Consider a geometric sequence where the first term is 768 and the second term is 576.

Find the least value of n such that the nth term of the sequence is less than 7. $\rightarrow U_n < 7$

$r = \frac{576}{768} \rightarrow r = 75$

$U_n = U_1 r^{n-1}$

$7 = 768 (.75)^{n-1}$

$n = 17.33$

so $n = 18$

15a. *[2 marks]*

The first three terms of an arithmetic sequence are $u_1 = 0.3,\ u_2 = 1.5,\ u_3 = 2.7$.

Find the common difference.

$15 - 3 = 12$

15b. *[2 marks]*

Find the 30th term of the sequence.

$U_{30} = 3 + 12(30-1) = 351$

15c. *[2 marks]*

Find the sum of the first 30 terms.

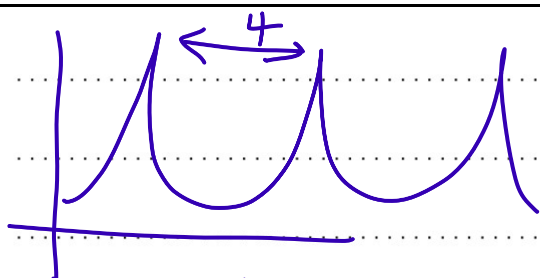
$S_{30} = \frac{30}{2}(3 + 351)$

16a. *[4 marks]*

Let $f(x) = e^{2\sin\left(\frac{\pi x}{2}\right)}$, for $x > 0$.

The kth maximum point on the graph of f has x-coordinate x_k where $k \in \mathbb{Z}^{+}$.

Given that $x_{k+1} = x_k + a$, find a.



$k = 4$

The distance between max points $= 4$

16b. *[4 marks]*

Hence find the value of n such that $\sum_{k=1}^{n} x_k = 861$.

$x_1 = 1$ arithmetic

$x_2 = 5$ $\quad S_n = \frac{n}{2}(u_1 + u_n)$

$x_3 = 9$ $\quad 861 = \frac{n}{2}(\underbrace{1+}_{u_1 +} \underbrace{1 + 4(n-1)}_{u_n})$

$n = 21$

17a. *[3 marks]*

if you have interest compounded anually, etc $\rightarrow$ $A = P\left(1+\frac{r}{n}\right)^{nt}$

A population of rare birds, P_t, can be modelled by the equation $P_t = P_0 e^{kt}$, where P_0 is the initial population, and t is measured in decades. After one decade, it is estimated that $\frac{P_1}{P_0} = 0.9$. $t=1$

(i) Find the value of k.

(ii) Interpret the meaning of the value of k.

(i) $P_1 = P_0 e^{k(1)}$ $\quad .9 = e^k$

$\frac{P_1}{P_0} = e^k$ $\quad \ln(.9) = k$

$k = -.105$

(ii) since $k < 0$, population decreases

17b. *[5 marks]*

Find the least number of **whole** years for which $\frac{P_t}{P_0} < 0.75$.

$P_t = P_0 e^{kt}$

$\frac{P_t}{P_0} = e^{kt}$

$.75 = e^{\ln(.9)t}$

$\ln(.75) = \ln(.9)t$

$t = \frac{\ln(.75)}{\ln(.9)} = 2.73$

$\boxed{28}$