final exam review stuff

Name

1. A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of 4/9 meter per second, at what rate, in meters per second, is the person walking?

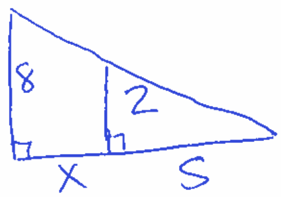
(A) 4/27

(B) 4/9

(C) 3/4

(D) 4/3

(E) 16/9



$\frac{x+s}{8} = \frac{s}{2}$

$2x + 2s = 8s$

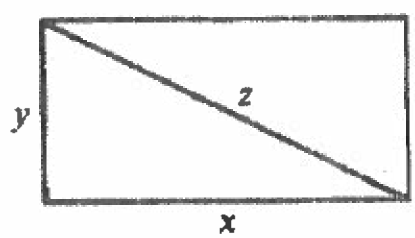
$2x = 6s$

$x = 3s$

$\frac{dx}{dt} = 3\frac{ds}{dt}$

$= 3\left(\frac{4}{9}\right) = \frac{4}{3}$

2.



$3^2 + 4^2 = z^2$

$z = 5$

The sides of the rectangle above increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

$x^2 + y^2 = z^2$

$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 2z \cdot \frac{dz}{dt}$

$(4)\left(3\frac{dy}{dt}\right) + (3)\left(\frac{dy}{dt}\right) = (5)(1)$

$15\frac{dy}{dt} = 5 \qquad \frac{dy}{dt} = \frac{1}{3}$

$\frac{dx}{dt} = 3\left(\frac{1}{3}\right) = 1$

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(A) $\frac{1}{3}$

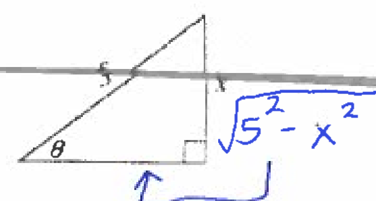
(B) 1

(C) 2

(D) $\sqrt{5}$

(E) 5

3.



In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

(A) 3

(B) $\frac{15}{4}$

(C) 4

(D) 9

(E) 12

$\sin\theta = \frac{x}{5}$

$\cos\theta = \frac{\sqrt{25-x^2}}{5}$

$\cos\theta \frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$

$= \frac{\sqrt{25-9}}{5}$

$\left(\frac{4}{5}\right)(3) = \frac{1}{5}\frac{dx}{dt}$

$= \frac{4}{5}$

$12 = \frac{dx}{dt}$

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4. The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

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(A) 0.4

(B) 0.6

(C) 0.7

(D) 1.3

(E) 1.4

$y - y_1 = m(x - x_1)$

$y - 1 = 4(x-2)$

$f(1.9) \approx 4(1.9-2)+1$

$4(-.1)+1$

$-.4+1$

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5. Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval [0,3]? $3^3 - 3(3)^2 = 0$

(A) 0 only

(B) 2 only

(C) 3 only

(D) 0 and 3

(E) 2 and 3

$f'(x) = 3x^2 - 6x$

$\frac{f(3)-f(0)}{3-0} = \frac{0-0}{3} = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$0, 2$

endpt. of interval doesn't satisfy

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doesn't say it's differentiable

6. Let f be a function defined and continuous on the closed interval [a,b]. If f has a relative maximum at c and $a<c<b$, which of the following statements must be true?

I. $f'(c)$ exists. (Not necessarily → $f'(x)$ could be undefined)

II. If $f'(c)$ exists, then $f'(c)=0$ → Yes!

III. If $f'(c)$ exists, then $f''(c) \leq 0$ ✓

2nd Derivative Test

critical pt. at top of ∩ cc down

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(A) II only

(B) III only

(C) I and II only

(D) I and III only

(E) II and III only

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7. The $\lim_{h \to 0} \frac{\tan 3(x+h) - \tan(3x)}{h}$ is

L'H or just recognize it $f'(x)$ of $f(x) = \tan(3x)$

$f'(x) = \sec^2(3x) \cdot 3$

(A) 0

(B) $3\sec^2(3x)$

(C) $\sec^2(2x)$

(D) $3\cot(3x)$

(E) nonexistent

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8. $\lim_{x \to 0} \frac{\sin x \cos x}{x}$

L'H.

$\frac{\sin x \cdot (-\sin x) + \cos x \cdot \cos x}{1}$

$\frac{0 \cdot (-0) + 1 \cdot 1}{1} = 1$

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(A) −1

(B) 0

(C) 1

(D) $\frac{\pi}{4}$

(E) nonexistent

9. $\lim_{h\to 0} \frac{e^h - 1}{2h}$ is

L'H.

$\frac{e^h}{2} = \frac{1}{2}$

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) e

(E) nonexistent