Series Review Stuff

1. Which of the following series diverge?

I. $\sum_{n=0}^{\infty}\left(\frac{\sin 2}{\pi}\right)^n$

II. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}$

III. $\sum_{n=1}^{\infty}\left(\frac{e^n}{e^n+1}\right)$

(A) III only

(B) I and II only

(C) I and III only

(D) II and III only ✔

(E) I, II, and III

---

2. What is the value of $\sum_{n=1}^{\infty}\frac{2^{n+1}}{3^n}$?

(A) 1

(B) 2

(C) 4 ✔

(D) 6

(E) The series diverges.

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

Series Review Stuff

3. Which of the following series converge to 2?

I. $\sum_{n=1}^{\infty} \frac{2n}{n+3}$

II. $\sum_{n=1}^{\infty} \frac{-8}{(-3)^n}$

III. $\sum_{n=0}^{\infty} \frac{1}{2^n}$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only ✔

4. The sum of the infinite geometric series $\frac{3}{2} + \frac{9}{16} + \frac{27}{128} + \frac{81}{1{,}024} + \cdots$ is

(A) 1.60

(B) 2.35

(C) 2.40 ✔

(D) 2.45

(E) 2.50

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

Series Review Stuff

5. What are all values of p for which $\int_1^{\infty} \frac{1}{x^{2p}}\, dx$ converges?

(A) $p < -1$

(B) $p > 0$

(C) $p > \frac{1}{2}$ ✔

(D) $p > 1$

(E) There are no values of p for which this integral converges.

6. What are all values of p for which the infinite series $\sum_{n=1}^{\infty} \frac{n}{n^p + 1}$ converges?

(A) $p > 0$

(B) $p \geq 1$

(C) $p > 1$

(D) $p \geq 2$

(E) $p > 2$ ✔

7. Which of the following series converge?

$I. \sum_{n=1}^{\infty} \frac{1}{n^2}$ $\quad II. \sum_{n=1}^{\infty} \frac{1}{n}$ $\quad III. \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

Series Review Stuff

(A) I only

(B) III only

(C) I and II only

(D) I and III only ✔

(E) I, II, and III

---

8. Consider the series $\sum_{n=1}^{\infty} \frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

(A) $\lim_{n \to \infty} \frac{e}{n!} < 1$

(B) $\lim_{n \to \infty} \frac{n!}{e} < 1$

(C) $\lim_{n \to \infty} \frac{n+1}{e} < 1$

(D) $\lim_{n \to \infty} \frac{e}{n+1} < 1$ ✔

(E) $\lim_{n \to \infty} \frac{e}{(n+1)!} < 1$

---

9. Which of the following series converges for all real numbers x ?

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

Series Review Stuff

(A) $\sum_{n=1}^{\infty} \frac{x^n}{n}$

(B) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

(C) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

(D) $\sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$ ✓

(E) $\sum_{n=1}^{\infty} \frac{n! x^n}{e^n}$

---

10. What are all values of *x* for which the series $\sum_{n=1}^{\infty} \frac{n3^n}{x^n}$ converges?

(A) All *x* except *x* = 0

(B) $|x| = 3$

(C) $-3 \leq x \leq 3$

(D) $|x| > 3$ ✓

(E) The series diverges for all *x*.

The function *f* is defined by the power series

$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + ... + \frac{(-1)^n x^{2n}}{(2n+1)!} + ...$

for all real numbers *x*.

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

Series Review Stuff

11. Show that $1-\frac{1}{3!}$ approximates *f*(1) with error less than $\frac{1}{100}$.

Please respond on separate paper, following directions from your teacher.

Part B

1 point is earned for correctly showing error bound < $\frac{1}{100}$ $f(1)=1-\frac{1}{3!}+\frac{1}{5!}-\frac{1}{7!}+\ldots+\frac{(-1)^n}{(2n+1)!}+\ldots$

This is an alternating series whose terms decrease in absolute value with limit 0. Thus, the error is less than the first omitted term, so $\left|f(1)-(1-\frac{1}{3!})\right|\leq\frac{1}{5!}=\frac{1}{120}<\frac{1}{100}$.

0	1

The student response earns one of the following points:

1 point is earned for correctly showing error bound < $\frac{1}{100}$

$f(1)=1-\frac{1}{3!}+\frac{1}{5!}-\frac{1}{7!}+\ldots+\frac{(-1)^n}{(2n+1)!}+\ldots$

This is an alternating series whose terms decrease in absolute value with limit 0. Thus, the error is less than the first omitted term, so $\left|f(1)-(1-\frac{1}{3!})\right|\leq\frac{1}{5!}=\frac{1}{120}<\frac{1}{100}$.

Let *f* be the function given by $f(x)=e^{-2x^2}$.

12. Let *g* be the function given by the sum of the first four nonzero terms of the power series for *f*(*x*) about *x*=0 . Show that |*f*(*x*)−*g*(*x*)| <0.02 for −0.6 ≤ *x* ≤ 0.6 .

Please respond on separate paper, following directions from your teacher.

Part C

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

Series Review Stuff

1 point is earned for correctly alternating series bound of $\frac{16x^8}{4!}$

$$f(x) - g(x) = \frac{16x^8}{4!} - \frac{32x^{16}}{5!} + \cdots$$

1 point is earned for correctly using *x* = 0.6

This is an alternating series for each *x*, since powers of *x* are even.

Also, $\left|\frac{a_n+1}{a_n}\right| = \frac{2}{n+1}x^2 < 1 \text{ for } -0.6 \le x \le 0.6$ so terms are decreasing in absolute value

1 point is used for the correct conclusion

Thus $\left|f(x) - g(x)\right| \le \frac{16x^8}{4!} \le \frac{16(0.6)^8}{4!}$
$= 0.011\cdots < 0.02$

0	1	2	3 ✔

The student response earns three of the following points:

1 point is earned for correctly alternating series bound of $\frac{16x^8}{4!}$

$$f(x) - g(x) = \frac{16x^8}{4!} - \frac{32x^{16}}{5!} + \cdots$$

1 point is earned for correctly using *x* = 0.6

This is an alternating series for each *x*, since powers of *x* are even.

Also, $\left|\frac{a_n+1}{a_n}\right| = \frac{2}{n+1}x^2 < 1 \text{ for } -0.6 \le x \le 0.6$ so terms are decreasing in absolute value

1 point is used for the correct conclusion

Thus $\left|f(x) - g(x)\right| \le \frac{16x^8}{4!} \le \frac{16(0.6)^8}{4!}$
$= 0.011\cdots < 0.02$

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

Series Review Stuff

13. For a series *S*, let $s = 1 - \frac{1}{9} + \frac{1}{2} - \frac{1}{25} + \frac{1}{4} - \frac{1}{49} + \frac{1}{8} - \frac{1}{81} + \frac{1}{16} - \frac{1}{121} + \ldots + a_n + \ldots$ where

$$a_n = \begin{cases} \dfrac{1}{2^{(n-1)/2}} & \text{if } n \text{ is odd} \\ \dfrac{-1}{(n+1)^2} & \text{if } n \text{ is even.} \end{cases}$$

Which of the following statements are true?

I. *S* converges because the terms of *S* alternate and $\lim_{n\to\infty} a_n = 0$

II. *S* diverges because it is not true that $|a_{n+1}| < |a_n|$ for all *n*.

III. *S* converges although it is not true that $|a_{n+1}| < |a_n|$ for all *n*.

(A) None

(B) I only

(C) II only

(D) III only ✔

(E) I and III only

---

14. If $f(x) = \sum_{k=1}^{\infty} (\sin^2 x)^k$, then *f*(1) is

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

Series Review Stuff

(A) 0.369

(B) 0.585

(C) 2.400

(D) 2.426 ✔

(E) 3.426

---

15. Which of the following statements is true about the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$?

(A) The series converges conditionally. ✔

(B) The series converges absolutely.

(C) The series converges but neither conditionally nor absolutely.

(D) The series diverges.

---

16. Which of the following series is conditionally convergent?

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

Series Review Stuff

(A) $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$

(B) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{17+n}{\sqrt{n}}$

(C) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{17+\sqrt{n}}{n}$ ✔

(D) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{17}\right)^n$

---

17. For what values of p is the series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^p + 2}$ conditionally convergent?

(A) $0 < p \leq 1$

(B) $p > 1$

(C) $1 < p \leq 2$ only ✔

(D) $p > 2$ only

---

18. Which of the following statements is true?

(A) The series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$ diverges by the alternating series test.

(B) The series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{4\sqrt{n}}{2+\sqrt{n}}$ converges by the alternating series test.

(C) The series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos(n\pi)}{n^2}$ converges by the alternating series test.

(D) The series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{4n}{9+n^2}$ converges by the alternating series test. ✔

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

Series Review Stuff

19. The alternating series test can be used to show convergence for which of the following series?

1. $1-\frac{1}{4}+\frac{1}{9}-\frac{1}{16}+\frac{1}{25}-\frac{1}{36}+\cdots+a_n+\cdots$, where $a_n=(-1)^{n+1}\frac{1}{n^2}$
2. $\sin 1-\frac{\sin 2}{4}+\frac{\sin 3}{9}-\frac{\sin 4}{16}+\frac{\sin 5}{25}-\frac{\sin 6}{36}+\cdots+b_n+\cdots$, where $b_n=(-1)^{n+1}\frac{\sin n}{n^2}$
3. $\frac{1}{\sqrt{2}+1}-\frac{1}{\sqrt{2}-1}+\frac{1}{\sqrt{3}+1}-\frac{1}{\sqrt{3}-1}+\frac{1}{\sqrt{4}+1}-\frac{1}{\sqrt{4}-1}+\cdots+c_n+\cdots$, where

$$c_n=\begin{cases}\frac{1}{\sqrt{k+1}+1} & \text{if } n=2k-1\\ -\frac{1}{\sqrt{k+1}-1} & \text{if } n=2k\end{cases}$$

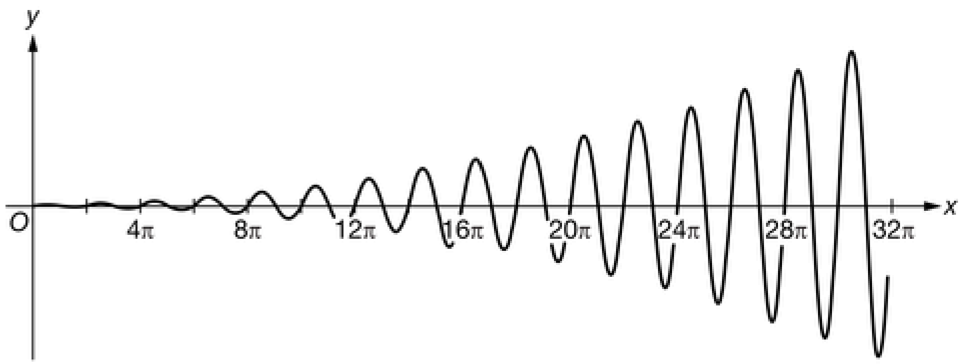
(A) I only ✔

(B) II only

(C) I and II only

(D) I and III only

20.



Graph of g

Let f be the function defined by $f(x)=\frac{2+\cos x}{x^2}$. The derivative of f is $f'(x)=-\frac{x^2\sin x+2x(2+\cos x)}{x^4}$. The graph of the function g defined by $g(x)=x^2\sin x+2x(2+\cos x)$ is shown above for $0\le x\le 100$. Let $b_n=f(n)$ for all integers $n\ge 1$. Which of the following statements about the series $\sum_{n=1}^{\infty}(-1)^{n+1}b_n$ is true?

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

Series Review Stuff

(A) The series converges by the alternating series test.

(B) The alternating series test cannot be used to determine convergence because the series is not alternating.

(C) The alternating series test cannot be used to determine convergence because $\lim_{n\to\infty} b_n \neq 0$.

(D) The alternating series test cannot be used to determine convergence because the terms b_n are not decreasing. ✓

---

21. Which of the following is not a p-series?

(A) $\sum_{n=1}^{\infty} n^{-4}$

(B) $\sum_{n=1}^{\infty} \frac{1}{n}$

(C) $\sum_{n=1}^{\infty} \frac{1}{n^e}$

(D) $\sum_{n=1}^{\infty} \frac{1}{e^n}$ ✓

---

22. Which of the following statements about the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ is true?

(A) The series diverges by the $n\text{th}$ term test.

(B) The series diverges by comparison to the series $\sum_{n=1}^{\infty} \frac{1}{n}$.

(C) The series diverges by limit comparison to the series $\sum_{n=1}^{\infty} \frac{1}{n}$. ✓

(D) The series diverges by limit comparison to the series $\sum_{n=1}^{\infty} n$.

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

Series Review Stuff

23. Which of the following series can be used with the limit comparison test to determine whether the series $\sum_{n=1}^{\infty} \frac{4^n}{5^n - n^2}$ converges or diverges?

(A) $\sum_{n=1}^{\infty} \frac{1}{n}$

(B) $\sum_{n=1}^{\infty} \frac{1}{4^n}$

(C) $\sum_{n=1}^{\infty} \frac{1}{5^n}$

(D) $\sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n$ ✔

24. If b and t are real numbers such that $0 < |t| < |b|$, which of the following infinite series has sum $\frac{1}{b^2+t^2}$?

(A) $\frac{1}{b^2} \sum_{k=0}^{\infty} \left(\frac{t^2}{b^2}\right)^k$

(B) $\frac{1}{b^2} \sum_{k=0}^{\infty} (-1)^k \left(\frac{t^2}{b^2}\right)^k$ ✔

(C) $b^2 \sum_{k=0}^{\infty} \left(\frac{t^2}{b^2}\right)^k$

(D) $b^2 \sum_{k=0}^{\infty} (-1)^k \left(\frac{t^2}{b^2}\right)^k$

25. If $a_n = \cos\left(\frac{\pi}{n}\right)$ for $n = 1, 2, \ldots$, which of the following statements about $\sum_{n=0}^{\infty} a_n$ must be true?

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

Series Review Stuff

(A) The series converges and $\lim_{n\to\infty} a_n = 0$.

(B) The series diverges and $\lim_{n\to\infty} a_n = 0$.

(C) The series converges and $\lim_{n\to\infty} a_n \neq 0$.

(D) The series diverges and $\lim_{n\to\infty} a_n \neq 0$. ✓

---

26. What is the sum of the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}}$?

(A) $\frac{-2}{e^2-2e}$

(B) $\frac{-2}{e^2+2e}$ ✓

(C) $\frac{-2}{e+2}$

(D) $\frac{e}{e+2}$

(E) The series diverges.

---

27. What is the value of $\sum_{n=0}^{\infty} \left(-\frac{2}{3}\right)^n$?

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

(A) -2

(B) $-\frac{2}{5}$

(C) $\frac{3}{5}$ ✓

(D) 3

(E) The series diverges.

---

28. Let f be a positive, continuous, decreasing function. If $\int_1^\infty f(x)dx = 5$, which of the following statements about the series $\sum_{n=1}^{\infty} f(n)$ must be true?

(A) $\sum_{n=1}^{\infty} f(n) = 0$

(B) $\sum_{n=1}^{\infty} f(n)$ converges, and $\sum_{n=1}^{\infty} f(n) < 5$

(C) $\sum_{n=1}^{\infty} f(n) = 5$

(D) $\sum_{n=1}^{\infty} f(n)$ converges, and $\sum_{n=1}^{\infty} f(n) > 5$ ✓

(E) $\sum_{n=1}^{\infty} f(n)$ diverges

Let an=1nln☐n for n≥3.

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

Series Review Stuff

29. Consider the infinite series $\sum_{n=3}^{\infty}(-1)^{n+1}a_n = \frac{1}{3\ln 3} - \frac{1}{4\ln 4} + \frac{1}{5\ln 5} - \cdots$. Identify the properties of this series that guarantee the series coverage. Explain why the sum of this series is less than $\frac{1}{3}$.

Please respond on separate paper, following directions from your teacher.

Part B

The response can earn up to 2 points:
1 point: properties
1 point: explanation

The terms in this alternating series decrease in absolute value and lim n→∞ 1 nlnn =0 . Therefore, the Alternating Series Test guarantees that this series converges. Furthermore,

$$\frac{1}{3In\ 3} - \frac{1}{4In\ 4} < \text{Sum} < \frac{1}{3In\ 3} < \frac{1}{3}$$

Therefore, the sum of the series is less than 1 3 .

0	1	2

The response can earn up to 2 points:
1 point: properties
1 point: explanation

The terms in this alternating series decrease in absolute value and lim n→∞ 1 nlnn =0 . Therefore, the Alternating Series Test guarantees that this series converges. Furthermore,

$$\frac{1}{3In\ 3} - \frac{1}{4In\ 4} < \text{Sum} < \frac{1}{3In\ 3} < \frac{1}{3}$$

Therefore, the sum of the series is less than 1 3 .

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.

Series Review Stuff

The function g has derivatives of all orders, and the Maclaurin series for g is

$$\sum_{n=0}^{\infty}(-1)^{n}\frac{x^{2n+1}}{2n+3}=\frac{x}{3}-\frac{x^3}{5}+\frac{x^5}{7}-\cdots.$$

30. The Maclaurin series for g evaluated at $x = 1/2$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g(1/2)$ using the first two nonzero terms of this series is 17/120. Show that this approximation differs from $g(1/2)$ by less than 1/200.

Please respond on separate paper, following directions from your teacher.

Part B

One point is earned for uses the third term as an error bound

One point is earned for error bound

$$\left|g\left(\frac{1}{2}\right)-\frac{17}{120}\right|<\frac{\left(\frac{1}{2}\right)^5}{7}=\frac{1}{224}<\frac{1}{200}$$

0	1	2 ✔

The student earns all of the following points:

One point is earned for uses the third term as an error bound

One point is earned for error bound

$$\left|g\left(\frac{1}{2}\right)-\frac{17}{120}\right|<\frac{\left(\frac{1}{2}\right)^5}{7}=\frac{1}{224}<\frac{1}{200}$$

Copyright © 2017. The College Board. These materials are part of a College Board program. Use or distribution of these materials online or in print beyond your school's participation in the program is prohibited.