

### Solving a 3x3 Matrix by Row Reduction

Solve each system.

1)  $-2x + 3z = 1$   
 $-5x + 4y + z = 25$   
 $4x - 4y - 4z = -20$

$$\begin{array}{ccc|c} -2 & 0 & 3 & 1 \\ -5 & 4 & 1 & 25 \\ 4 & -4 & -4 & -20 \end{array}$$

Swap  $R_1 \leftrightarrow R_3$

$$\begin{array}{ccc|c} 4 & -4 & -4 & -20 \\ -5 & 4 & 1 & 25 \\ -2 & 0 & 3 & 1 \end{array} \quad -4$$

$$\begin{array}{ccc|c} 1 & -1 & -1 & -5 \\ -5 & 4 & 1 & 25 \\ -2 & 0 & 3 & 1 \end{array}$$

$$\begin{array}{ccc|c} 1 & -1 & -1 & -5 \\ 0 & -1 & -4 & 0 \end{array}$$

$5R_1 + R_2$

$$\begin{array}{ccc|c} 0 & -2 & 1 & -9 \end{array}$$

$2R_1 + R_3$

$$\begin{array}{ccc|c} 1 & -1 & -1 & -5 \\ 0 & 1 & 4 & 0 \\ 0 & -2 & 1 & -9 \end{array} \quad R_2(-1)$$

$$\begin{array}{ccc|c} 1 & -1 & -1 & -5 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 9 & -9 \end{array} \quad 2R_2 + R_3$$

$$\begin{array}{ccc|c} 1 & -1 & -1 & -5 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & -1 \end{array}$$

$$\begin{array}{ccc|c} 1 & -1 & -1 & -5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array} \quad -4R_3 + R_2$$

$$\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array} \quad R_3 + R_2 + R_1$$

$(x, y, z) = (-2, 4, -1)$

$$2) -6x + 4y = 18$$

$$4x + 2y + 2z = -8$$

$$2x - 3y - 2z = -10$$

$$\begin{array}{ccc|c} -6 & 4 & 0 & 18 \\ 4 & 2 & 2 & -8 \\ 2 & -3 & -2 & -10 \end{array}$$

$$\begin{array}{ccc|c} -3 & 2 & 0 & 9 \quad R_1/2 \\ 4 & 2 & 2 & -8 \\ 2 & -3 & -2 & -10 \end{array}$$

$$\begin{array}{ccc|c} 1 & 4 & 2 & 1 \quad R_2+R_1 \\ 4 & 2 & 2 & -8 \\ 2 & -3 & -2 & -10 \end{array}$$

$$\begin{array}{ccc|c} 1 & 4 & 2 & 1 \\ 0 & -14 & -6 & -12 \quad -4R_1+R_2 \\ 0 & -11 & -6 & -12 \quad -2R_1+R_3 \end{array}$$

$$\begin{array}{ccc|c} 1 & 4 & 2 & 1 \\ 0 & -3 & 0 & 0 \quad -R_3+R_2 \\ 0 & -11 & -6 & -12 \end{array}$$

$$\begin{array}{ccc|c} 1 & 4 & 2 & 1 \\ 0 & 1 & 0 & 0 \quad R_2/-3 \\ 0 & -11 & -6 & -12 \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & 2 & 1 \quad -4R_2+R_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -6 & -12 \quad R_2+R_3 \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \quad R_3/-6 \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & 0 & -3 \quad -2R_3+R_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}$$

$$(x, y, z) = (-3, 0, 2)$$

3)  $3x + y - 4z = 16$  ✓

$x + y - z = 3$  ✓

$5x - y - 4z = 16$  ✓

go ahead and row swap

$$\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 3 & 1 & -4 & 16 \\ 5 & -1 & -4 & 16 \end{array}$$

$R_3 + R_2$

$$\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & 1 & -5 \end{array}$$

$-3R_1 + R_2$

$$\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & -2 & -1 & 7 \\ -5R_1 + R_3 & 0 & -6 & 1 \end{array}$$

$R_2 / -2$

$$\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -5 \end{array}$$

$-5R_1 + R_3$

$$\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & -2 & -1 & 7 \\ 0 & -6 & 1 & 1 \end{array}$$

$R_3 + R_1$

$$\begin{array}{ccc|c} 1 & 1 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -5 \end{array}$$

$-3R_2 + R_3$

$$\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & -2 & -1 & 7 \\ 0 & 0 & 4 & -20 \end{array}$$

$-R_2 + R_1$

$$\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -5 \end{array}$$

$R_3 / 4$

$$\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & -2 & -1 & 7 \\ 0 & 0 & 1 & -5 \end{array}$$

$R_3 / 4$

$$\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & -2 & -1 & 7 \\ 0 & 0 & 1 & -5 \end{array}$$

$(x, y, z) = (-1, -1, -5)$

4)  $5x - 3y - 2z = 21$   
 $-4x - 3y + 2z = -4$   
 $x + y + 2z = 13$

$$\begin{array}{ccc|c} 5 & -3 & -2 & 21 \end{array}$$

$$\begin{array}{ccc|c} -4 & -3 & 2 & -4 \end{array}$$

$$\begin{array}{ccc|c} 1 & 1 & 2 & 13 \end{array}$$

$$-R_1 + R_3 \quad \begin{array}{ccc|c} 0 & 7 & 2 & -4 \end{array}$$

$$\begin{array}{ccc|c} 1 & -6 & 0 & 17 \end{array}$$

$$-5R_3 + R_2 \quad \begin{array}{ccc|c} 0 & -34 & 0 & 68 \end{array}$$

$$\begin{array}{ccc|c} 0 & 7 & 2 & -4 \end{array}$$

$$\begin{array}{ccc|c} 1 & -6 & 0 & 17 \end{array}$$

$$\begin{array}{ccc|c} 1 & -6 & 0 & 17 \end{array} \quad R_2 + R_1$$

$$R_2/34$$

$$\begin{array}{ccc|c} 0 & 1 & 0 & -2 \end{array}$$

$$\begin{array}{ccc|c} -4 & -3 & 2 & -4 \end{array}$$

$$\begin{array}{ccc|c} 0 & 7 & 2 & -4 \end{array}$$

$$\begin{array}{ccc|c} 1 & 1 & 2 & 13 \end{array}$$

$$\begin{array}{ccc|c} 1 & -6 & 0 & 17 \end{array}$$

$$\begin{array}{ccc|c} 0 & 1 & 0 & -2 \end{array}$$

$$-7R_2 + R_3 \quad \begin{array}{ccc|c} 0 & 0 & 2 & 10 \end{array}$$

$$\begin{array}{ccc|c} 1 & -6 & 0 & 17 \end{array}$$

$$\begin{array}{ccc|c} 0 & 1 & 10 & 48 \end{array} \quad 4R_3 + R_2$$

$$R_3/2$$

$$\begin{array}{ccc|c} 1 & -6 & 0 & 17 \end{array}$$

$$\begin{array}{ccc|c} 0 & 1 & 0 & -2 \end{array}$$

$$\begin{array}{ccc|c} 0 & 0 & 1 & 5 \end{array}$$

$$\begin{array}{ccc|c} 1 & 1 & 2 & 13 \end{array}$$

$$6R_2 + R_1$$

$$\begin{array}{ccc|c} 1 & 0 & 0 & 5 \end{array}$$

$$\begin{array}{ccc|c} 0 & 1 & 0 & -2 \end{array}$$

$$\begin{array}{ccc|c} 0 & 0 & 1 & 5 \end{array}$$

$$(x, y, z) = (5, -2, 5)$$