

Solving a 3x3 Matrix by Row Reduction

Solve each system.

1) $-2x + 3z = 1$ ✓
 $-5x + 4y + z = 25$ ✓
 $4x - 4y - 4z = -20$ ✓

$$\begin{array}{ccc|c} -2 & 0 & 3 & 1 \\ -5 & 4 & 1 & 25 \\ 4 & -4 & -4 & -20 \end{array}$$

$$\begin{array}{ccc|c} 1 & -1 & -1 & -5 \\ 0 & -1 & -4 & 0 \\ 0 & 0 & 9 & -9 \end{array}$$

$-2R_2 + R_3$

$$\begin{array}{ccc|c} 4 & -4 & -4 & -20 \\ -5 & 4 & 1 & 25 \\ -2 & 0 & 3 & 1 \end{array}$$

$$\begin{array}{ccc|c} 1 & -1 & -1 & -5 \\ 0 & -1 & -4 & 0 \\ 0 & 0 & 1 & -1 \end{array}$$

$R_3/9$

$$\begin{array}{ccc|c} 1 & -1 & -1 & -5 \\ -5 & 4 & 1 & 25 \\ -2 & 0 & 3 & 1 \end{array}$$

$$\begin{array}{ccc|c} 1 & -1 & -1 & -5 \\ 0 & -1 & 0 & -4 \\ 0 & 0 & 1 & -1 \end{array}$$

$4R_3 + R_2$

$$\begin{array}{ccc|c} 1 & -1 & -1 & -5 \\ 0 & -1 & -4 & 0 \\ 0 & -2 & 1 & -9 \end{array}$$

$R_2 + 5R_1$

$$\begin{array}{ccc|c} 1 & -1 & -1 & -5 \\ 0 & 0 & 1 & -1 \\ 1 & -1 & -1 & -5 \end{array}$$

$R_2/10$

$$\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}$$

$R_3 + 2R_1$

$$\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{array}$$

$R_2/10$

$$\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}$$

$R_1 + R_2 + R_3$

$x = -2$
 $y = 4$ $z = -1$

$$\begin{aligned} \checkmark 2) & -6x + 4y = 18 \\ \checkmark & 4x + 2y + 2z = -8 \\ \checkmark & 2x - 3y - 2z = -10 \end{aligned}$$

$$\begin{array}{ccc|c} -6 & 4 & 0 & 18 \\ 4 & 2 & 2 & -8 \\ 2 & -3 & -2 & -10 \end{array} \rightarrow \begin{array}{ccc|c} -3 & 2 & 0 & 9 \\ 2 & 1 & 1 & -4 \\ 2 & -3 & -2 & -10 \end{array}$$

$$\begin{array}{ccc|c} -3 & 2 & 0 & 9 \\ 2 & 1 & 1 & -4 \\ R_3 - R_2 & 0 & -4 & -3 \end{array} \begin{array}{ccc|c} -3 & 2 & 0 & 9 \\ 0 & 7 & 3 & 6 \\ 0 & -4 & -3 & -6 \end{array}$$

$$\begin{array}{ccc|c} -3 & 2 & 0 & 9 \\ 0 & 1 & 0 & 0 \\ 0 & -4 & -3 & -6 \end{array}$$

$$\begin{array}{ccc|c} -3 & 2 & 0 & 9 \\ \frac{2}{3}R_1 + R_2 & 0 & \frac{7}{3} & 2 \\ 0 & -4 & -3 & -6 \end{array}$$

$$\begin{array}{ccc|c} -3 & 2 & 0 & 9 \\ R_3 + R_2 & 0 & 3 & 0 \\ 0 & -4 & -3 & -6 \end{array}$$

$$\begin{array}{ccc|c} -3 & 2 & 0 & 9 \\ 0 & 1 & 0 & 0 \\ 4R_2 + R_3 & 0 & 0 & -3 \end{array}$$

$$\begin{array}{ccc|c} -3 & 2 & 0 & 9 \\ 0 & 1 & 0 & 0 \\ R_3 / -3 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{ccc|c} R_1 / -3 & 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}$$

$$\begin{array}{ccc|c} -2R_2 + R_1 & -3 & 0 & 0 & 9 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}$$

$$\begin{aligned} x &= -3 \\ y &= 0 \\ z &= 2 \end{aligned}$$

~~$$\begin{aligned} 3) \quad & 3x + y - 4z = 16 \\ & x + y - z = 3 \\ & 5x - y - 4z = 16 \end{aligned}$$~~

$$\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & -2 & -1 & 7 \end{array}$$

$$\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 3 & 1 & -4 & 16 \\ 5 & -1 & -4 & 16 \end{array}$$

$$-3R_2 + R_3 \quad \begin{array}{ccc|c} 0 & 0 & 4 & -20 \end{array}$$

$$R_3/4 \quad \begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & -2 & -1 & 7 \\ 0 & 0 & 1 & -5 \end{array}$$

$$\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ -3R_1 + R_2 & 0 & -2 & 7 \\ -5R_1 + R_3 & 0 & -6 & 1 \end{array}$$

$$\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ R_3 + R_2 & 0 & -2 & 0 \\ 0 & 0 & 1 & -5 \end{array}$$

$$\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ R_2/2 & 0 & 1 & -1 \\ 0 & 0 & 1 & -5 \end{array}$$

$$\begin{array}{ccc|c} R_1 - R_2 + R_3 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -5 \end{array}$$

$$x = -1 \quad y = -1 \quad z = -5$$

$$\begin{aligned}
 4) \quad & 5x - 3y - 2z = 21 \\
 & -4x - 3y + 2z = -4 \\
 & x + y + 2z = 13
 \end{aligned}$$

$$\begin{array}{ccc|c}
 1 & 1 & 2 & 13 \\
 -4 & -3 & 2 & -4 \\
 5 & -3 & -2 & 21
 \end{array}$$

$$\begin{array}{ccc|c}
 1 & 1 & 2 & 13 \\
 R_1 + R_2 & 0 & 1 & 10 & 48 \\
 -5R_1 + R_3 & 0 & -8 & -12 & -44
 \end{array}$$

$$\begin{array}{ccc|c}
 1 & 1 & 2 & 13 \\
 0 & 1 & 10 & 48 \\
 8R_2 + R_3 & 0 & 0 & 68 & 340 \\
 R_3 & 0 & 0 & 1 & 5
 \end{array}$$

$$\begin{array}{ccc|c}
 1 & 1 & 2 & 13 \\
 -10R_3 + R_2 & 0 & 1 & 0 & -2 \\
 0 & 0 & 1 & 5
 \end{array}$$

$$\begin{array}{ccc|c}
 R_1 - R_2 - 2R_3 & 1 & 0 & 0 & 5 \\
 0 & 1 & 0 & -2 \\
 0 & 0 & 1 & 5
 \end{array}$$

$$\boxed{x = 5, y = -2, z = 5}$$

$$\begin{array}{r}
 648 \\
 \underline{8} \\
 384 \\
 \underline{44} \\
 340
 \end{array}$$

Answers to Solving a 3x3 Matrix by Row Reduction

1) $(-2, 4, -1)$

2) $(-3, 0, 2)$

3) $(-1, -1, -5)$

4) $(5, -2, 5)$

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 $-5x + 4y + z = 25$
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 $(-2, 4, -1)$

$$2) -6x + 4y = 18$$

$$4x + 2y + 2z = -8$$

$$2x - 3y - 2z = -10$$

$$(-3, 0, 2)$$

$$\begin{aligned} 3) \quad & 3x + y - 4z = 16 \\ & x + y - z = 3 \\ & 5x - y - 4z = 16 \\ & \quad (-1, -1, -5) \end{aligned}$$

$$\begin{aligned} 4) \quad & 5x - 3y - 2z = 21 \\ & -4x - 3y + 2z = -4 \\ & x + y + 2z = 13 \\ & (5, -2, 5) \end{aligned}$$

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