Pre Calculus Name_____

Solving Matrix Equations Notes

Date_____ Period____

1) What is the determinant of this 2x2 matrix?

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $ad - bc$

Evaluate the determinant of each matrix.

2) $\begin{bmatrix} 1 & -3 \\ -4 & 3 \end{bmatrix}$

$(3)(1) - (-4)(-3)$

$3 - 12$

$= -9$

3) $\begin{bmatrix} -4 & -4 \\ 2 & 2 \end{bmatrix}$

$-8 - (-8)$

$-8 + 8$

$= 0$

Evaluate [illegible]

4) $\begin{vmatrix} 0 & 5 \\ -2 & -1 \end{vmatrix}$

$0 - (-10)$

$= 10$

5) $\begin{vmatrix} -5 & 4 \\ 3 & 3 \end{vmatrix}$

$-15 - 12$

$= -27$

For each matrix state if an inverse exists. Inverses do not exist if the determinant is zero.

6) $\begin{bmatrix} -3 & -6 \\ -2 & -9 \end{bmatrix}$ yes

$27 - 12 = 15$

7) $\begin{bmatrix} -2 & 4 \\ 7 & -14 \end{bmatrix}$ No

$28 - 28 = 0$

8) What is the inverse of the following 2x2 matrix?

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Find the inverse of each matrix.

9) $\begin{bmatrix} 0 & 4 \\ 3 & 6 \end{bmatrix} = A$

$\det A = 0 - 12 = -12$

$A^{-1} = \frac{1}{-12}\begin{bmatrix} 6 & -4 \\ -3 & 0 \end{bmatrix}$

10) $\begin{bmatrix} -2 & 5 \\ -7 & -7 \end{bmatrix} = A$

$\det A = 14 + 35 = 49$

$A^{-1} = \frac{1}{49}\begin{bmatrix} -7 & -5 \\ 7 & -2 \end{bmatrix}$

Solve each equation or state if there is no unique solution.

11) $\begin{bmatrix} -6 & -4 \\ -5 & -5 \end{bmatrix} C = \begin{bmatrix} 28 \\ -15 \end{bmatrix}$ $C = \begin{bmatrix} -8 \\ -5 \end{bmatrix}$

$A^{-1} = \frac{1}{30-20}\begin{bmatrix} -5 & -4 \\ -5 & -6 \end{bmatrix}$

$C = \frac{1}{10}\begin{bmatrix} -5 & -4 \\ -5 & -6 \end{bmatrix}\begin{bmatrix} 28 \\ -15 \end{bmatrix}$

$C = \frac{1}{10}\begin{bmatrix} (-5)(28)+(-4)(-15) \\ (-5)(28)+(-6)(-15) \end{bmatrix} = \frac{1}{10}\begin{bmatrix} -140+60 \\ -140+90 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} -80 \\ -50 \end{bmatrix}$

12) $\begin{bmatrix} -9 & 9 \\ -5 & 5 \end{bmatrix} Y = \begin{bmatrix} 18 & 27 \\ 10 & 15 \end{bmatrix}$

$\det A = -45 - (-45) = 0$

No unique solution.

13) $\begin{bmatrix} 5 \\ -19 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ -7 & 11 \end{bmatrix} B + \begin{bmatrix} -5 \\ -2 \end{bmatrix}$

$- \begin{bmatrix} -5 \\ -2 \end{bmatrix}$

$C \begin{bmatrix} 10 \\ -17 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ -7 & 11 \end{bmatrix} B$ ← A

$\det A = 22 - 28 = -6$

$A^{-1} = \frac{-1}{6} \begin{bmatrix} 11 & 4 \\ 7 & 2 \end{bmatrix}$

$B = A^{-1} C$

$= -\frac{1}{6} \begin{bmatrix} 11 & 4 \\ 7 & 2 \end{bmatrix} \begin{bmatrix} 10 \\ -17 \end{bmatrix}$

$= -\frac{1}{6} \begin{bmatrix} (11)(10)+(4)(-17) \\ (7)(10)+2(-17) \end{bmatrix} = -\frac{1}{6} \begin{bmatrix} 110-68 \\ 70-34 \end{bmatrix} = -\frac{1}{6} \begin{bmatrix} 42 \\ 36 \end{bmatrix} = \begin{bmatrix} -7 \\ -6 \end{bmatrix}$

14) $\begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} A + \begin{bmatrix} -6 & -7 \\ 5 & -11 \end{bmatrix} = \begin{bmatrix} 4 & -22 \\ 1 & -17 \end{bmatrix}$

$- \begin{bmatrix} -6 & -7 \\ 5 & -11 \end{bmatrix} - \begin{bmatrix} -6 & -7 \\ 5 & -11 \end{bmatrix}$

B →

$\begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} A = \begin{bmatrix} 10 & -15 \\ -4 & -6 \end{bmatrix}$

$\det B = 12 - 4 = 8$

$B^{-1} = \frac{1}{8} \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}$

$A = \frac{1}{8} \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 10 & -15 \\ -4 & -6 \end{bmatrix}$

$= \frac{1}{8} \begin{bmatrix} 40-8 & -60-12 \\ 20-12 & -30-18 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 32 & -72 \\ 8 & -48 \end{bmatrix}$

$A = \begin{bmatrix} 4 & -9 \\ 1 & -6 \end{bmatrix}$

For each matrix state if an inverse exists.

15) $\begin{bmatrix} 5 & -5 & 5 \\ -1 & 3 & 0 \\ -2 & 2 & -2 \end{bmatrix}$

$5(-6-0) + 5(2-0) + 5(-2+6)$

$5(-6) + 5(2) + 5(4)$

$-30 + 10 + 20 = \boxed{0}$

No, no inverse b/c the determinant = 0

Find the inverse of each matrix.

16) $\begin{bmatrix} 0 & -3 & 1 \\ -3 & 5 & 2 \\ 1 & -4 & 3 \end{bmatrix}$

$\det \rightarrow 0(15+8) + 3(-9-2) + 1(12-5)$

$\rightarrow 0 + 3(-11) + 7 = -33 + 7 = -26$

$\begin{bmatrix} 0 & -3 & 1 \\ -3 & 5 & -4 \\ 1 & 2 & 3 \end{bmatrix}$

$A^{-1} = \frac{-1}{26} \begin{bmatrix} 23 & 5 & -11 \\ 11 & -1 & -3 \\ 7 & -3 & -9 \end{bmatrix}$

Solve each system.

17) $-48x - 75y = -21$
$7x + 25y = -11$

A

① $\begin{bmatrix} -48 & -75 \\ 7 & 25 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -21 \\ -11 \end{bmatrix}$

② $\det A = (-48)(25) - (-75)(7) = -675$

③ $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-675}\begin{bmatrix} 25 & 75 \\ -7 & -48 \end{bmatrix}\begin{bmatrix} -21 \\ -11 \end{bmatrix}$

$= -\frac{1}{675}\begin{bmatrix} 25(-21) + 75(-11) \\ -7(-21) + -48(11) \end{bmatrix} = -\frac{1}{675}\begin{bmatrix} -1350 \\ 675 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

18) $5x + 4y - 4z = 6$
$-2y + z = 10$
$6x + 6y + 5z = 22$

$\begin{bmatrix} 5 & 4 & -4 \\ 0 & -2 & 1 \\ 6 & 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 22 \end{bmatrix}$

A → $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ← X, $\begin{bmatrix} 6 \\ 10 \\ 22 \end{bmatrix}$ ← B

$X = A^{-1}B \rightarrow X = \begin{bmatrix} 6 \\ -4 \\ 2 \end{bmatrix}$

19) $-6y - 6z = 0$
$-5x + 5y = 10$
$4x - y + 3z = -8$

$y + z = 0$
$+ \quad x - y = -2$
$x + z = -2$

$x = -2 - z$

$4(-2 - z) - y + 3z = -8$

$-8 - 4z - y + 3z = -8$

$-z - y = 0$

(No unique solution)

Infinite solutions

Pre Calculus Name_____

Solving Matrix Equations Notes

Date_____ Period____

1) What is the determinant of this 2x2 matrix?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$ad - bc$

Evaluate the determinant of each matrix.

2) $\begin{bmatrix} 1 & -3 \\ -4 & 3 \end{bmatrix}$

-9

3) $\begin{bmatrix} -4 & -4 \\ 2 & 2 \end{bmatrix}$

0

Evaluate each determinant.

4) $\begin{vmatrix} 0 & 5 \\ -2 & -1 \end{vmatrix}$

10

5) $\begin{vmatrix} -5 & 4 \\ 3 & 3 \end{vmatrix}$

-27

For each matrix state if an inverse exists. Inverses do not exist if the determinant is zero.

6) $\begin{bmatrix} -3 & -6 \\ -2 & -9 \end{bmatrix}$

Yes

7) $\begin{bmatrix} -2 & 4 \\ 7 & -14 \end{bmatrix}$ No

8) What is the inverse of the following 2x2 matrix?

$$\frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Find the inverse of each matrix.

9) $\begin{bmatrix} 0 & 4 \\ 3 & 6 \end{bmatrix}$

$$-\frac{1}{12} \cdot \begin{bmatrix} 6 & -4 \\ -3 & 0 \end{bmatrix}$$

10) $\begin{bmatrix} -2 & 5 \\ -7 & -7 \end{bmatrix}$

$$\frac{1}{49} \cdot \begin{bmatrix} -7 & -5 \\ 7 & -2 \end{bmatrix}$$

Solve each equation or state if there is no unique solution.

11) $\begin{bmatrix} -6 & 4 \\ 5 & -5 \end{bmatrix} C = \begin{bmatrix} 28 \\ -15 \end{bmatrix}$

$$\begin{bmatrix} -8 \\ -5 \end{bmatrix}$$

12) $\begin{bmatrix} -9 & 9 \\ -5 & 5 \end{bmatrix} Y = \begin{bmatrix} 18 & 27 \\ 10 & 15 \end{bmatrix}$

No unique solution

13) $\begin{bmatrix} 5 \\ -19 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ -7 & 11 \end{bmatrix} B + \begin{bmatrix} -5 \\ -2 \end{bmatrix}$

$\begin{bmatrix} -7 \\ -6 \end{bmatrix}$

14) $\begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} A + \begin{bmatrix} -6 & -7 \\ 5 & -11 \end{bmatrix} = \begin{bmatrix} 4 & -22 \\ 1 & -17 \end{bmatrix}$

$\begin{bmatrix} 4 & -9 \\ 1 & -6 \end{bmatrix}$

For each matrix state if an inverse exists.

15) $\begin{bmatrix} 5 & -5 & 5 \\ -1 & 3 & 0 \\ -2 & 2 & -2 \end{bmatrix}$

No

Find the inverse of each matrix.

16) $\begin{bmatrix} 0 & -3 & 1 \\ -3 & 5 & 2 \\ 1 & -4 & 3 \end{bmatrix}$

$-\frac{1}{26} \cdot \begin{bmatrix} 23 & 5 & -11 \\ 11 & -1 & -3 \\ 7 & -3 & -9 \end{bmatrix}$

Solve each system.

17) $-48x - 75y = -21$
$7x + 25y = -11$

$(2, -1)$

18) $5x + 4y - 4z = 6$
$-2y + z = 10$
$6x + 6y + 5z = 22$

$(6, -4, 2)$

19) $-6y - 6z = 0$
$-5x + 5y = 10$
$4x - y + 3z = -8$

No unique solution