$A = [\quad]$ det A or $|A|$

Pre Calculus Name________________________________

Solving Matrix Equations Notes

Date________________ Period____

1) What is the determinant of this 2x2 matrix?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$ad - bc$

Evaluate the determinant of each matrix.

2) $\begin{bmatrix} 1 & -3 \\ -4 & 3 \end{bmatrix}$

$(1)(3) - (-4)(-3)$

$3 - 12$

-9

3) $\begin{bmatrix} -4 & -4 \\ 2 & 2 \end{bmatrix}$

$(-4)(2) - (-4)(2)$

$-8 - -8 = 0$

Evaluate each determinant.

4) $\begin{vmatrix} 0 & 5 \\ -2 & -1 \end{vmatrix}$

$0 - -10$

10

5) $\begin{vmatrix} -5 & 4 \\ 3 & 3 \end{vmatrix}$

$-15 - 12 = -27$

det 0 → matrix is non-invertible

For each matrix state if an inverse exists. Inverses do not exist if the determinant is zero.

6) $\begin{bmatrix} -3 & -6 \\ -2 & -9 \end{bmatrix}$ yes, inverse exists

$27 - 12 = 15$

7) $\begin{bmatrix} -2 & 4 \\ 7 & -14 \end{bmatrix}$ No inverse exists

$28 - 28 = 0$

8) What is the inverse of the following 2x2 matrix?

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Find the inverse of each matrix.

9) $\begin{bmatrix} 0 & 4 \\ 3 & 6 \end{bmatrix} = A$

$$\frac{1}{0-12}\begin{bmatrix} 6 & -4 \\ -3 & 0 \end{bmatrix} = A^{-1}$$

10) $\begin{bmatrix} -2 & 5 \\ -7 & -7 \end{bmatrix} = A$

$$A^{-1} = \frac{1}{14+35}\begin{bmatrix} -7 & -5 \\ 7 & -2 \end{bmatrix}$$

Solve each equation or state if there is no unique solution.

11) $\underbrace{\begin{bmatrix} -6 & 4 \\ 5 & -5 \end{bmatrix}}_{A} C = \underbrace{\begin{bmatrix} 28 \\ -15 \end{bmatrix}}_{y}$ $\qquad C = \begin{bmatrix} -8 \\ -5 \end{bmatrix}$

$Ac = y$

$A^{-1}Ac = A^{-1}y$

$\boxed{C = A^{-1}y}$

$$\underbrace{\frac{1}{30-20}\begin{bmatrix} -5 & -4 \\ -5 & -6 \end{bmatrix}}_{A^{-1}}\underbrace{\begin{bmatrix} 28 \\ -15 \end{bmatrix}}_{y} = \frac{1}{10}\begin{bmatrix} -140+60 \\ -140+90 \end{bmatrix}$$

$$\frac{1}{10}\begin{bmatrix} -80 \\ -50 \end{bmatrix}$$

12) $\begin{bmatrix} -9 & 9 \\ -5 & 5 \end{bmatrix} Y = \begin{bmatrix} 18 & 27 \\ 10 & 15 \end{bmatrix}$

det of A is $(5)(-9)-(-5)(9)$

$-45+45$

$=0$

$\boxed{\text{No unique solution}}$

13) $\begin{bmatrix} 5 \\ -19 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ -7 & 11 \end{bmatrix} B + \begin{bmatrix} -5 \\ -2 \end{bmatrix}$

$-\begin{bmatrix} -5 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 10 \\ -17 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ -7 & 11 \end{bmatrix} B$

$\frac{1}{22-28} \begin{bmatrix} 11 & 4 \\ 7 & 2 \end{bmatrix} \begin{bmatrix} 10 \\ -17 \end{bmatrix} = B$

$\frac{1}{-6} \begin{bmatrix} 110 - 68 \\ 70 - 34 \end{bmatrix} \rightarrow \frac{1}{-6} \begin{bmatrix} 42 \\ 36 \end{bmatrix} \rightarrow B = \begin{bmatrix} -7 \\ -6 \end{bmatrix}$

14) $\begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} A + \begin{bmatrix} -6 & -7 \\ 5 & -11 \end{bmatrix} = \begin{bmatrix} 4 & -22 \\ 1 & -17 \end{bmatrix}$

$\begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} A = \begin{bmatrix} 10 & -15 \\ -4 & -6 \end{bmatrix}$

$A = \frac{1}{8} \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 10 & -15 \\ -4 & -6 \end{bmatrix}$

$\downarrow$

$\frac{1}{8} \begin{bmatrix} 32 & -72 \\ 8 & -48 \end{bmatrix}$

$A = \begin{bmatrix} 4 & -9 \\ 1 & -6 \end{bmatrix}$

For each matrix state if an inverse exists.

15) $\begin{bmatrix} 5 & -5 & 5 \\ -1 & 3 & 0 \\ -2 & 2 & -2 \end{bmatrix}$ $\quad 5\begin{vmatrix} 3 & 0 \\ 2 & -2 \end{vmatrix} + 5\begin{vmatrix} -1 & 0 \\ -2 & -2 \end{vmatrix} + 5\begin{vmatrix} -1 & 3 \\ -2 & 2 \end{vmatrix}$

$5(-6-0)+5(2-0)+5(-2+6) = -30+10+20 \boxed{=0}$

Find the inverse of each matrix.

16) $\begin{bmatrix} 0 & -3 & 1 \\ -3 & 5 & 2 \\ 1 & -4 & 3 \end{bmatrix}$ $\quad -\frac{1}{26} \begin{bmatrix} 23 & 5 & -11 \\ 11 & -1 & -3 \\ 7 & -3 & -9 \end{bmatrix}$

Solve each system.

17) $-48x - 75y = -21$
$7x + 25y = -11$

$$\underbrace{\begin{bmatrix} -48 & -75 \\ 7 & 25 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} -21 \\ -11 \end{bmatrix}}_{b}$$

ask calculator to calculate

$$\boxed{X = A^{-1} \cdot b}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \text{ or } (2, -1)$$

18) $5x + 4y - 4z = 6$
$-2y + z = 10$
$6x + 6y + 5z = 22$

$$\underbrace{\begin{bmatrix} 5 & 4 & -4 \\ 0 & -2 & 1 \\ 6 & 6 & 5 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 6 \\ 10 \\ 22 \end{bmatrix}}_{b}$$

$$A^{-1} b$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 2 \end{bmatrix}$$

19) $-6y - 6z = 0$
$-5x + 5y = 10$
$4x - y + 3z = -8$

$$\begin{bmatrix} 0 & -6 & -6 \\ -5 & 5 & 0 \\ 4 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ -8 \end{bmatrix}$$

No unique solution (the calculators knows this because $\det A = 0$)