

Solving Trig Equations Notes

Date _____

Period _____

Find all solutions to each equation in radians.

$$1) \frac{-3\sqrt{2}}{6} = \frac{6\cos\theta}{6}$$

$$-\frac{\sqrt{2}}{2} = \cos\theta$$

$$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \cos^{-1}\cos\theta$$

$$\left\{\frac{3\pi}{4}, \frac{5\pi}{4}\right\} = \theta$$

$$+ 2\pi n$$

$$\{n = \text{all integers}\}$$

$$2) \frac{-2 + \tan\theta}{+2} = \frac{-6 - \sqrt{3}}{3} + \frac{6}{3}$$

$$\tan\theta = -\frac{\sqrt{3}}{3}$$

$$\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$$

$$\left\{-\frac{\pi}{6} + \pi n\right\}$$

$$\left\{\frac{5\pi}{6} + \pi n\right\} \text{ or } \left\{\frac{11\pi}{6} + \pi n\right\}$$

$$3) \frac{-2\sqrt{3}}{-2} = \frac{-2\cos\theta}{-2}$$

$$\sqrt{3} = \cos\theta$$

$$\cos^{-1}\sqrt{3} = \theta$$

↑ undefined so

there is no solution

$$4) -1 + \sin -4\theta = \frac{-2 - \sqrt{3}}{2} + \frac{2}{2}$$

$$\sin(-4\theta) = -\frac{\sqrt{3}}{2}$$

$$(-4\theta) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$\frac{-4\theta}{-4} = \left\{ \frac{-\pi}{3}, \frac{4\pi}{3} \right\} + \frac{2\pi n}{-4}$$

$$\theta = \left\{ \frac{+\pi}{12}, \frac{-\pi}{3} \right\} - \frac{\pi n}{2}$$

$$5) -4 + \tan \frac{\theta}{3} = -5 + 4$$

$$\tan\left(\frac{\theta}{3}\right) = -1$$

$$\frac{\theta}{3} = \tan^{-1} -1$$

$$\frac{\theta}{3} = -\frac{\pi}{4} + \pi n$$

$$\theta = -\frac{3\pi}{4} + 3\pi n$$

$$6) -1 + \sin\left(\theta + \frac{11\pi}{6}\right) = \frac{-2 + \sqrt{2}}{2} + \frac{2}{2}$$

$$\sin\left(\theta + \frac{11\pi}{6}\right) = \frac{\sqrt{2}}{2}$$

$$\theta + \frac{11\pi}{6} = \sin^{-1} \frac{\sqrt{2}}{2}$$

$$\theta + \frac{11\pi}{6} = \left\{ \frac{\pi}{4}, \frac{3\pi}{4} \right\} + 2\pi n$$

$$\frac{-11\pi}{6} \quad \frac{-11\pi}{6} \quad \frac{-11\pi}{6} \quad \rightarrow$$

$$\frac{1}{4} - \frac{11}{6} = \frac{3}{12} - \frac{22}{12} = \frac{-19}{12}$$

$$\frac{3}{4} - \frac{11}{6} = \frac{9}{12} - \frac{22}{12} = \frac{-13}{12}$$

Solve each equation for $0 \leq \theta < 2\pi$.

$$7) \frac{4 = \sin^2 \theta + 3}{-3 \quad -3}$$

$$\sqrt{1} = \sqrt{\sin^2 \theta}$$

$$\pm 1 = \sin \theta$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$8) 3\cos^2 \theta = -1 + 4\cos^2 \theta$$

$$\frac{-3\cos^2 \theta \quad -3\cos^2 \theta}{-3\cos^2 \theta \quad -3\cos^2 \theta}$$

$$0 = -1 + \cos^2 \theta$$

$$0 = \cos^2 \theta - 1$$

$$0 = (\cos \theta + 1)(\cos \theta - 1)$$

$$\begin{array}{cc} \downarrow & \downarrow \\ \cos \theta = -1 & \cos \theta = 1 \end{array}$$

$$\theta = \{ \pi, 0 \}$$

$$9) 2\cos \theta \sin \theta = -\cos \theta$$

$$\frac{+\cos \theta \quad +\cos \theta}{+\cos \theta \quad +\cos \theta}$$

$$2\cos \theta \sin \theta + \cos \theta = 0$$

$$\cos \theta (2\sin \theta + 1) = 0$$

↓

↓

$$\cos \theta = 0 \quad \sin \theta = -\frac{1}{2}$$

↓

↓

$$\theta = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$

$$\theta = \left\{ \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$

$[0, 2\pi)$

10) $\sin \theta \tan \theta - \sqrt{3} \sin \theta = 0$

$$\sin \theta (\tan \theta - \sqrt{3}) = 0$$

$$\sin \theta = 0 \quad \tan \theta = \sqrt{3}$$

$$\theta = \{0, \pi\} \quad \theta = \left\{ \frac{\pi}{3}, \frac{4\pi}{3} \right\}$$

11) $-4\sin^2 \theta = 1 - 3\sin \theta - 2\sin^2 \theta$

$$0 = 1 - 3\sin \theta + 2\sin^2 \theta$$

$$0 = 2\sin^2 \theta - 3\sin \theta + 1$$

$$0 = (2\sin \theta - 1)(\sin \theta - 1)$$

$$\sin \theta = \frac{1}{2} \quad \sin \theta = 1$$

$$\theta = \left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\} \quad \theta = \left\{ \frac{\pi}{2} \right\}$$

12) $4\cos \theta - 4\cos^2 \theta = 1$

$$4\cos \theta - 4\cos^2 \theta - 1 = 0$$

$$0 = 4\cos^2 \theta - 4\cos \theta + 1$$

$$0 = (2\cos \theta - 1)(2\cos \theta - 1)$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$$

Solving Trig Equations Notes

Find all solutions to each equation in radians.

1) $-3\sqrt{2} = 6\cos \theta$

$$\left\{ \frac{3\pi}{4} + 2\pi n, \frac{5\pi}{4} + 2\pi n \right\}$$

2) $-2 + \tan \theta = \frac{-6 - \sqrt{3}}{3}$

$$\left\{ \frac{11\pi}{6} + \pi n \right\}$$

3) $-2\sqrt{3} = -2\cos \theta$

No solution.

$$4) -1 + \sin -4\theta = \frac{-2 - \sqrt{3}}{2}$$

$$\left\{ -\frac{5\pi}{12} - \frac{\pi n}{2}, -\frac{\pi}{3} - \frac{\pi n}{2} \right\}$$

$$5) -4 + \tan \frac{\theta}{3} = -5$$

$$\left\{ \frac{21\pi}{4} + 3\pi n \right\}$$

$$6) -1 + \sin \left(\theta + \frac{11\pi}{6} \right) = \frac{-2 + \sqrt{2}}{2}$$

$$\left\{ -\frac{19\pi}{12} + 2\pi n, -\frac{13\pi}{12} + 2\pi n \right\}$$

Solve each equation for $0 \leq \theta < 2\pi$.

7) $4 = \sin^2 \theta + 3$

$$\left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$

8) $3\cos^2 \theta = -1 + 4\cos^2 \theta$

$$\{0, \pi\}$$

9) $2\cos \theta \sin \theta = -\cos \theta$

$$\left\{ \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6} \right\}$$

$$10) \sin \theta \tan \theta - \sqrt{3} \sin \theta = 0$$

$$\left\{ 0, \frac{\pi}{3}, \pi, \frac{4\pi}{3} \right\}$$

$$11) -4\sin^2 \theta = 1 - 3\sin \theta - 2\sin^2 \theta$$

$$\left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6} \right\}$$

$$12) 4\cos \theta - 4\cos^2 \theta = 1$$

$$\left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$$