

## Solving Trig Equations Notes

Find all solutions to each equation in radians.

$$1) \frac{-3\sqrt{2}}{6} = \frac{6\cos\theta}{6}$$

$$-\frac{\sqrt{2}}{2} = \cos\theta$$

$$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \theta$$

$$\theta = \left\{ \frac{3\pi}{4}, \frac{5\pi}{4} \right\} + 2\pi n$$

(n represent integers)

$$2) -2 + \tan\theta = \frac{-6 - \sqrt{3}}{3}$$

$$\tan\theta = \frac{-6 - \sqrt{3}}{3} + \frac{6}{3}$$

$$\tan\theta = -\frac{\sqrt{3}}{3}$$

$$\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$$

$$\theta = \left\{ \frac{5\pi}{6} \right\} + \pi n$$

$$3) \frac{-2\sqrt{3}}{-2} = \frac{-2\cos\theta}{-2}$$

$$\sqrt{3} = \cos\theta$$

$$\cos^{-1}\sqrt{3} = \theta$$

No solution

$$4) -1 + \sin(-4\theta) = \frac{-2 - \sqrt{3}}{2} + \frac{2}{2}$$

$$\sin(-4\theta) = \frac{-\sqrt{3}}{2}$$

$$-4\theta = \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$

$$\frac{-4\theta}{-4} = \left\{ \frac{-\pi}{3}, \frac{4\pi}{3} \right\} \frac{+2\pi n}{-4}$$

$$\theta = \left\{ \frac{\pi}{12}, -\frac{\pi}{3} \right\} - \frac{\pi n}{2}$$

$$5) -4 + \tan \frac{\theta}{3} = -5$$

$$\tan\left(\frac{\theta}{3}\right) = -1$$

$$\frac{\theta}{3} = \tan^{-1}(-1)$$

$$\frac{\theta}{3} = \left\{ -\frac{\pi}{4} \right\} + \pi n$$

$$\theta = \left\{ -\frac{3\pi}{4} \right\} + 3\pi n$$

$$6) -1 + \sin\left(\theta + \frac{11\pi}{6}\right) = \frac{-2 + \sqrt{2}}{2} + \frac{2}{2}$$

$$\sin\left(\theta + \frac{11\pi}{6}\right) = \frac{\sqrt{2}}{2}$$

$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \theta + \frac{11\pi}{6}$$

$$\left\{ \frac{\pi}{4}, \frac{3\pi}{4} \right\} + 2\pi n = \theta + \frac{11\pi}{6}$$

$$\theta = \left\{ -\frac{19\pi}{12}, -\frac{13\pi}{12} \right\} + 2\pi n$$

$$\frac{3}{3} \cdot \frac{1}{4} - \frac{11}{6} \cdot \frac{2}{2} = \frac{3-22}{12} = -\frac{19}{12}$$

$$\frac{3}{3} \cdot \frac{3}{4} - \frac{11}{6} \cdot \frac{2}{2} = \frac{9-22}{12} = -\frac{13}{12}$$

Solve each equation for  $0 \leq \theta < 2\pi$ .

7)  $4 = \sin^2 \theta + 3$

$$1 = \sin^2 \theta$$

$$\pm \sqrt{1} = \sin \theta$$

$$\theta = \sin^{-1}(\pm 1)$$

$$\theta = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$

8)  $3\cos^2 \theta = -1 + 4\cos^2 \theta$

$$\frac{-3\cos^2 \theta \quad -3\cos^2 \theta}{-3\cos^2 \theta \quad -3\cos^2 \theta}$$

$$0 = -1 + \cos^2 \theta$$

$$\sqrt{\cos^2 \theta} = \sqrt{1}$$

$$\cos \theta = \pm 1$$

$$\theta = \cos^{-1}(\pm 1)$$

$$\theta = \{0, \pi\}$$

9)  $2\cos \theta \sin \theta = -\cos \theta$

$$2\cos \theta \sin \theta + \cos \theta = 0$$

$$\cos \theta (2\sin \theta + 1) = 0$$

$$\cos \theta = 0 \quad 2\sin \theta + 1 = 0$$

$$\theta = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$

$$2\sin \theta = -1$$

$$\sin \theta = -\frac{1}{2}$$

$$\theta = \left\{ \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$

$$10) \sin \theta \tan \theta - \sqrt{3} \sin \theta = 0$$

$$\sin \theta (\tan \theta - \sqrt{3}) = 0$$

$$\sin \theta = 0 \quad \tan \theta = \sqrt{3}$$

$$\theta = \{0, \pi\} \quad \theta = \left\{ \frac{\pi}{3}, \frac{4\pi}{3} \right\}$$

$$11) -4\sin^2 \theta = 1 - 3\sin \theta - 2\sin^2 \theta$$

$$0 = 1 - 3\sin \theta + 2\sin^2 \theta$$

$$0 = 2\sin^2 \theta - 3\sin \theta + 1$$

$$0 = (2\sin \theta - 1)(\sin \theta - 1)$$

$$\sin \theta = \frac{1}{2} \quad \sin \theta = 1$$

$$\theta = \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2} \right\}$$

$$12) 4\cos \theta - 4\cos^2 \theta = 1$$

$$4\cos \theta - 4\cos^2 \theta - 1 = 0$$

$$4\cos^2 \theta - 4\cos \theta + 1 = 0$$

$$(2\cos \theta - 1)^2 = 0$$

$$\cos \theta = \frac{1}{2} \quad \theta = \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$$