

Trig Equations Review

Find all the solutions of the equation.

Date: _____

* Calculator use
only on 3, 5, 10

#1 - 3

$$1.) 2\cos^2 x + (2 - \sqrt{3})\cos x - \sqrt{3} = 0$$

$$2\cos^2 x + 2\cos x - \sqrt{3}\cos x - \sqrt{3} = 0$$

$$2\cos x (\cos x + 1) - \sqrt{3} (\cos x + 1) = 0$$

$$(2\cos x - \sqrt{3})(\cos x + 1) = 0$$

$$\cos x = \frac{\sqrt{3}}{2} \quad \cos x = -1$$

Use \cos^{-1} or \arccos

$$x = \left\{ \frac{\pi}{6}, \frac{11\pi}{6} \right\} + 2\pi n \quad x = \{ \pi \} + 2\pi n$$

$$2.) \cot^2 x - 3 = 0$$

$$\sqrt{\cot^2 x} = \sqrt{3}$$

$$\cot x = \pm \sqrt{3}$$

$$\tan x = \pm \frac{\sqrt{3}}{3}$$

$$x = \left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\} + \pi n$$

$$3.) \sec^5 x - 25 \sec x = 0$$

$$\sec x (\sec^4 x - 25) = 0$$

$$\sec x (\sec^2 x + 5)(\sec^2 x - 5) = 0$$

$$\cancel{\sec x} = 0$$

$$\sec^2 x = 5$$

extraneous

$$\sec x = \pm\sqrt{5} \longrightarrow \cos x = \pm\frac{1}{\sqrt{5}}$$

Find all the solutions of the equation in the interval $[0, 2\pi)$. #4-5

$$4.) 2 \sin 2x + 2 \cos x = \sqrt{3} + 2\sqrt{3} \sin x \quad \sin(2x) = 2 \sin x \cos x$$

$$4 \sin x \cos x + 2 \cos x - 2\sqrt{3} \sin x - \sqrt{3} = 0$$

$$2 \cos x (2 \sin x + 1) - \sqrt{3} (2 \sin x + 1) = 0$$

$$(2 \cos x - \sqrt{3})(2 \sin x + 1) = 0$$

$$\cos x = \frac{\sqrt{3}}{2} \quad \sin x = -\frac{1}{2}$$

$$x = \left\{ \frac{\pi}{6}, \frac{11\pi}{6} \right\} \quad x = \left\{ \frac{7\pi}{6} \right\}$$

$$5.) 6 \sec^2 x + 13 \sec x - 28 = 0$$

$$(2 \sec x + 7)(3 \sec x - 4) = 0$$

$$\sec x = -\frac{7}{2} \quad \sec x = \frac{4}{3}$$

$$\cos x = -\frac{2}{7} \quad \cos x = \frac{3}{4}$$

$$x = \left\{ 41.410, 106.602, 253.398, 318.59 \right\}$$

Solving Trigonometric Equations Review

Find all solutions to each equation in radians. #6, 7

6) $1 - \frac{1}{5} \cdot \cos \theta = \frac{4}{5}$

$$-\frac{1}{5} \cos \theta = -\frac{1}{5}$$

$$\cos \theta = 1$$

$$\theta = \{0\} + 2\pi n$$

7) $-1 - 3\csc\left(-3\theta + \frac{5\pi}{4}\right) = -3 - 2\csc\left(-3\theta + \frac{5\pi}{4}\right)$

$$+1 + 2\csc\left(-3\theta + \frac{5\pi}{4}\right) = +1 + 2\csc\left(-3\theta + \frac{5\pi}{4}\right)$$

$$+ \csc\left(-3\theta + \frac{5\pi}{4}\right) = +2$$

$$\sin\left(-3\theta + \frac{5\pi}{4}\right) = \frac{1}{2}$$

$$-3\theta + \frac{5\pi}{4} = \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\} + 2\pi n$$

$$\frac{2}{2} \frac{\pi}{6} - \frac{5\pi}{4} \cdot \frac{3}{3} \rightarrow \frac{2\pi - 15\pi}{12} \rightarrow \frac{-13\pi}{12}$$

$$\frac{2}{2} \frac{5\pi}{6} - \frac{5\pi}{4} \cdot \frac{3}{3} \rightarrow \frac{10\pi - 15\pi}{12} \rightarrow \frac{-5\pi}{12}$$

$$\frac{-3\theta}{-3} = \left\{\frac{-13\pi}{12}, \frac{-5\pi}{12}\right\} + \frac{2\pi n}{-3}$$

$$\theta = \left\{\frac{13\pi}{36}, \frac{5\pi}{36}\right\} - \frac{2\pi n}{3}$$

Solve each equation for $0 \leq \theta < 360$. #8

8) $-4 - \frac{7}{3} \cdot \tan(-3\theta + 45) = -\frac{13}{3} - 2\tan(-3\theta + 45)$

$$+4 + 2\tan(\cdot) \downarrow +4 + 2\tan(\cdot)$$

$\left(\frac{6}{3}\right) \qquad \qquad \left(\frac{12}{3}\right)$

$$-\frac{1}{3} \tan(-3\theta + 45) = -\frac{1}{3}$$

$$\tan(-3\theta + 45) = 1$$

$$-3\theta + 45 = 45 + 180n$$

$$-3\theta = 180n$$

$$\theta = -60n$$

$$\theta = \{0, 60, 120, 180, 240, 300\}$$

$$9.) \left(\sqrt{\frac{1}{2}(1 - \cos x)} = \frac{\sqrt{3}}{2} \right)^2 \quad x \in [0, 2\pi)$$

$$\frac{1}{2}(1 - \cos x) = \frac{3}{4}$$

$$1 - \cos x = \frac{3}{2}$$

$$-\cos x = \frac{1}{2} \rightarrow \cos x = -\frac{1}{2} \rightarrow x = \left\{ \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$$

$$10.) 6 \cos x = -5 \sin x \quad x \in [0, 360)$$

$$\frac{-5 \cos x}{-5 \cos x} = \frac{-5 \sin x}{-5 \cos x}$$

$$\frac{6}{-5} = \tan x$$

$$x = \{129.806^\circ, 309.806^\circ\}$$

$$x = \tan^{-1}\left(\frac{6}{-5}\right) \quad \left(\begin{array}{l} \text{this means} \\ x \text{ will be} \\ \text{in Q2, Q4} \end{array} \right)$$

$$11.) \frac{1 - \sin \theta}{\cos \theta} = 1 \quad \theta \in [0, 360)$$

$$(1 - \sin \theta)^2 = (\cos \theta)^2$$

$$1 - 2\sin \theta + \sin^2 \theta = \cos^2 \theta$$

$$1 - 2\sin \theta + \sin^2 \theta = 1 - \sin^2 \theta$$

$$2\sin^2 \theta - 2\sin \theta = 0$$

$$2\sin \theta (\sin \theta - 1) = 0$$

$$\sin \theta = 0 \quad \sin \theta = 1$$

$$\theta = \{0, 90, 180\}$$

$$12.) 3 \tan\left(\frac{x}{2} + \pi\right) = \sqrt{27} \quad x \in [0, 2\pi)$$

$$\frac{3 \tan\left(\frac{x}{2} + \pi\right)}{3} = \frac{3\sqrt{3}}{3}$$

$$\tan\left(\frac{x}{2} + \pi\right) = \sqrt{3}$$

$$\frac{x}{2} + \pi = \frac{\pi}{3} + \pi n$$

$$-\pi \downarrow = -\pi$$

$$\left(\frac{x}{2} = -\frac{2\pi}{3} + \pi n\right)^* 2$$

$$x = -\frac{4\pi}{3} + 2\pi n$$

$$x = \left\{ \frac{2\pi}{3} \right\}$$