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Name

1. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval [0,2] ?

(A) $\frac{26}{9}$

(B) $\frac{52}{9}$

(C) $\frac{26}{3}$

(D) $\frac{52}{3}$

(E) 24

---

2. If the function *f* given by $f(x) = x^3$ has an average value of 9 on the closed interval [0, *k*], then *k* =

(A) 3

(B) $3^{\frac{1}{2}}$

(C) $18^{\frac{1}{3}}$

(D) $36^{\frac{1}{4}}$

(E) $36^{\frac{1}{3}}$

---

3. The acceleration of a particle moving along the *x*-axis at time *t* is given by $a(t) = 6t - 2$. If the velocity is 25 when *t*=3 and the position is 10 when *t*=1, then the position *x(t)*=

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(A) $9t^2 + 1$

(B) $3t^2 - 2t + 4$

(C) $t^3 - t^2 + 4t + 6$

(D) $t^3 - t^2 + 9t - 20$

(E) $36t^3 - 4t^2 - 77t + 55$

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4. If the velocity of a particle moving along the *x*-axis is $v(t) = 2t - 4$ and if at *t*=0 its position is 4, then at any time *t* its position *x*(*t*) is

(A) t^2-4*t*

(B) t^2-4*t*-4

(C) t^2-4*t*+4

(D) 2t^2-4*t*

(E) 2t^2-4*t*+4

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5. The acceleration α of a body moving in a straight line is given in terms of time *t* byα=8−6t. If the velocity of the body is 25 at *t* = 1 and if *s(t)* is the distance of the body from the origin at time *t*, what is *s*(4) - *s*(2) ?

(A) 20

(B) 24

(C) 28

(D) 32

(E) 42

---

6. A particle moves along the *x*-axis. The velocity of the particle at time t is $6t - t^2$. What is the total distance traveled by the particle from time $t = 0$ to $t = 3$?

(A) 3

(B) 6

(C) 9

(D) 18

(E) 27

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7. A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

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(A) $e^2 - 1$

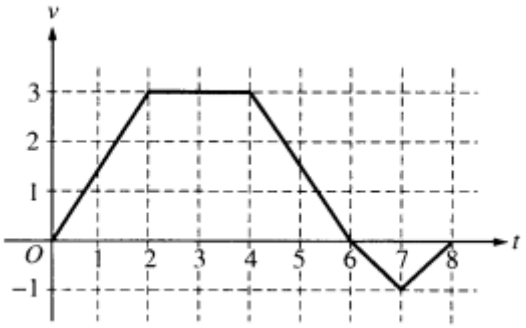
(B) $e - 1$

(C) $2e$

(D) e^2

(E) $\frac{e^3}{3}$

8.



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.
What is the total distance the bug traveled from $t = 0$ to $t = 8$

(A) 14

(B) 13

(C) 11

(D) 8

(E) 6

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9. A particle travels in a straight line with a constant acceleration of 3 meters per second per second. If the velocity of the particle is 10 meters per second at time 2 seconds, how far does the particle travel during the time interval when its velocity increases from 4 meters per second to 10 meters per second?

(A) 20 m

(B) 14 m

(C) 7 m

(D) 6 m

(E) 3 m

---

10. The rate of change of the volume, *V*, of water in a tank with respect to time, *t*, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) $V(t) = k\sqrt{t}$

(B) $V(t) = k\sqrt{V}$

(C) $\frac{dV}{dt} = k\sqrt{t}$

(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

(E) $\frac{dV}{dt} = k\sqrt{V}$

---

11. If $P(t)$ is the size of a population at time *t*, which of the following differential equations describes linear growth in the size of the population?

(A) $\frac{dP}{dt} = 200$

(B) $\frac{dP}{dt} = 200t$

(C) $\frac{dP}{dt} = 100t^2$

(D) $\frac{dP}{dt} = 200P$

(E) $\frac{dP}{dt} = 100P^2$

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12. A rumor spreads among a population of *N* people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If *p* denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time *t*, where *k* is a positive constant?

(A) $\frac{dp}{dt} = kp$

(B) $\frac{dp}{dt} = kp\,(N - p)$

(C) $\frac{dp}{dt} = kp\,(p - N)$

(D) $\frac{dp}{dt} = kt\,(N - t)$

(E) $\frac{dp}{dt} = kt\,(t - N)$

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13. If dy/dx=y$\sec^2$x and *y*=5 when *x*=0, then *y*=

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(A) $e^{\tan x} + 4$

(B) $e^{\tan x} + 5$

(C) $5e^{\tan x}$

(D) $\tan x + 5$

(E) $\tan x + 5e^{x}$

---

14. A curve has slope $2x + 3$ at each point (x,y) on the curve. Which of the following is an equation for this curve if it passes through the point (1,2) ?

(A) $y = 5x - 3$

(B) $y = x^2 + 1$

(C) $y = x^2 + 3x$

(D) $y = x^2 + 3x - 2$

(E) $y = 2x^2 + 3x - 3$

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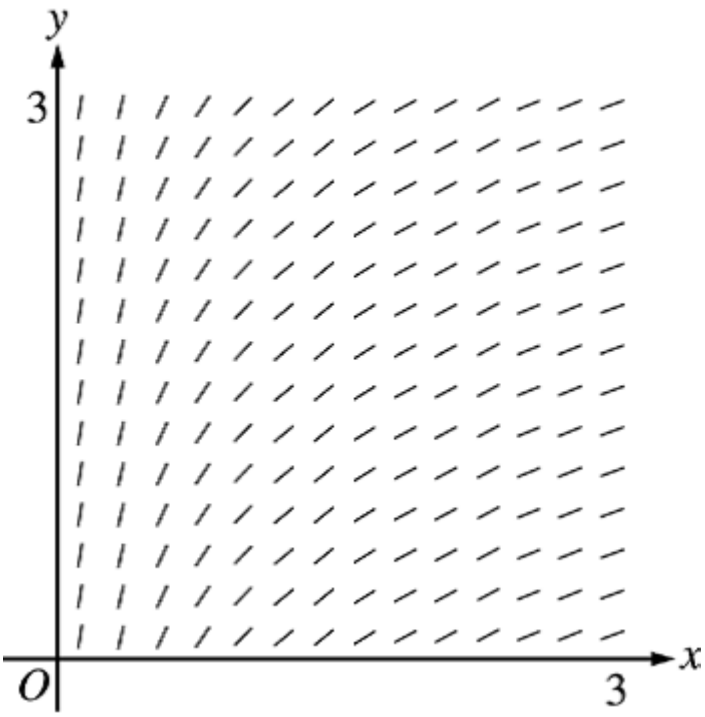
15.



The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$

(B) $y = e^x$

(C) $y = e^{-x}$

(D) $y = \cos x$

(E) $y = \ln x$

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16. Which of the following is a slope field for the differential equation $\frac{dy}{dx} = \frac{x}{y}$?

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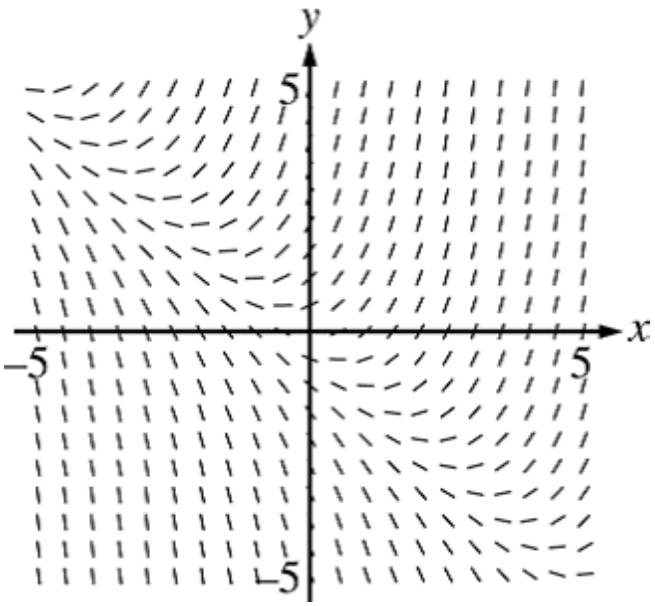
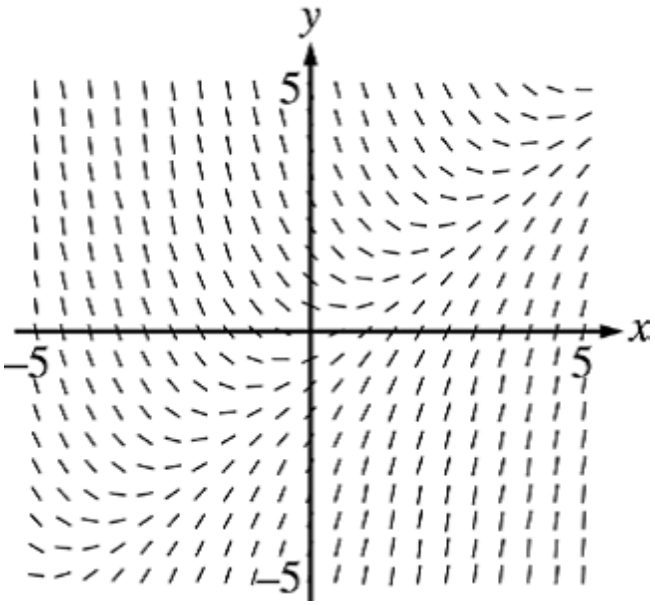
A



B



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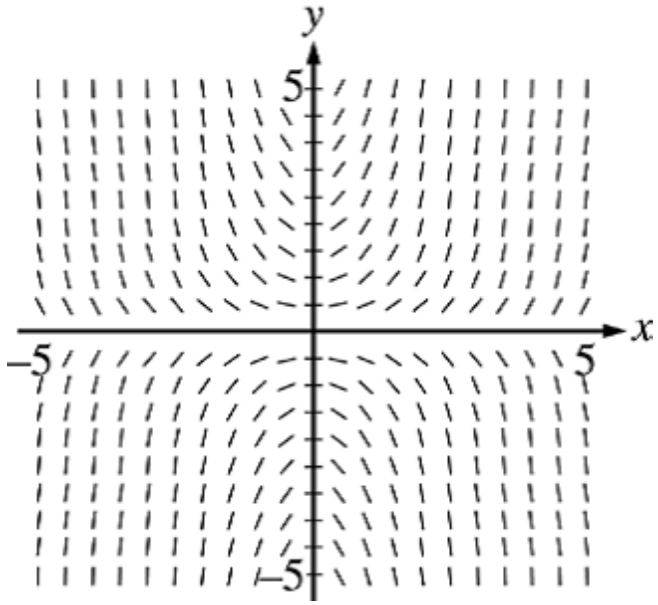
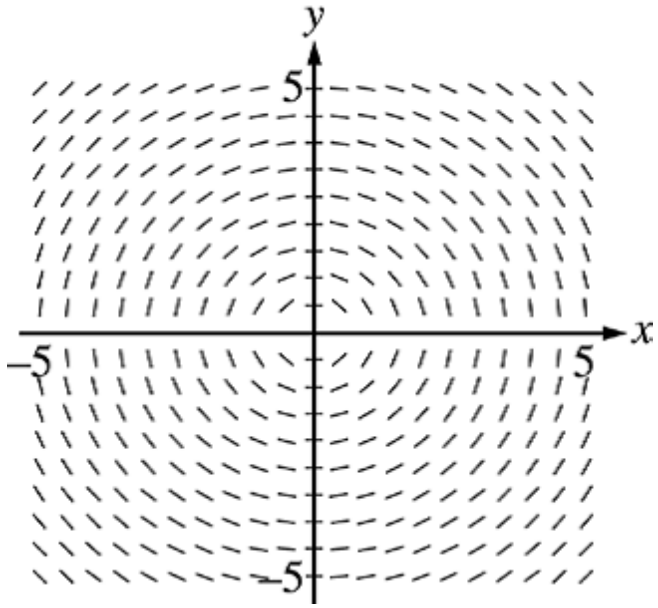
C



D



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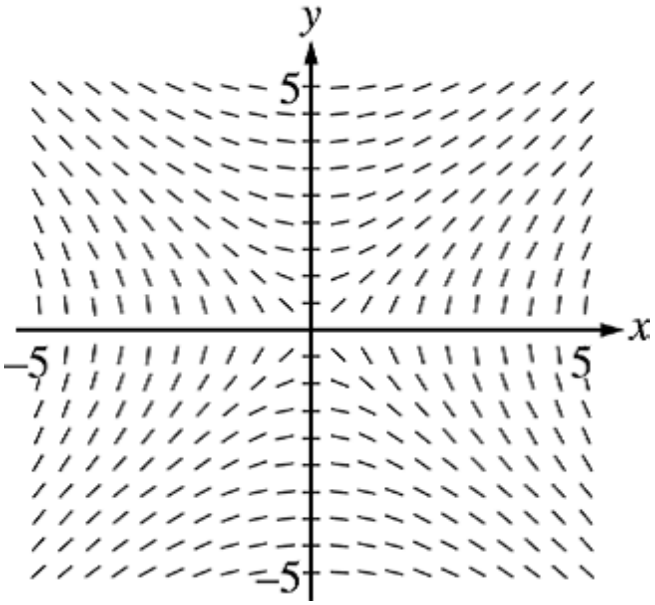
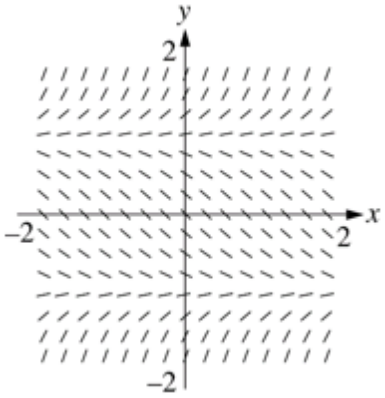
E

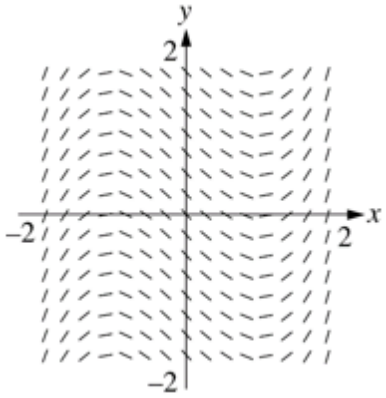


17. Which of the following could be the slope field for the differential equation $\frac{dy}{dx} = y^2 - 1$?

A



B



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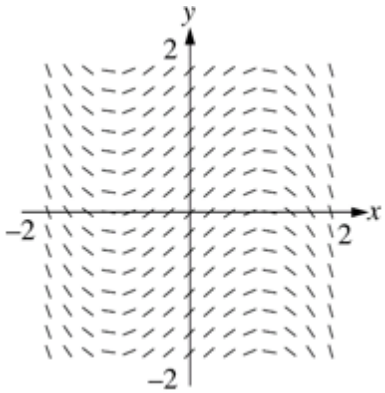
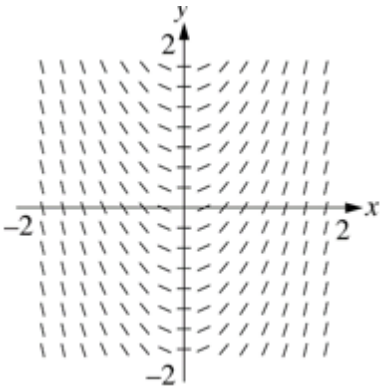
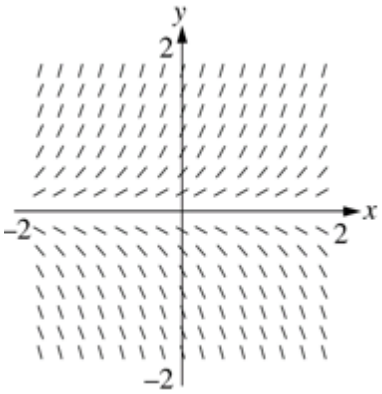
C



D



E



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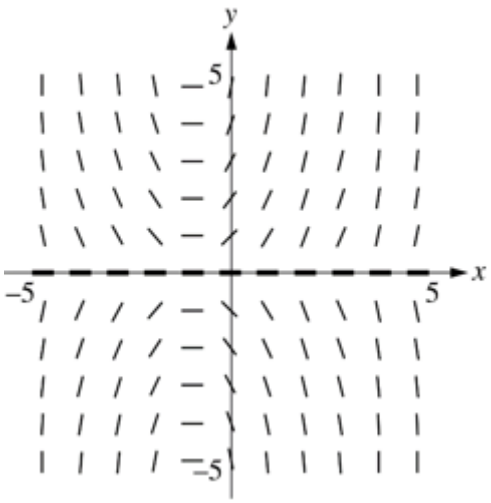
18.



Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = xy$

(B) $\frac{dy}{dx} = xy - y$

(C) $\frac{dy}{dx} = xy + y$

(D) $\frac{dy}{dx} = xy + x$

(E) $\frac{dy}{dx} = (x+1)^3$

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19. If $\frac{dy}{dx} = \sin x \cos^2 x$ and if *y*=0 when $x = \frac{\pi}{2}$, what is the value of *y* when *x*=0 ?

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(A) -1

(B) $-\frac{1}{3}$

(C) 0

(D) $\frac{1}{3}$

(E) 1

---

20. If dy/dx=2y^2 and if *y*=-1 when *x*=1, then when *x*=2, *y*=

(A) −2/3

(B) −1/3

(C) 0

(D) 1/3

(E) 2/3

---

21. Which of the following is the solution to the differential equation $\frac{dy}{dx} = 2\sin x$ with the initial condition $y(\pi) = 1$?

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(A) $y = 2\cos x + 3$

(B) $y = 2\cos x - 1$

(C) $y = -2\cos x + 3$

(D) $y = -2\cos x + 1$

(E) $y = -2\cos x - 1$

---

22. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$ with the initial condition $y(3) = -2$?

(A) $y = 2e^{-9+x^3/3}$

(B) $y = -2e^{-9+x^3/3}$

(C) $y = \sqrt{\frac{2x^3}{3}}$

(D) $y = \sqrt{\frac{2x^3}{3} - 14}$

(E) $y = -\sqrt{\frac{2x^3}{3} - 14}$

---

23. If $\frac{dy}{dt} = ky$ and *k* is a nonzero constant, then *y* could be

(A) $2e^{kty}$

(B) $2e^{kt}$

(C) $e^{kt}+3$

(D) $kty+5$

(E) $\frac{1}{2}ky^2+\frac{1}{2}$

24. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

$$f(x)=\begin{cases}\sqrt{9-x^2} & \text{for } -3\le x\le 0\\ -x+3\cos\left(\frac{\pi x}{2}\right) & \text{for } 0<x\le 4\end{cases}$$

Let f be the function defined above.

(a) Find the average rate of change of f on the interval $-3\le x\le 4$.

Please respond on separate paper, following directions from your teacher.

(b) Write an equation for the line tangent to the graph of f at $x = 3$.

Please respond on separate paper, following directions from your teacher.

(c) Find the average value of f on the interval $-3 \leq x \leq 4$.

Please respond on separate paper, following directions from your teacher.

(d) Must there be a value of x at which $f(x)$ attains an absolute maximum on the closed interval $-3 \leq x \leq 4$? Justify your answer.

Please respond on separate paper, following directions from your teacher.

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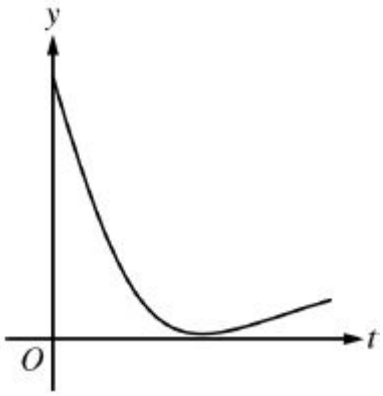
25. 5. During a chemical reaction, the function $y = f(t)$ models the amount of a substance present, in grams, at time t seconds. At the start of the reaction $(t = 0)$, there are 10 grams of the substance present. The function $y = f(t)$ satisfies the differential equation $\frac{dy}{dt} = -0.02y^2$.

(a) Use the line tangent to the graph of $y = f(t)$ at $t = 0$ to approximate the amount of the substance remaining at time $t = 2$ seconds.

Please respond on separate paper, following directions from your teacher.

(b) Using the given differential equation, determine whether the graph of *f* could resemble the following graph. Give a reason for your answer.



Please respond on separate paper, following directions from your teacher.

(c) Find an expression for $y = f(t)$ by solving the differential equation $\frac{dy}{dt} = -0.02y^2$ with the initial condition $f(0) = 10$.

Please respond on separate paper, following directions from your teacher.

(d) Determine whether the amount of the substance is changing at an increasing or a decreasing rate. Explain your reasoning.

Please respond on separate paper, following directions from your teacher.

26. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

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Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Consider the differential equation $\frac{dy}{dx} = 10 - 2y$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 0$. Use the tangent line to approximate $f(0.5)$.

Please respond on separate paper, following directions from your teacher.

(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(0, 2)$. Is the graph of $y = f(x)$ concave up or concave down at the point $(0, 2)$? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 2$.

Please respond on separate paper, following directions from your teacher.

(d) For the particular solution $y = f(x)$ found in part (c), find $\lim_{x \to \infty} f(x)$.

Please respond on separate paper, following directions from your teacher.

27. **CALCULUS AB**

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SECTION II, Part A

Time - 30 minutes

Number of questions - 2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

t (minutes)	0	1	5	6	8
$g(t)$ (cubic feet per minute)	12.8	15.1	20.5	18.3	22.7

1. Grain is being added to a silo. At time $t = 0$, the silo is empty. The rate at which grain is being added is modeled by the differentiable function g, where $g(t)$ is measured in cubic feet per minute for $0 \le t \le 8$ minutes. Selected values of $g(t)$ are given in the table above.

(a) Using the data in the table, approximate $g'(3)$. Using correct units, interpret the meaning of $g'(3)$ in the context of the problem.

Please respond on separate paper, following directions from your teacher.

(b) Write an integral expression that represents the total amount of grain added to the silo from time $t = 0$ to time $t = 8$. Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate the integral.

Please respond on separate paper, following directions from your teacher.

(c) The grain in the silo is spoiling at a rate modeled by $w(t) = 32 \cdot \sqrt{\sin\left(\frac{\pi t}{74}\right)}$, where $w(t)$ is measured in cubic feet per minute for $0 \le t \le 8$ minutes. Using the result from part (b), approximate the amount of unspoiled grain remaining in the silo at time $t = 8$.

Please respond on separate paper, following directions from your teacher.

(d) Based on the model in part (c), is the amount of unspoiled grain in the silo increasing or decreasing at time $t = 6$? Show the work that leads to your answer.

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Please respond on separate paper, following directions from your teacher.