tegrals 1 Test Review

Name Key

1. What is the average value of y for the part of the curve $y = 3x - x^2$, which is the *first quadrant*?

A -6

B -2

C $\frac{3}{2}$

D $\frac{9}{4}$

E $\frac{9}{2}$

$y = x(3-x)$

$\downarrow \quad \downarrow$

$0 \quad 3$

$$\frac{1}{3-0}\int_0^3 (3x - x^2)\,dx$$

$$\frac{1}{3}\left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^3$$

$$\frac{1}{3}\left(\frac{3}{2}(3)^2 - \frac{1}{3}(3)^3\right) - (0) = \frac{1}{3}\left(\frac{27}{2} - 9\right) \rightarrow \frac{1}{3}\left(\frac{9}{2}\right) \rightarrow \frac{3}{2}$$

---

2. If the function f given by $f(x) = x^3$ has an average value of 9 on the closed interval $[0, k]$, then $k =$

A 3

B $3\frac{1}{2}$

C $18\frac{1}{3}$

D $36\frac{1}{4}$

E $36\frac{1}{3}$

$$\frac{1}{k-0}\int_0^k x^3 = 9$$

$$\frac{1}{k}\left[\frac{1}{4}x^4\right]_0^k$$

$$\frac{1}{k}\left(\frac{1}{4}k^4 - 0\right)$$

$$\frac{1}{4k}(k^4) = \frac{1}{4}k^3 = 9 \rightarrow k^3 = 36 \quad k = \sqrt[3]{36}$$

---

3. The average (mean) value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

$$\frac{1}{2-0}\int_0^2 \sqrt{x}\,dx \qquad \sqrt{x} = x^{\frac{1}{2}}$$

$$\frac{1}{2}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^2$$

$$\frac{1}{2}\left(\frac{2}{3}(2)^{\frac{3}{2}} - \frac{2}{3}(0)^{\frac{3}{2}}\right) = \frac{1}{3}\sqrt{8} = \frac{1}{3}2\sqrt{2} = \frac{2\sqrt{2}}{3}$$

[redacted]

(A) $\frac{1}{3}\sqrt{2}$

(B) $\frac{1}{2}\sqrt{2}$

(C) $\frac{2}{3}\sqrt{2}$

(D) 1

(E) $\frac{4}{3}\sqrt{2}$

---

4. The average value of 1/x on the closed interval [1,3] is

(A) 12

(B) 23

(C) ln2/2

(D) ln3/2

(E) ln3

$$\frac{1}{3-1}\int_{\phi}^{3} \frac{1}{x}\,dx$$

$$\frac{1}{2}\ln|x|\Big|_1^3$$

$$\frac{1}{2}\ln(3) - \frac{1}{2}\ln(1)$$

$$= \frac{1}{2}\ln(3)$$

---

5. $\frac{d}{dx}\left(\int_0^{x^3} \ln\,(t^2+1)\;dt\right) =$

$$\ln\left((x^3)^2+1\right)\left(3x^2\right)$$

(A) $\frac{2x^3}{x^6+1}$

(B) $\frac{3x^2}{x^6+1}$

(C) $\ln\left(x^6+1\right)$

(D) $2x^3\ln\left(x^6+1\right)$

(E) $3x^2\ln\left(x^6+1\right)$

---

6. For all $x > 1$, if $f(x) = \int_t^x \frac{1}{t}dt$, then $f'(x) =$

(A) 1

$f'(x) = \frac{1}{x}$

(B) $\frac{1}{x}$

(C) $\ln x - 1$

(D) $\ln x$

(E) e^x

---

7. Let g be a function with first derivative given by $g'(x) = \int_0^x e^{-t^2}\, dt$. Which of the following must be true on the interval $0 < x < 2$?

$g''(x) = e^{-x^2}$

↓

also always positive so g is ccup

e^{-t^2} is always positive so $\int_0^2$ will be positive

and $g' = \int_0^x$ so g' is positive so g is increasing

(A) g is increasing, and the graph of g is concave up.

(B) g is increasing, and the graph of g is concave down.

(C) g is decreasing, and the graph of g is concave up.

(D) g is decreasing, and the graph of g is concave down.

(E) g is decreasing, and the graph of g has a point of inflection on $0 < x < 2$.

---

8. $\frac{d}{dx}\left(\int_0^{x^2} \sin\left(t^3\right)\, dt\right) =$

$\sin((x^2)^3)(2x)$

(A) $-\cos\left(x^6\right)$

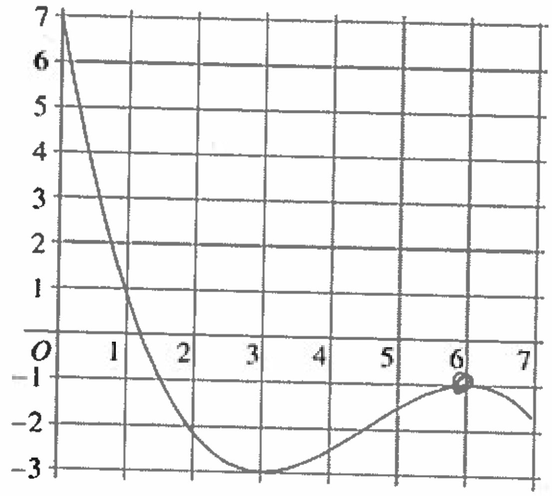
(B) $\sin\left(x^3\right)$

(C) $\sin\left(x^6\right)$

(D) $2x\sin\left(x^3\right)$

(E) $2x\sin\left(x^6\right)$

9.



Graph of f

The graph of the function f shown in the figure above has horizontal tangents at $x = 3$ and $x = 6$. If $g(x) = \int_0^{2x} f(t)\,dt$, what is the value of $g'(3)$?

(A) 0

(B) -1

(C) -2

(D) -3

(E) -6

$g'(x) = f(2x) \cdot (2)$

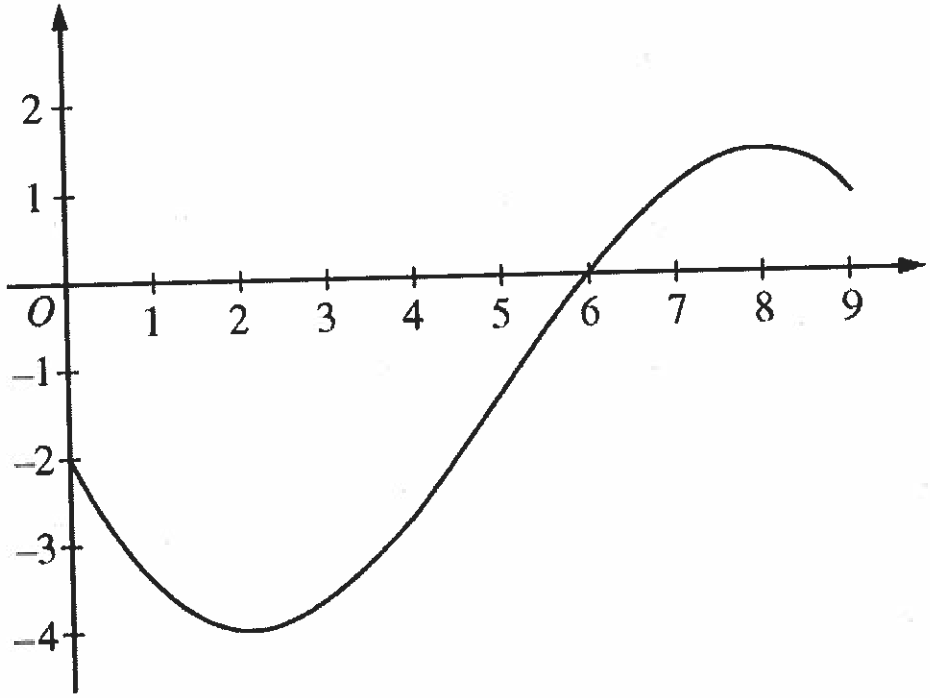
$g'(3) = f(2(3)) \cdot 2$

$f(6) \cdot 2$

$(-1) \cdot 2$

-2

10.



Graph of f

The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

- (A) $h(6) < h'(6) < h''(6)$
- (B) $h(6) < h''(6) < h'(6)$
- (C) $h'(6) < h(6) < h''(6)$
- (D) $h''(6) < h(6) < h'(6)$
- (E) $h''(6) < h'(6) < h(6)$

$h(6) = \int_0^6 f(t) = -$

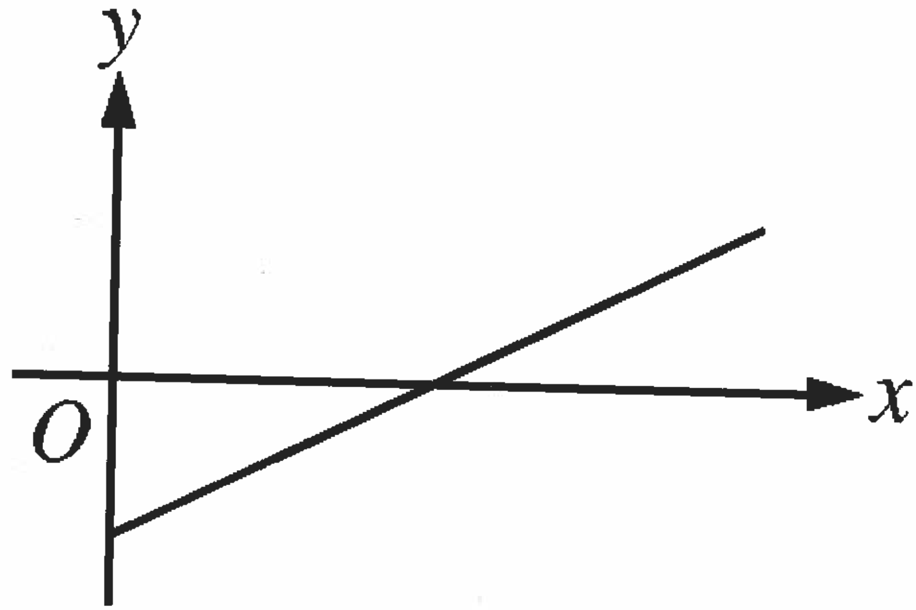
$h'(x) = f(x)$

$h'(6) = f(6) = 0$

$h''(x) = f'(x)$

$h''(6) = f'(6) = +$

11.

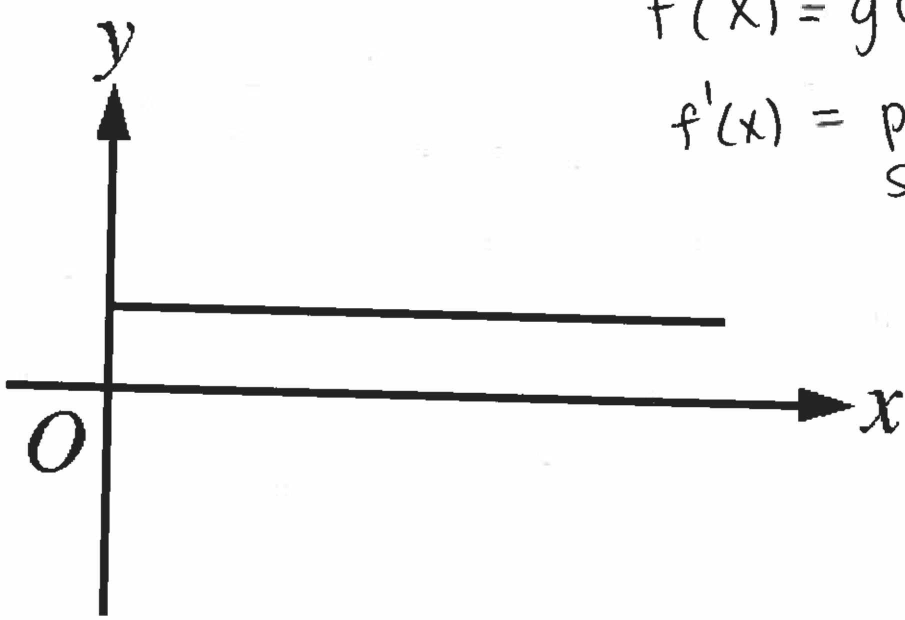


Graph of f

The figure above shows the graph of f. If $f(x) = \int_2^x g(t)\, dt$, which of the following could be the graph of $y = g(x)$?

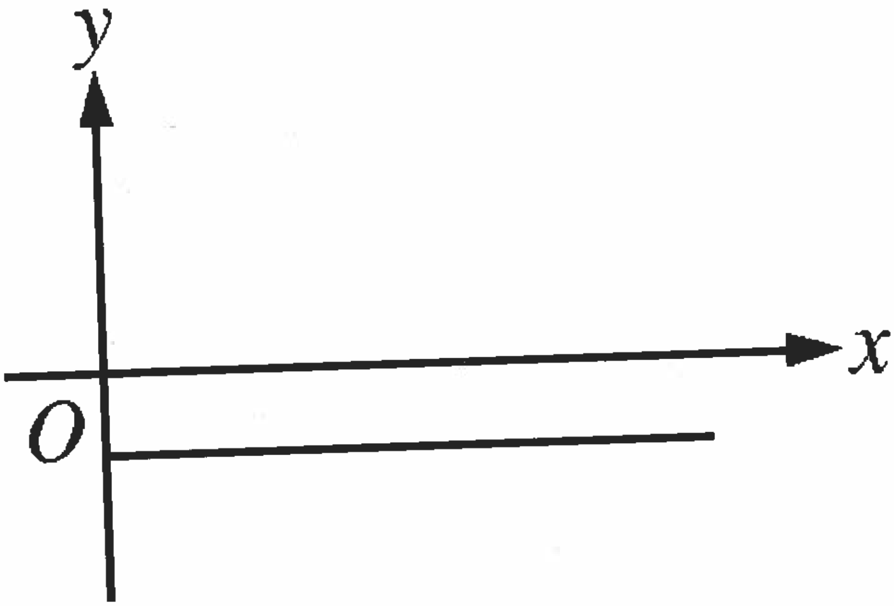
$f'(x) = g(x)$

$f'(x) =$ positive constant since $f(x)$ is linear.

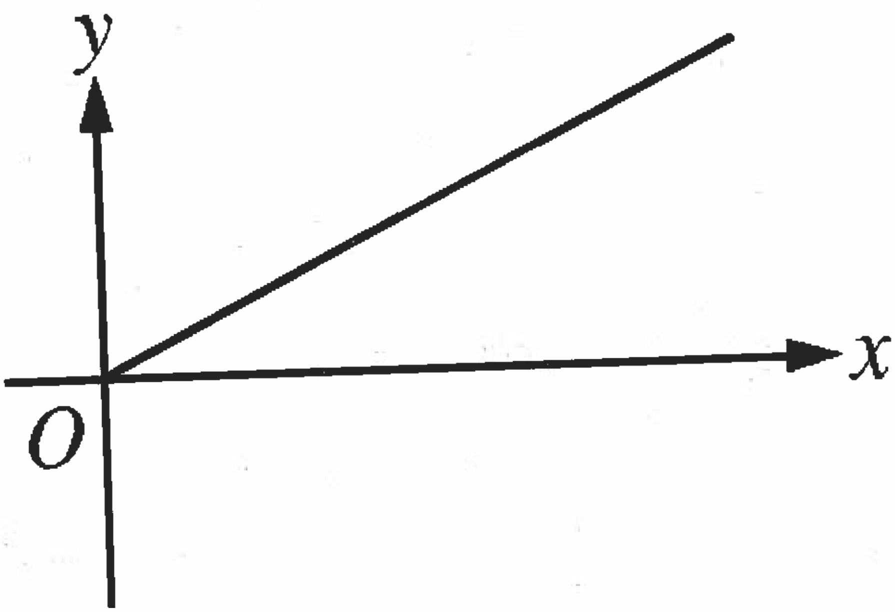


So, $g(x)$ will be a horizontal Line at a positive y-value

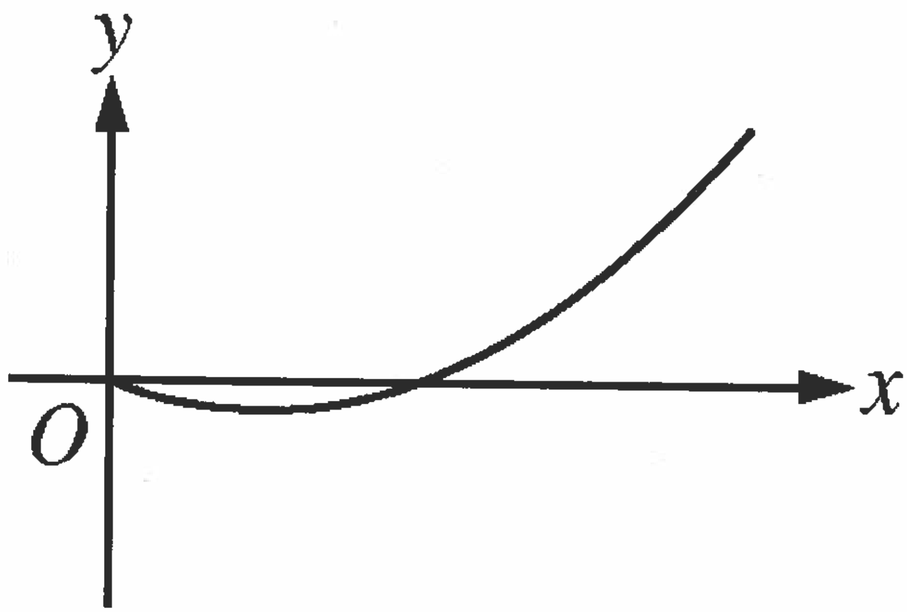
Ⓑ


y
x
O

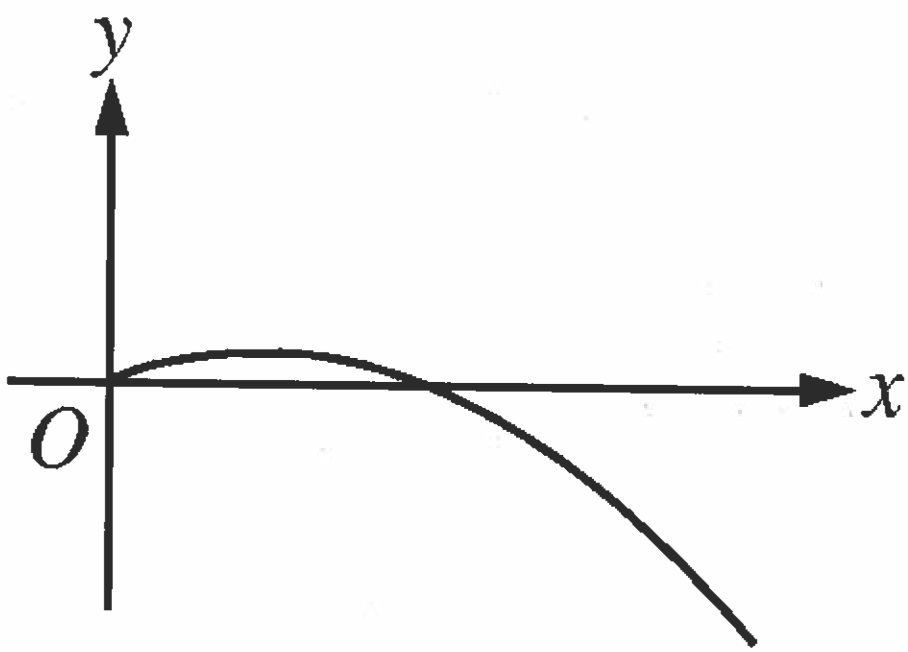
Ⓒ


y
x
O

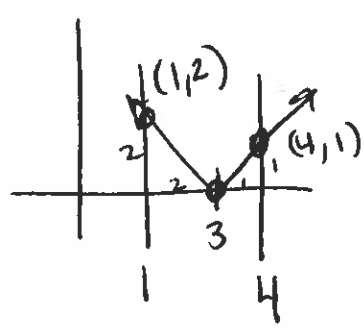
(D)



(E)



12. $\int_1^4 |x-3|\,dx =$



$\frac{1}{2}(2)(2) + \frac{1}{2}(1)(1)$

$2 + \frac{1}{2}$

$\frac{5}{2}$

(A) $-\frac{3}{2}$

(B) $\frac{3}{2}$

(C) $\frac{5}{2}$

(D) $\frac{9}{2}$

(E) 5

---

13. If $\int_{1}^{10} f(x)dx = 4$ and $\int_{10}^{3} f(x)dx = 7$, then $\int_{1}^{3} f(x)dx =$

(A) -3

(B) 0

(C) 3

(D) 10

(E) 11

$$\int_1^3 f(x)dx + \int_3^{10} f(x)dx = \int_1^{10} f(x)dx$$

$$\downarrow \quad + (-7) = 4$$

$$\downarrow \quad = 4 + 7 = 11$$

---

14. The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3. \end{cases}$ What is the value of $\int_{1}^{5} f(x)\ dx$?

$$\int_1^3 2dx + \int_3^5 (x-1)\,dx$$

$$2x\Big|_1^3 \qquad \left(\tfrac{1}{2}x^2 - x\right)\Big|_3^5$$

$$2(3) - 2(1) \qquad \left(\tfrac{1}{2}(5)^2 - 5\right) - \left(\tfrac{1}{2}(3)^2 - 3\right)$$

$$4 \qquad \frac{25}{2} - 5 - \frac{9}{2} + 3 \rightarrow \frac{16}{2} - 2 = 6$$

$$4 + 6 = 10$$

(A) 2

(B) 6

(C) 8

(D) 10

(E) 12

15.

Given $f(x) = \begin{cases} x+1 & \text{for } x < 0 \\ \cos\pi & \text{for } x \geq 0 \end{cases}$ $\int_{-1}^{1} f(x)dx =$

$\cos\pi = -1 \to$ constant

(A) $\frac{1}{2} + \frac{1}{\pi}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2} - \frac{1}{\pi}$

(D) $\frac{1}{2}$

(E) $-\frac{1}{2} + \pi$

$$\int_{-1}^{0}(x+1)\,dx + \int_{0}^{1}(\cos\pi)\,dx$$

$$\left[\frac{1}{2}x^2 + x\right]_{-1}^{0} + \left[-x\right]_{0}^{1}$$

$$\left(\frac{1}{2}(0)^2 + 0\right) - \left(\frac{1}{2}(-1)^2 - 1\right) + (-1 + 0)$$

$$-\frac{1}{2} + 1 - 1 = -\frac{1}{2}$$

(solution online in other file is incorrect)

16.

If f is a linear function and $0<a<b$, then $\int_{a}^{b} f''(x)dx =$ $f'(b) - f'(a)$ (FTOC 1)

↳ if f is linear $f'(x)$ is a constant so

$f'(b) = f'(a)$

so $f'(b) - f'(a) = 0$

(A) 0

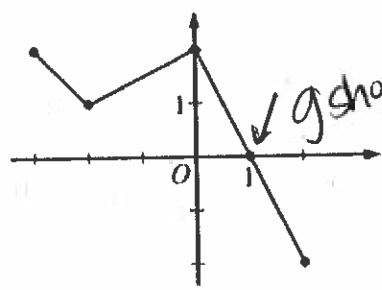
(B) 1

(C) $\frac{ab}{2}$

(D) b-a

(E) $\frac{b^2-a^2}{2}$

17.



g should be maxed here

Graph of f

The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\, dt$, which of the following values is greatest?

(A) g(-3)

(B) g(-2)

(C) g(0)

(D) g(1)

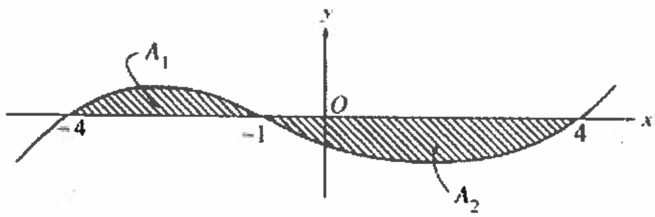
(E) g(2)

$g'(x) = f(x)$

g increases if $f > 0$

g decreases if $f < 0$

18.



The graph of $y=f(x)$ is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2,

$$\int_{-4}^{4} f(x)dx - 2\int_{-1}^{4} f(x)dx =$$

$A_1 - A_2 - 2(-A_2)$

$A_1 - A_2 + 2A_2$

$A_1 + A_2$

(A) A_1

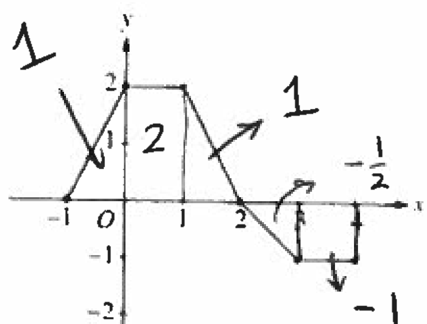
(B) $A_1 - A_2$

(C) $2A_1 - A_2$

(D) $A_1 + A_2$

(E) $A_1 + 2A_2$

19.



The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of

$$\int_{-1}^{4} f(x)dx?$$

$1 + 2 + 1 - \frac{1}{2} - 1$

$2\frac{1}{2}$

[redacted]

(A) 1

(B) 2.5

(C) 4

(D) 5.5

(E) 8

---

20. If $\int_0^k (2kx - x^2)dx = 18$, then $k =$

(A) -9

(B) -3

(C) 3

(D) 9

(E) 18

$$\left[\frac{2kx^2}{2} - \frac{x^3}{3}\right]_0^K$$

$$\left(K^3 - \frac{K^3}{3}\right) - (0 - 0)$$

$$\frac{2}{3}K^3 = 18 \qquad K^3 = 27$$

$$k = 3$$

---

21. $\int_0^1 (3x - 2)^2 dx =$

$$\int_0^1 9x^2 - 12x + 4$$

$$\left[3x^3 - 6x^2 + 4x\right]_0^1$$

$$(3 - 6 + 4) - (0) = 1$$

(A) $-\frac{7}{3}$

(B) $-\frac{7}{9}$

(C) $\frac{1}{9}$

(D) 1

(E) 3

---

22. $\int_0^{\frac{\pi}{4}} \sin x\,dx =$

$-\cos x \Big|_0^{\pi/4}$

$-\cos\frac{\pi}{4} + \cos 0$

$-\frac{\sqrt{2}}{2} + 1$

(A) $-\frac{\sqrt{2}}{2}$

(B) $\frac{\sqrt{2}}{2}$

(C) $-\frac{\sqrt{2}}{2} - 1$

(D) $-\frac{\sqrt{2}}{2} + 1$

(E) $\frac{\sqrt{2}}{2} - 1$

---

23. $\int_1^2 \frac{x-4}{x^2}dx$

$\frac{x}{x^2} - \frac{4}{x^2}$

$\int \frac{1}{x} - 4x^{-2}$

$\left[\ln|x| + 4x^{-1}\right]_1^2$

$\left(\ln 2 + \frac{4}{2}\right) - \left(\ln 1 + \frac{4}{1}\right)$

$\ln 2 + 2 - \ln 1 - 4$

$\ln 2 - 2$

(A) $-\frac{1}{2}$

(B) ln2-2

(C) ln2

(D) 2

(E) ln2+2

---

24. $\int_0^1 \sqrt{x}(x+1)dx =$

$x\sqrt{x} + \sqrt{x}$

$\int x^{\frac{3}{2}} + x^{\frac{1}{2}}$

$\left[\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}}\right]_0^1$

$\left(\frac{2}{5}(1)^{\frac{5}{2}} + \frac{2}{3}(1)^{\frac{3}{2}}\right) - (0+0)$

$\frac{3}{3}\cdot\frac{2}{5} + \frac{2}{3}\cdot\frac{5}{5} = \frac{6+10}{15}$

(A) 0

(B) 1

(C) $\frac{16}{15}$

(D) $\frac{7}{5}$

(E) 2

---

25. What are all values of k for which $\int_{-3}^{k} x^2\,dx = 0$?

$\frac{1}{3}x^3 \Big|_{-3}^{K}$

$\frac{1}{3}K^3 - \frac{1}{3}(-3)^3 = 0$

$\frac{1}{3}k^3 + 9 = 0$

$\frac{1}{3}k^3 = -9$

$k^3 = -27$

$k = -3$

(A) -3

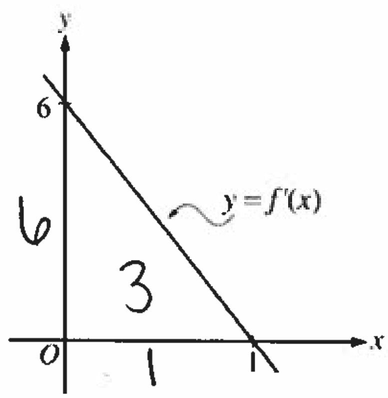
(B) 0

(C) 3

(D) -3 and 3

(E) -3, 0, 3

26.



The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

(A) 0

$f(0) + \int_0^x f'(t)\,dt = f(x)$

(B) 3

$f(0) + \int_0^1 f'(x)\,dx = f(1)$

(C) 6

$5 + 3 = 8$

(D) 8

(E) 11

27. $\int \sec^2 x dx =$

$\tan x + C$

(A) $\tan x + c$

(B) $\csc^2 x + c$

(C) $\cos^2 x + c$

(D) $\frac{\sec^3 x}{3} + c$

(E) $2\sec^2 x \tan x + c$

---

28. If the second derivative of f is given by $f''(x) = 2x - \cos x$, which of the following could be f(x)?

(A) $\frac{x^3}{3} + \cos x - x + 1$

(B) $\frac{x^3}{3} - \cos x - x + 1$

(C) $x^3 + \cos x - x + 1$

(D) $x^2 - sinx + 1$

(E) $x^2 + \sin x + 1$

$f'(x) = x^2 - \sin x + C$

$f(x) = \frac{1}{3}x^3 + \cos x + Cx + d$

new constant

---

29. $\int_1^e \frac{x^2+1}{x} dx =$

$\int x + \frac{1}{x}$

$\left[\frac{1}{2}x^2 + \ln|x|\right]_1^e$

$\left(\frac{1}{2}e^2 + \ln e\right) - \left(\frac{1}{2}(1)^2 + \ln 1\right)$

$\frac{1}{2}e^2 + 1 - \frac{1}{2} - 0$

$\frac{1}{2}e^2 + \frac{1}{2}$

(A) $\frac{e^2-1}{2}$

(B) $\frac{e^2+1}{2}$

(C) $\frac{e^2+2}{2}$

(D) $\frac{e^2-1}{e^2}$

(E) $\frac{2e^2-8e+6}{3e}$

30.

x	2	3	5	8	13
$f(x)$	6	−2	−1	3	9

The function f is continuous on the closed interval [2,13] and has values as shown in the table above. Using the intervals [2,3], [3,5], [5,8], and[8,13] what is the approximation of $\int_2^{13} f(x)\,dx$ obtained from a left Riemann sum?

(A) 6

(B) 14

(C) 28

(D) 32

(E) 50

$(1)(6)+(2)(-2)+(3)(-1)+(5)(3)$

$6-4-3+15$

$21-7$

14

31.

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

$L(t) \to$ liters $R(t) \to$ liters/hr

(A) 64.9

(B) 68.2

(C) 114.9

(D) 116.6

(E) 118.2

$$L(t) = 50 + \int_4^t R(t)\,dt$$

$$L(15) = 50 + \int_4^{15} R(t)\,dt$$

$$(3)(6.2) + (5)(5.9) + (3)(5.6)$$

$$50 + 18.6 + 29.5 + 16.8$$

32.

x	2	5	10	14
$f(x)$	12	28	34	30

The function f is continuous on the closed interval [2,14] and has values as shown in the table above. Using the subintervals [2,5], [5,10], and [10,14], what is the approximation of $\int_2^{14} f(x)\,dx$ found by using a right Riemann sum?

$$(3)(28) + (5)(34) + (4)(30)$$

$$84 + 170 + 120$$

(A) 296

(B) 312

(C) 343

(D) 374

(E) 390

---

33. If the average value of a continuous function f on the interval $[-2, 4]$ is 12, what is $\int_{-2}^{4} \frac{f(x)}{8} dx$?

(A) $\frac{3}{2}$

(B) 3

(C) 9

(D) 72

$$\frac{\int_{-2}^{4} f(x)dx}{4 - -2} = 12$$

$$\int_{-2}^{4} f(x)\,dx = (12)(6)$$

$$= 72$$

$$\frac{\int_{-2}^{4} f(x)}{8}$$

$$\frac{72}{8} = 9$$

---

34. Let f be a differentiable function such that $f(0) = -5$ and $f'(x) \leq 3$ for all x. Of the following, which is not a possible value for $f(2)$?

$$f(0) + \int_{0}^{2} f'(x)dx$$

$$f(0) + \int_{0}^{2} 3\,dx$$

$$-5 + 3x\Big|_{0}^{2}$$

$$-5 + 3(2) - 3(0) = 1 \rightarrow \text{biggest value possible}$$

(A) -10

(B) -5

(C) 0

(D) 1

(E) 2

---

35. Which of the following is an equation for the line tangent to the graph of $y = 3 - \int_{-1}^{x} e^{-t^3}\,dt$ at the point where x = -1 ?

(A) y - 3 = -3e(x + 1)

(B) y - 3 = -e(x + 1)

(C) y - 3 = 0

(D) y - 3 = -1/e(x + 1)

(E) y - 3 = 3e(x + 1)

$\frac{dy}{dx} = -e^{-x^3}$

$y(-1) = 3 - \int_{-1}^{-1} e^{-t^3}$

$= 3 - 0$

$= 3$

$\left.\frac{dy}{dx}\right|_{x=-1} = -e^{-(-1)^3}$

$= -e$

$y - 3 = -e(x+1)$

---

36. Let g be the function defined by $g(x) = \int_{-1}^{x} \frac{t^3 - t^2 - 6t}{\sqrt{t^2+7}}\,dt$. On which of the following intervals is g decreasing?

$g'(x) = \frac{x^3 - x^2 - 6x}{\sqrt{x^2+7}}$

$-\quad + \quad - \quad +$

$-2 \quad 0 \quad 3$

$x^2 + 7 > 0$

$x^3 - x^2 - 6x = 0$

$x(x^2 - x - 6) = 0$

$x(x-3)(x+2) = 0$

$x = 3, -2, 0$

(A) $x \le -2 \; and \; 0 \le x \le 3$

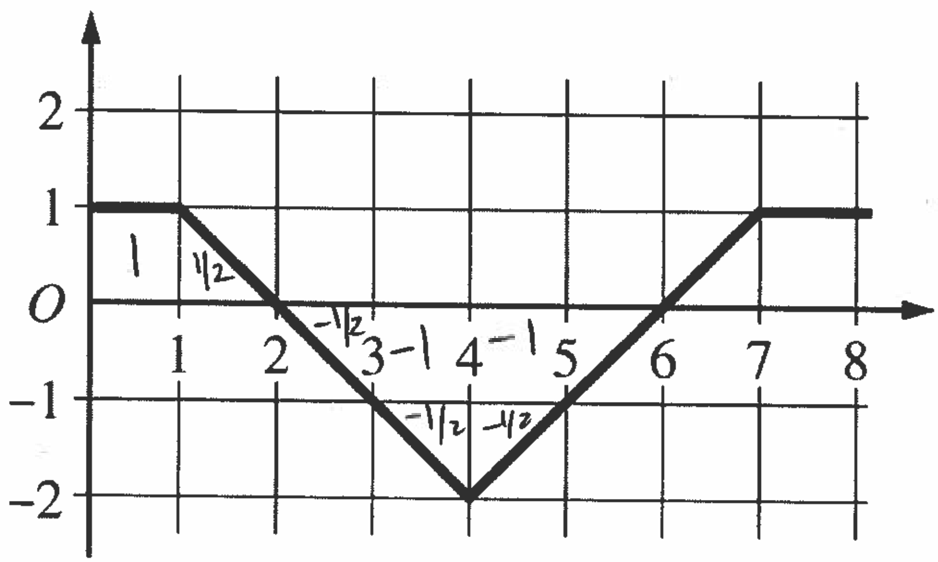
(B) $x \le -2 \; and \; x \ge 3$

(C) $-2 \le x \le 0 \; and \; x \ge 3$

(D) $-2 \le x \le 3$

(E) $x \le -1$

37.



Graph of f

The graph of the function f in the figure above consists of four line segments. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. Which of the following is an equation of the line tangent to the graph of g at $x = 5$?

$g'(x) = f(x)$
$g'(5) = f(5)$
$= -1$

$g(5) = \int_0^5 f(t)dt = -2$

$y + 2 = -1(x - 5)$

(A) $y+1=x-5$

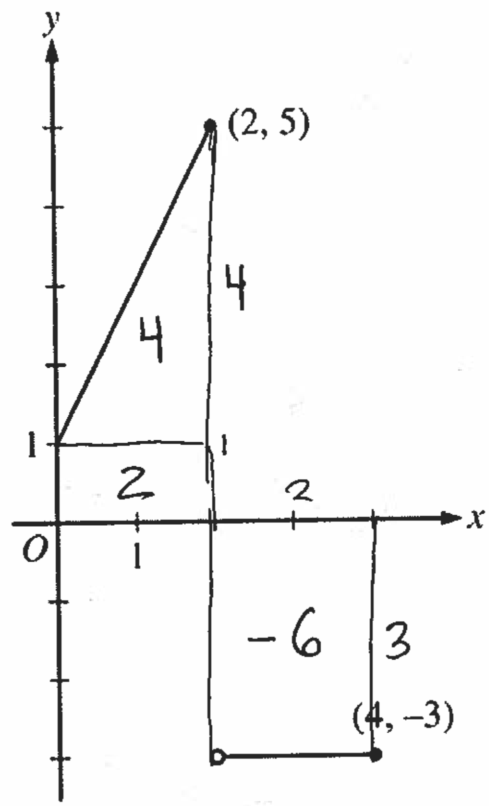
(B) $y-2=x-5$

(C) $y-2=-1\,(x-5)$

(D) $y+2=x-5$

(E) $y+2=-1\,(x-5)$

38.



Graph of f

The graph of f is shown above for $0 \le x \le 4$. What is the value of $\int_0^4 f\,(x)\ dx$?

$4+2-6=0$

(A) -1

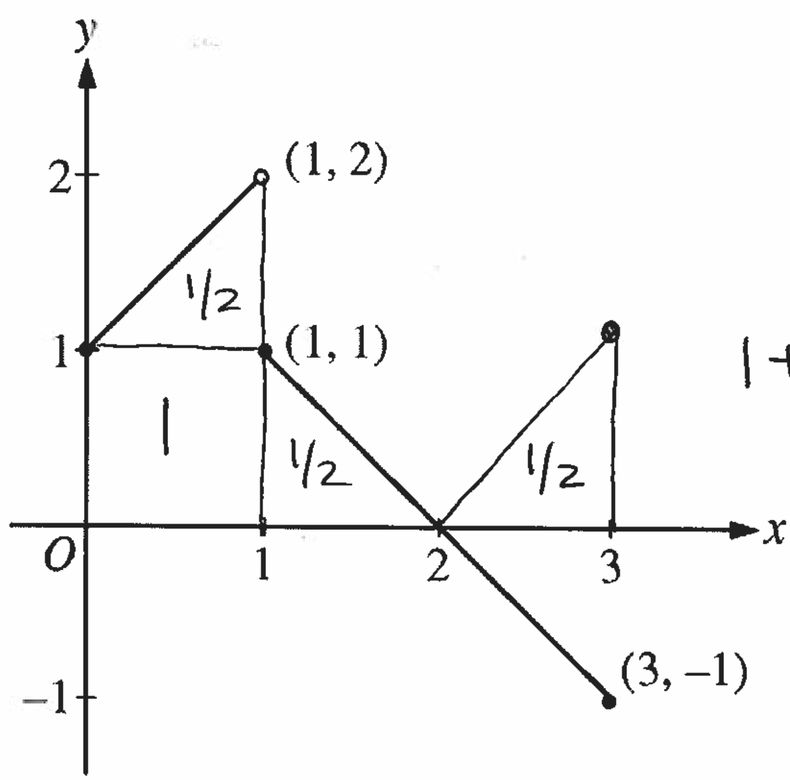
(B) 0

(C) 2

(D) 6

(E) 12

39.



$1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

Graph of f

The graph of the function f consists of two line segments, as shown in the figure above. The value of $\int_0^3 |f(x)|\, dx$ is

(A) $-\frac{3}{2}$

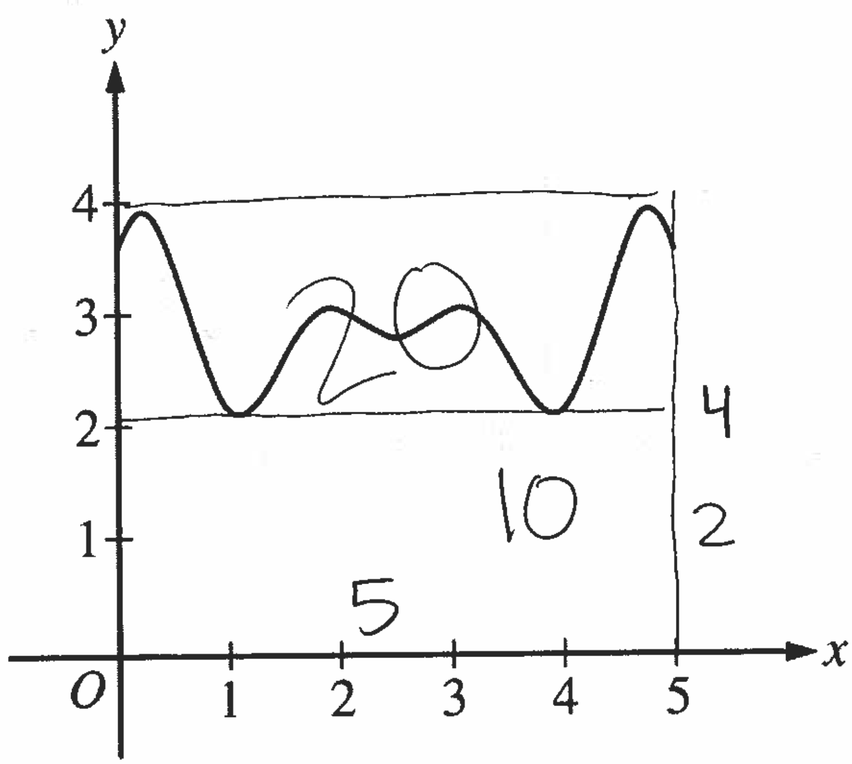
(B) $\frac{1}{2}$

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$

(E) nonexistent

40.



Graph of f'

The graph of f', the derivative of f, is shown in the figure above. If $f(0) = 20$, which of the following could be the value of $f(5)$?

$$f(5) = f(0) + \int_0^5 f'(x)\,dx$$

estimate as big rectangle

$20 + 20 = 40$

obviously overestimate

rest on back

(A) 15

(B) 20

(C) 25

(D) 35

(E) 40

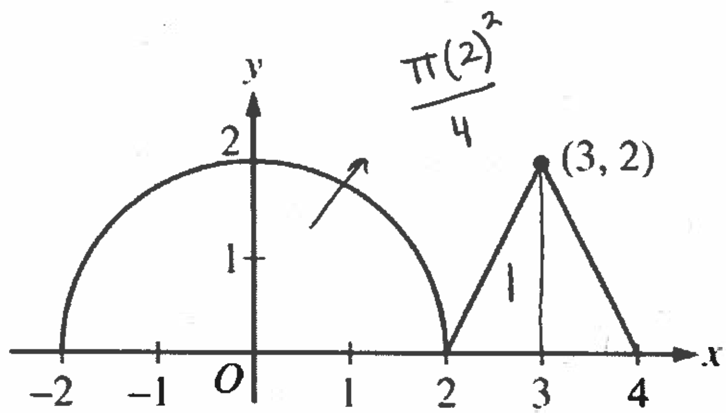
$20 + 10 = 30$

Obviously underestimate

So its between

30 and 40

41.



Graph of g'

The graph of g', the first derivative of the function g, consists of a semicircle of radius 2 and two line segments, as shown in the figure above. If $g(0) = 1$, what is $g(3)$?

(A) $\pi + 1$

(B) $\pi + 2$

(C) $2\pi + 1$

(D) $2\pi + 2$

$g(0) + \int_0^3 g' = g(3)$

$1 + \pi + 1$

42. $\displaystyle\int_{-2}^{1} (8x^3 - 3x^2)\,dx =$

(A) -561

(B) -90

(C) -39

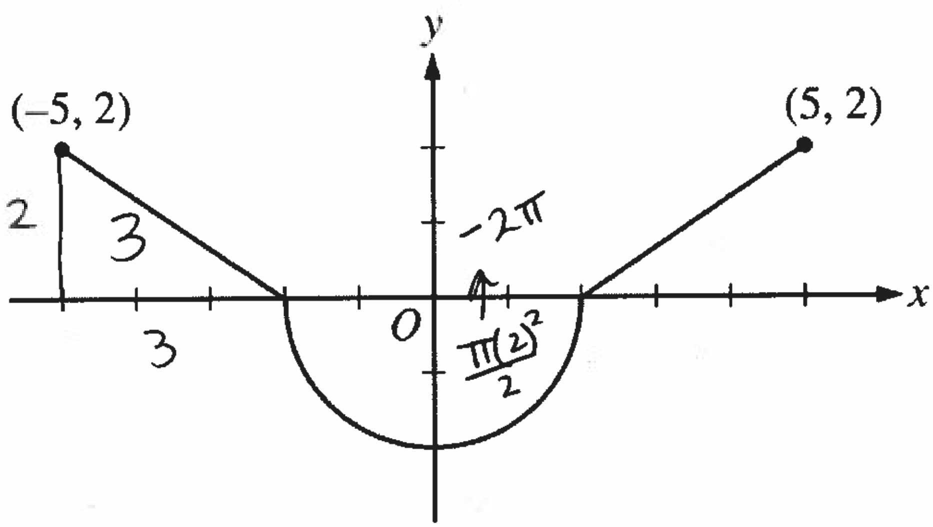
(D) 81

$$\left[\frac{8x^4}{4} - \frac{3x^3}{3}\right]_{-2}^{1}$$

$$\left(2(1)^4 - 1(1)^3\right) - \left(2(-2)^4 - 1(-2)^3\right)$$

$$2 - 1 - 32 - 8$$

43.



Graph of f'

The graph of f', the derivative of a function f, consists of two line segments and a semicircle, as shown in the figure above. If $f(2) = 1$, then $f(-5) =$

$$f(-5) + \int_{-5}^{2} f'(x)\,dx = f(2)$$

$$f(-5) + 3 - 2\pi = 1$$

$$f(-5) = 1 - 3 + 2\pi$$

(A) $2\pi - 2$

(B) $2\pi - 3$

(C) $2\pi - 5$

(D) $6 - 2\pi$

(E) $4 - 2\pi$

44.

e is a constant!

$$\int (e^x + e)\, dx =$$

$e^x + ex + C$

(A) $e^x + C$

(B) $2e^x + C$

(C) $e^x + e + C$

(D) $e^{x+1} + ex + C$

(E) $e^x + ex + C$

---

45. $\int 2^x\, dx =$

$$2^x \cdot \frac{1}{\ln 2} + C$$

(A) $2^x + C$

(B) $(ln\ 2)\ 2^x + C$

(C) $\frac{2^x}{\ln 2} + C$

(D) $\frac{2^{x+1}}{x+1} + C$

46.

x	0	2	4	6
$f(x)$	-22	-6	2	2
$f'(x)$	10	6	2	-2

Selected values of the twice-differentiable function f and its derivative f' are given in the table above. What is the value of $\int_0^6 f'(x)\,dx$?

(A) -12

$= f(6) - f(0)$

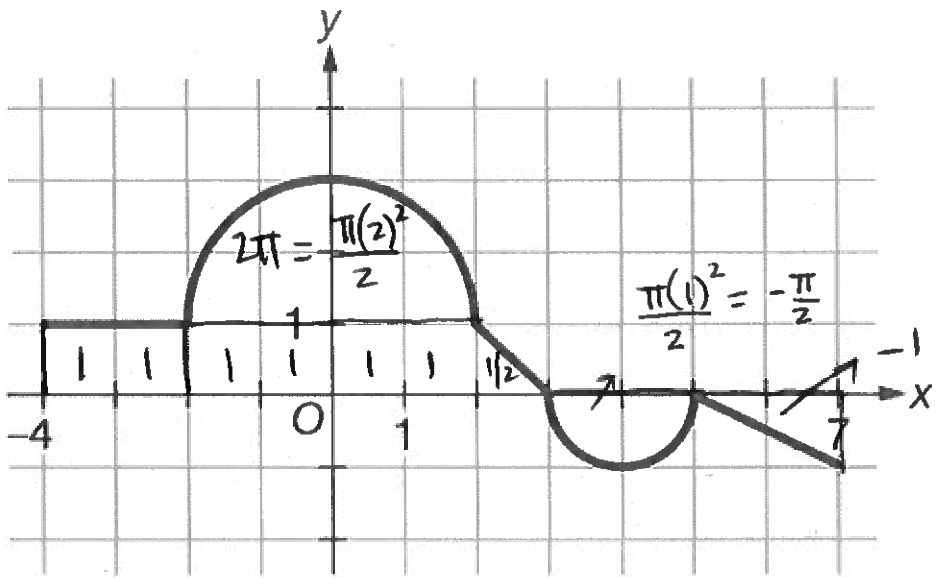
(B) 12

$= 2 - -22$

(C) 24

(D) 36

47.



Graph of f

The graph of the function f on the interval $-4 \le x \le 7$ consists of three line segments and two semicircles, as shown in the figure above. What is the value of $\int_{-4}^{7} f(x)\,dx$?

(A) $\frac{3}{2}\pi + \frac{3}{2}$

(B) $\frac{3}{2}\pi + \frac{11}{2}$

(C) $\frac{5}{2}\pi + \frac{7}{2}$

(D) $\frac{5}{2}\pi + \frac{15}{2}$

$\frac{13}{2} + 2\pi - \frac{\pi}{2} - 1$

$\frac{11}{2} + \frac{3\pi}{2}$

---

48. If $\int_{-1}^{3} (2g(x) + 4)\,dx = 22$ and $\int_{10}^{-1} g(x)\,dx = 12$, then $\int_{3}^{10} g(x)\,dx =$

split up

$\int_{-1}^{3} g(x) + \int_{3}^{10} g(x) = \int_{-1}^{10} g(x)$

$2\int_{-1}^{3} g(x)dx + \underbrace{\int_{-1}^{3} 4\,dx} = 22$

$4x\Big|_{-1}^{3}$

$= 12 - 4(-1)$

$= 16$

$2\int_{-1}^{3} g(x)dx + 16 = 22$

$2\int_{-1}^{3} g(x)dx = 6$

$\int_{-1}^{3} g(x)dx = 3$

(A) -21

(B) -15

(C) -9

(D) 9

$$3 + \int_3^{10} g(x)\,dx = -12$$

$$\int_3^{10} g(x)\,dx = -15$$

49.

x	0	1	2	3	4	5	6
$f(x)$	0	5	2	-1	-2	0	3

The function f is continuous on the closed interval [0, 6] and has values as shown in the table above. Using the intervals [0,2], [2,4], and [4,6], what is the approximation of $\int_0^6 f(x)dx$ obtained from a midpoint Riemann sum?

$$2(5) + 2(-1) + 2(0)$$

$$10 - 2 + 0$$

$$8$$

(A) 0

(B) 3

(C) 4

(D) 6

(E) 8

50. The average value of a function f over the interval $[-1, 2]$ is -4, and the average value of f over the interval $[2, 7]$ is 8. What is the average value of f over the interval $[-1, 7]$?

$$\frac{1}{2--1}\int_{-1}^{2} f(x)dx = -4$$

$$\frac{1}{3}\int_{-1}^{2} f(x)dx = -4$$

$$\int_{-1}^{2} f(x)dx = -12$$

$$\frac{1}{5}\int_{2}^{7} f(x) = 8$$

$$\int_{2}^{7} f(x) = 40$$

$$\frac{1}{8}\int_{-1}^{7} f(x) = \frac{1}{8}(-12+40) = \frac{1}{8}(28) = \frac{7}{2}$$

(A) $\frac{1}{2}$

(B) 2

(C) $\frac{7}{2}$

(D) 14

---

51. Let f be the function given by $f(x) = \int_{10}^{x} (-t^2 + 2t + 3)\, dt$. On what intervals is f increasing?

(A) $(-\infty, 1]$

(B) $[-1, 3]$

(C) $[1, \infty)$

(D) $(-\infty, -1]$ and $[3, \infty)$

$f'(x) = -x^2 + 2x + 3$

$0 = -(x^2 - 2x - 3)$

$-(x-3)(x+1)$

$-$ $+$ $-$

-1 3

---

52. Which of the following is a left Riemann sum approximation of $\int_{1}^{7} (4 \ln x + 2)\, dx$ with n subintervals of equal length?

Ⓐ $\sum_{k=1}^{n}\left(4\ln\left(1+\frac{k-1}{n}\right)+2\right)\frac{1}{n}$ X

$7-1=6$

Ⓑ $\sum_{k=1}^{n}\left(4\ln\left(\frac{6k}{n}\right)+2\right)\frac{6}{n}$

Ⓒ $\sum_{k=1}^{n}\left(4\ln\left(1+\frac{6(k-1)}{n}\right)+2\right)\frac{6}{n}$

LRAM

1st value at $x=1$

$4\ln 1+2 = 2$

Ⓓ $\sum_{k=1}^{n}\left(4\ln\left(1+\frac{6k}{n}\right)+2\right)\frac{6}{n}$

---

53. Which of the following is a left Riemann sum approximation of $\int_2^8 \cos(x^2)\,dx$ with n subintervals of equal length?

Ⓐ $\sum_{k=1}^{n}\left(\cos\left(2+\frac{k-1}{n}\right)^2\right)\frac{1}{n}$ X

$8-2=6$

LRAM

$\cos((2)^2)$

$\cos 4$

Ⓑ $\sum_{k=1}^{n}\left(\cos\left(\frac{6k}{n}\right)^2\right)\frac{6}{n}$

Ⓒ $\sum_{k=1}^{n}\left(\cos\left(2+\frac{6(k-1)}{n}\right)^2\right)\frac{6}{n}$ ✓

Ⓓ $\sum_{k=1}^{n}\left(\cos\left(2+\frac{6k}{n}\right)^2\right)\frac{6}{n}$