AP Calculus BC Name________________________________

Test 1 Review

Date________________ Period____

Evaluate each integral or state that it diverges (show all necessary steps to justify- do not simply put diverges).

1) $\int \frac{\sin^{-1} x}{\sqrt{1 - x^2}}\, dx$

2) $\int \frac{\ln x}{x^3}\, dx$

3) $\int x^3 \cos 2x\, dx$

4) $\int \frac{\sec^2 \ln x}{x}\, dx$

5) $\int_{-8}^{0} \frac{1}{x^{\frac{1}{3}}}\, dx$

6) $\int_{2}^{4} \frac{1}{x - 2}\, dx$

7) $\int_{0}^{1} \frac{1}{\sqrt{1 - x^2}}\, dx$

8) $\int_{3}^{\infty} \frac{1}{x^4}\, dx$

9) $\int_{0}^{36} \frac{1}{\sqrt{36 - x}}\, dx$

10) $\int -\frac{4x}{\sqrt[3]{3x^2 + 4}}\, dx$

11) $\int_{-\infty}^{-2} \left(\frac{1}{x - 1} - \frac{1}{x + 1}\right) dx$

12) $\int \tan^{-1} 2x\, dx$

13) Consider the differential equation $\frac{dy}{dx} = y^2(2x + 2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

Use one of the comparison tests (showing the proper steps and justifications) to determine whether the given integral converges or diverges. You DO NOT need to find the actual value if it does converge.

14) $\int_{3}^{\infty} \frac{x}{x - 2}\, dx$

15) $\int_{1}^{\infty} \frac{1}{x + x^3}\, dx$

16) $\int_{2}^{\infty} \frac{\cos^2 x}{x^2}\, dx$

17) $\int_{4}^{\infty} \frac{3}{x - e^{-x}}\, dx$

18) Assume that f and $f'(x)$ have the values given in the table. Use Euler's method with 3 equal steps to approximate the value of $f(5.8)$.

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the given axis.

19) $x = \frac{y - 2}{2},\ x = \sqrt{y - 2}$

Axis: $y = 1$

20) $y = 1,\ \ y = (x + 2)^2,\ \ x = -2,\ \ x = -1$
Axis: $x = -2$

Evaluate each integral.

21) $\int (\ln\ 5x)^2\ dx$

22) $\int \cos 4x \cdot e^{-x}\ dx$

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Evaluate each integral or state that it diverges (show all necessary steps to justify- do not simply put diverges).

1) $\int \frac{\sin^{-1} x}{\sqrt{1 - x^2}}\, dx$

Use: $u = \sin^{-1} x$

$\int \frac{\sin^{-1} x}{\sqrt{1 - x^2}}\, dx = \frac{1}{2} \cdot \left(\sin^{-1} x\right)^2 + C$

2) $\int \frac{\ln x}{x^3}\, dx$

Use: $u = \ln x,\ dv = \frac{1}{x^3}dx$

$\int \frac{\ln x}{x^3}\, dx = -\frac{1}{2}x^{-2}\ln x - \frac{1}{4}x^{-2} + C$

3) $\int x^3 \cos 2x\, dx$

Use: $u = x^3,\ dv = \cos 2x dx$

$\int x^3 \cos 2x\, dx = \frac{1}{2}x^3 \sin 2x + \frac{3}{4}x^2 \cos 2x - \frac{3}{4}x \sin 2x - \frac{3}{8} \cdot \cos 2x + C$

4) $\int \frac{\sec^2 \ln x}{x}\, dx$ Use: $u = \ln x$

$\int \frac{\sec^2 \ln x}{x}\, dx = \tan \ln x + C$

5) $\displaystyle\int_{-8}^{0} \frac{1}{x^{\frac{1}{3}}}\,dx$

-6

6) $\displaystyle\int_{2}^{4} \frac{1}{x-2}\,dx$

diverges

7) $\displaystyle\int_{0}^{1} \frac{1}{\sqrt{1-x^2}}\,dx$

$\dfrac{\pi}{2}$

8) $\displaystyle\int_{3}^{\infty} \frac{1}{x^4}\,dx$ $\dfrac{1}{81}$

9) $\int_{0}^{36} \frac{1}{\sqrt{36 - x}}\, dx$

12

10) $\int -\frac{4x}{\sqrt[3]{3x^2 + 4}}\, dx$

$-(3x^2 + 4)^{\frac{2}{3}} + C$

11) $\int_{-\infty}^{-2} \left(\frac{1}{x - 1} - \frac{1}{x + 1}\right) dx$

$\ln 3$

12) $\int \tan^{-1} 2x\, dx$

$x\tan^{-1} 2x - \frac{1}{4} \cdot \ln\left|1 + 4x^2\right| + C$

13) Consider the differential equation $\frac{dy}{dx} = y^2(2x + 2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

$-\frac{11}{32}$

Use one of the comparison tests (showing the proper steps and justifications) to determine whether the given integral converges or diverges. You DO NOT need to find the actual value if it does converge.

14) $\int_{3}^{\infty} \frac{x}{x-2}\,dx$

diverges by DCT using $\frac{1}{x-2}$

15) $\int_{1}^{\infty} \frac{1}{x+x^3}\,dx$ converges by LCT using $\frac{1}{x^3}$

16) $\int_{2}^{\infty} \frac{\cos^2 x}{x^2}\, dx$

converges by DCT

17) $\int_{4}^{\infty} \frac{3}{x - e^{-x}}\, dx$

diverges by DCT

18) Assume that f and $f'(x)$ have the values given in the table. Use Euler's method with 3 equal steps to approximate the value of $f(5.8)$.

5.6

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the given axis.

19) $x = \frac{y - 2}{2},\ x = \sqrt{y - 2}$

Axis: $y = 1$

$$\pi \int_{0}^{2} \left((2x + 1)^2 - \left(x^2 + 1\right)^2 \right) dx$$

$$= \frac{104}{15}\pi \approx 21.782$$

20) $y = 1,\ \ y = (x + 2)^2,\ \ x = -2,\ \ x = -1$
Axis: $x = -2$

$$\pi \int_0^1 \left(\sqrt{y}\right)^2 \, dy$$
$$= \frac{1}{2}\pi \approx 1.571$$

Evaluate each integral.

21) $\int (\ln\ 5x)^2 \, dx$

Use: $u = (\ln\ 5x)^2,\ \ dv = dx$
$$\int (\ln\ 5x)^2 \, dx = x \cdot (\ln\ 5x)^2 - 2x \ln\ 5x + 2x + C$$

22) $\int \cos 4x \cdot e^{-x} \, dx$

Use: $u = e^{-x},\ \ dv = \cos 4x \, dx$
$$\int \cos 4x \cdot e^{-x} \, dx = \frac{4\sin 4x - \cos 4x}{17e^x} + C$$