1.) $\int x \ln x \, dx$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{x^2}{2}$

$$= \frac{x^2}{2} \cdot \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} \, dx$$

$$= \frac{x^2}{2} \cdot \ln x - \frac{1}{2}\int x \, dx$$

$$\boxed{= \frac{x^2}{2} \cdot \ln x - \frac{1}{4}x^2 + C}$$

2.) $\int x \sec^2 x \, dx$

$u = x \qquad dv = \sec^2 x \, dx$

$du = dx \qquad v = \tan x$

$$= x\tan x - \int \tan x \, dx$$

$\tan x = \frac{\sin x}{\cos x} \qquad u = \cos x$

$du = -\sin x \, dx$

$-\int \frac{1}{u} du \rightarrow -\ln|u|$

$$\boxed{= x\tan x + \ln|\cos x| + C}$$

3.) $\int x^3 e^{x^4+3} dx \rightarrow e^3 \int x^3 \cdot e^{x^4} dx$

$u = x^4$

$du = 4x^3 dx$

$\frac{du}{4} = x^3 dx$

$$\rightarrow e^3 \cdot \frac{1}{4}\int e^u du$$

$$\rightarrow \frac{e^3}{4} e^{x^4} + C \rightarrow \boxed{\frac{e^{x^4+3}}{4} + C}$$

4.) $\int x^2 \cdot \sqrt[4]{x}\,dx \to \int x^2 \cdot x^{1/4}\,dx \to \int x^{2\frac{1}{4}}\,dx$

$\to \int x^{\frac{9}{4}}\,dx \to \boxed{\frac{4}{13}x^{\frac{13}{4}} + C}$

5.) $\int x^2 \cos(6x)\,dx$

$u = x^2$		$dv = \cos 6x\,dx$
$2x$	$+$	$\frac{1}{6}\sin 6x$
2	$-$	$-\frac{1}{36}\cos 6x$
0	$+$	$-\frac{1}{216}\sin 6x$

$$\boxed{= \frac{x^2}{6}\sin 6x + \frac{x}{18}\cos 6x - \frac{1}{108}\sin 6x + C}$$

6.) $\int x\sqrt{x+3}\,dx$

$u = x$ $\quad dv = \sqrt{x+3}\,dx$

$du = dx$ $\quad v = \frac{2}{3}(x+3)^{\frac{3}{2}}$

$= \frac{2}{3}x(x+3)^{\frac{3}{2}} - \int \frac{2}{3}(x+3)^{\frac{3}{2}}\,dx$

$$\boxed{= \frac{2x}{3}(x+3)^{\frac{3}{2}} - \frac{2}{3}\cdot\frac{2}{5}(x+3)^{\frac{5}{2}} + C}$$

7.) $\int (3x^4+3)^2\,dx \to \int (9x^8 + 18x^4 + 9)\,dx$

$$\boxed{\to x^9 + \frac{18}{5}x^5 + 9x + C}$$

8.) $\int \frac{x^3+\ln x}{x}\,dx \rightarrow \int \frac{x^3}{x}dx + \int \frac{\ln x}{x}\,dx$ $u=\ln x \quad du=\frac{1}{x}dx$

$\rightarrow \int x^2\,dx + \int u\,du \rightarrow \boxed{\frac{x^3}{3}+\frac{(\ln x)^2}{2}+C}$

9.) $\int \sin^{-1}x\,dx$ $u=\sin^{-1}x \quad dv=dx$

$du=\frac{1}{\sqrt{1-x^2}} \quad v=x$

$\hookrightarrow x\sin^{-1}x-\int\frac{x}{\sqrt{1-x^2}}\,dx$ $u=1-x^2$ $du=-2x\,dx \nearrow -\frac{1}{2}du=x\,dx$

$\downarrow$

$x\sin^{-1}x+\frac{1}{2}\int u^{-\frac{1}{2}}\,du \rightarrow \boxed{x\sin^{-1}x+\sqrt{1-x^2}+C}$

10.) $\int_1^{\infty}\frac{dx}{\sqrt[3]{x}}$ p-series $a>0$ $p\leq 1 \ (1/3)$ so $\boxed{\text{diverges}}$

11.) $\int_0^{\infty}\frac{\tan^{-1}x+4}{1+x^2}\,dx \rightarrow \lim_{b\to\infty}\int_0^b \frac{\tan^{-1}x+4}{1+x^2}\,dx$ $u=\tan^{-1}x+4$ $du=\frac{1}{1+x^2}dx$

$\rightarrow \lim_{b\to\infty}\left[\frac{(\tan^{-1}x+4)^2}{2}\right]_0^b$ $\int u\,du \rightarrow \frac{u^2}{2}$

$\rightarrow \lim_{b\to\infty}\left(\frac{(\tan^{-1}b+4)^2}{2}-\frac{(\tan^{-1}0+4)^2}{2}\right) \rightarrow \frac{(\frac{\pi}{2}+4)^2}{2}-\frac{(0+4)^2}{2}$

$\rightarrow \boxed{\frac{(\frac{\pi}{2}+4)^2}{2}-8} \rightarrow \boxed{\text{* or } \frac{\pi^2}{8}+2\pi}$

12.) $\int_0^{49} \frac{dx}{\sqrt{49-x}} \rightarrow \lim_{x\to 49^-} \int_0^b \frac{dx}{\sqrt{49-x}}$

$u = 49-x$

$-du = +dx$

$-\int u^{-\frac{1}{2}} \rightarrow -2u^{\frac{1}{2}}$

$\rightarrow \lim_{x\to 49^-} \left[-2\sqrt{49-x}\right]_0^b$

$\rightarrow \lim_{x\to 49^-} \left(-2\sqrt{49-b} + 2\sqrt{49-0}\right) \rightarrow 0+14 \boxed{= 14}$

14.) $\int_0^{27} \frac{dx}{x^{2/3}} \rightarrow \lim_{a\to 0^+} \int_a^{27} \frac{dx}{x^{2/3}} \rightarrow \lim_{a\to 0^+} \left[3x^{\frac{1}{3}}\right]_a^{27}$

$\rightarrow \lim_{a\to 0^+} \left(3(27)^{1/3} - 3(a)^{\frac{1}{3}}\right) \rightarrow 9 - 0 \boxed{= 9}$

13.) $\int_{-\infty}^{\infty} 6xe^{-x^2}\,dx$

$u = -x^2 \rightarrow du = -2x\,dx \rightarrow -\frac{1}{2}du = x\,dx$

$6 \cdot -\frac{1}{2} \int e^u\,du \rightarrow -3e^u \rightarrow -3e^{-x^2}$

$\downarrow$

$\lim_{a\to -\infty} \int_a^0 6xe^{-x^2}dx + \lim_{b\to\infty} \int_0^b 6xe^{-x^2}dx$

$\downarrow \lim_{a\to -\infty} \left[-3e^{-x^2}\right]_a^0 + \lim_{b\to\infty} \left[-3e^{-x^2}\right]_0^b$

$\rightarrow \lim_{a\to -\infty} \left(-3e^0 + 3e^{-a^2}\right) + \lim_{b\to\infty} \left(-3e^{-b^2} + 3e^0\right)$

$\rightarrow -3 + 0 - 0 + 3 \boxed{= 0}$

15.) $\int_1^\infty \frac{dx}{x^4+7}$ | since $\lim_{x\to\infty} \dfrac{\frac{1}{x^4+7}}{\frac{1}{x^4}} \to \frac{x^4}{x^4+7} \to 1$ and $0<1<\infty$

and $\int_1^\infty \frac{1}{x^4}\,dx$ p-series $a>0$ $(a=1)$ $p>1$ $(p=4)$ converges (to $\frac{1}{4-1}$)

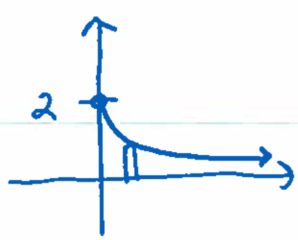
by LCT $\int_1^\infty \frac{dx}{x^4+7}$ also converges.

16.) $\int_5^\infty \frac{\ln x}{x}\,dx$ $0 \le \frac{1}{x} \le \frac{\ln x}{x}$ for $x \ge 5$

$\int_5^\infty \frac{1}{x}\,dx$ p-series $a>0$ $p=1$ so diverges

by DCT, $\int_5^\infty \frac{\ln x}{x}\,dx$ diverges

17.) $y = 2e^{-x}$



$$V = \pi\int_0^\infty (2e^{-x})^2\,dx \to \pi\int_0^\infty 4e^{-2x}\,dx$$

$$\to \lim_{b\to\infty} \pi\int_0^b 4e^{-2x}\,dx \to \lim_{b\to\infty} 4\pi\cdot\frac{-1}{2}\left[e^{-2x}\right]_0^b$$

$$\to \lim_{b\to\infty}\left(-2\pi\left(e^{-2b}-e^0\right)\right) \to -2\pi(0-1) = \boxed{2\pi}$$

18.) $\int \frac{6x-2}{x^2+11x+28}\,dx$

$x^2+11x+28 = (x+7)(x+4)$

$\frac{6x-2}{(x+7)(x+4)} = \frac{A(x+4)}{(x+7)(x+4)} + \frac{B(x+7)}{(x+4)(x+7)}$

$A(x+4)+B(x+7) = 6x-2$

$x \to -4 \quad 3B = -26 \quad B = \frac{-26}{3}$

$x \to -7 \quad -3A = -44 \quad A = \frac{44}{3}$

$\to \int \frac{44}{3(x+7)}\,dx + \int \frac{-26}{3(x+4)}\,dx \to \boxed{\frac{44}{3}\ln|x+7| - \frac{26}{3}\ln|x+4| + C}$

19.) $\int \frac{1}{3x^2+8x+4}\,dx$

$3x^2+8x+4 \to (3x+2)(x+2)$

$\frac{(x+2)\,A}{(x+2)(3x+2)} + \frac{B\,(3x+2)}{(x+2)(3x+2)} = \frac{1}{3x^2+8x+4}$

$A(x+2) + B(3x+2) = 1$

$A \to -2 \quad -4B = 1 \quad B = -1/4$

$B \to -2/3 \quad \frac{4}{3}A = 1 \quad A = 3/4$

$\to \int \frac{3}{4(3x+2)}\,dx + \int \frac{-1}{4(x+2)}\,dx \to \frac{3}{4}\cdot\frac{1}{3}\ln|3x+2| - \frac{1}{4}\ln|x+2| + C$

↓
Chain rule

$\boxed{\to \frac{1}{4}\ln|3x+2| - \frac{1}{4}\ln|x+2| + C}$

20.) $\frac{dy}{dx} = 3x + 2y + 1$

a) $\frac{d^2y}{dx^2} = 3 + 2\frac{dy}{dx} = 3 + 2(3x + 2y + 1) = 3 + 6x + 4y + 2$

$\boxed{= 5 + 6x + 4y}$

b.) $y = mx + b + e^{rx}$ so $\frac{dy}{dx} = m + re^{rx}$

$m + re^{rx} = 3x + 2(mx + b + e^{rx}) + 1$

$= 3x + 2mx + 2b + 2e^{rx} + 1$

$= (3 + 2m)x + 2e^{rx} + 1 + 2b$

↓

no x, so $3 + 2m = 0$

$\boxed{m = -\frac{3}{2}}$

$m = 1 + 2b$

$-\frac{3}{2} = 1 + 2b$

$2e^{rx} = re^{rx}$ so $\boxed{r = 2}$

$-\frac{5}{2} = 2b$ $\boxed{b = -\frac{5}{4}}$

c.)

old pt.	dx	$m = \frac{dy}{dx}$	dy = m dx	new pt
(0, −2)	.5	$3(0)+2(-2)+1 = -3$	$-3(.5) = -1.5$	(.5, −3.5)
(.5, −3.5)	.5	$3(.5)+2(-3.5)+1 = -4.5$	$-4.5(.5) = -2.25$	(1, −5.75)

$\boxed{f(1) \approx -5.75}$

d.) (0, k) | 1 | $3(0) + 2k + 1 = 2k + 1$ | $(2k+1)(1) = 2k+1$ | (1, 3k+1)

$3k + 1 = 0$

$\boxed{k = -1/3}$

$\frac{3-2}{2} = \frac{1}{2} = .5$

21.) The curve passing through $(2,0)$ satisfies the differential equation $\frac{dy}{dx} = 4x + y$. Find an approximation to $y(3)$ using Euler's Method with two equal steps.

old pt	dx	$m = \frac{dy}{dx}$	$dy = m \cdot dx$	new pt.
(2,0)	.5	$4(2)+0=8$	$8 \cdot .5 = 4$	(2.5, 4)
(2.5,4)	.5	$4(2.5)+4=14$	$14 \cdot .5 = 7$	(3, 11)

$\boxed{y(3) \approx 11}$

22.) Assume that f and f' have the values given in the table. Use Euler's Method with two equal steps to approximate the value of $f(4.4)$.

x	4	4.2	4.4
$f'(x)$	-0.5	-0.3	-0.1
$f(x)$	2	1.9	1.84

old pt	dx	$m = \frac{dy}{dx}$	$dy = m \cdot dx$	new pt
(4,2)	.2	-.5	-.1	(4.2, 1.9)
(4.2, 1.9)	.2	-.3	-.06	(4.4, 1.84)

$\boxed{f(4.4) \approx 1.84}$

23.) The table gives selected values for the derivative of a function f on the interval $-2 \le x \le 2$. If $f(-2) = 3$ and Euler's Method with a step size of 0.5 is used to approximate $f(2)$, what is the resulting approximation?

x	$f'(x)$
-2	-0.8
-1.5	-0.5
-1	-0.2
-0.5	0.4
0	0.9
0.5	1.6
1	2.2
1.5	3
2	3.7

old pt	dx	$m = \frac{dy}{dx}$	$dy = m \cdot dx$	new pt
(-2,3)	.5	-.8	-.8(.5) = -.4	
			-.5(.5) = -.25	
			-.2(.5) = -.1	
			.4(.5) = .2	
			.9(.5) = .45	
			1.6(.5) = .8	
			2.2(.5) = 1.1	
			3(.5) = 1.5	

$f(2) \approx 3 - (.4 + .25 + .1) + (.2 + .45 + .8 + 1.1 + 1.5$

$= 6.3$

24.) Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = x + 2y$ with the initial condition $f(0) = 1$. Use Euler's Method, starting at $x = 0$ with two steps of equal size to approximate $f(-0.6)$.

old pt	dx	$m = \frac{dy}{dx}$	$dy = m \cdot dx$	new pt.
(0,1)	-.3	0 + 2(1) = 2	2(-.3) = -.6	(-.3, .4)
(-.3, .4)	-.3	-.3 + 2(.4) = .5	.5(-.3) = -.15	(-.6, .25)

$\boxed{f(-.6) \approx .25}$