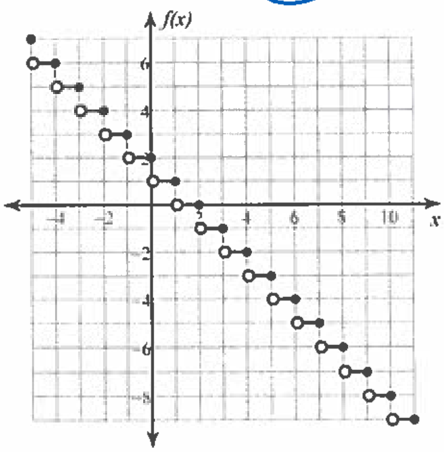
Calculus Name Key

Limits and Continuity Test Review *No Calculators* Date________________ Period____

Evaluate each limit.

1) $\lim_{x \to 3^+} \lfloor -x+2 \rfloor$ -2



-2

2) $\lim_{x \to 7} \dfrac{x-7}{2-\sqrt{x-3}} \cdot \dfrac{2+\sqrt{x-3}}{2+\sqrt{x-3}}$

-4

$\dfrac{(x-7)(2+\sqrt{x-3})}{4-(x-3)} \to \dfrac{(x-7)(2+\sqrt{x-3})}{-x+7}$

$\to \dfrac{-1\cancel{(7-x)}(2+\sqrt{x-3})}{\cancel{-x+7}}$

$\to -1(2+\sqrt{7-3})$

$\to -1(2+\sqrt{4})$

$\to -1(2+2) \to -4$

3) $\lim_{t \to -\infty} \dfrac{\sqrt{2t^2+2}}{4t+2} \to \dfrac{\pm\sqrt{2}\cdot \cancel{t}}{4\cancel{t}}$

EBM

$-\dfrac{\sqrt{2}}{4}$

since $\sqrt{\ }$ is $+$

and $4t+2$ @ $t \to -\infty$ is $-$

$\dfrac{+}{-} = -$, so $-\dfrac{\sqrt{2}}{4}$

4) $\lim_{x \to \frac{2\pi}{3}} \tan(x)$

$-\sqrt{3}$

$\tan\left(\dfrac{2\pi}{3}\right)$

$\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$

$\tan\theta = \dfrac{y}{x}$

$\tan\dfrac{2\pi}{3} = \dfrac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$

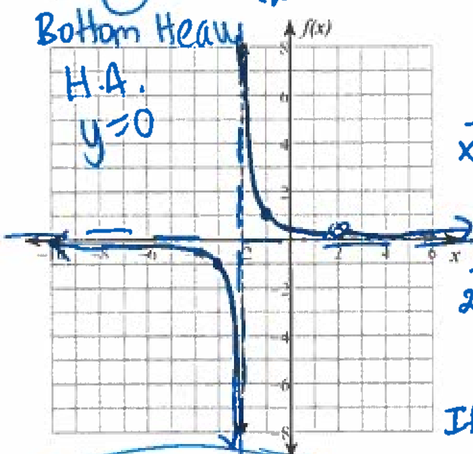
Evaluate each limit and sketch the function labeling asymptotes and holes.

5) $\lim_{x \to -2} \dfrac{x-2}{x^2-4} \to \dfrac{\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$

$x = 2$ Hole

Bottom Heavy

H.A. $y=0$



$\dfrac{1}{x+2} \to$ V.A. @ $x=-2$

$\dfrac{1}{2+2} = \dfrac{1}{4} \to$ y-value of Hole

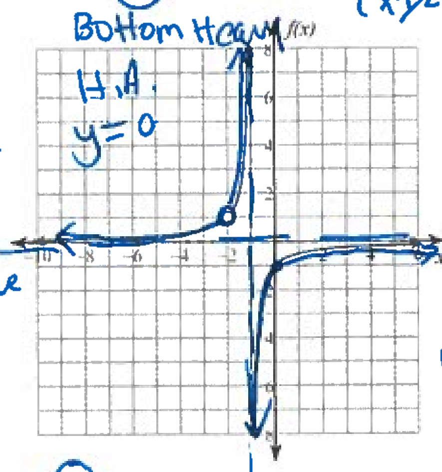
If I plug in -1, I get $+1$, if I plug in -3, I get -1.

Does not exist.

6) $\lim_{x \to -2} -\dfrac{x+2}{x^2+3x+2} \to -\dfrac{\cancel{(x+2)}}{\cancel{(x+2)}(x+1)} \to -\dfrac{1}{x+1}$

Bottom Heavy

H.A. $y=0$



Hole @ -2

$\dfrac{-1}{-2+1} \to 1$

V.A. @ $x=-1$

Plug in 0,

$\dfrac{-1}{0+1} = 1$

1

Find the intervals on which each function is continuous.

7) $f(x) = \frac{x+5}{x^2 - 3x}$ $\frac{x+5}{x(x-3)} \rightarrow x \neq 0, 3$

$(-\infty, 0) \cup (0, 3) \cup (3, \infty)$

$(-\infty, 0), (0, 3), (3, \infty)$

Determine if each function is continuous. If the function is not continuous, find the x-axis location of and classify each discontinuity.

8) $f(x) = \begin{cases} 2, & x \leq 1 \rightarrow 2 \\ \frac{x}{2} + \frac{1}{2}, & x > 1 \rightarrow \frac{1}{2} + \frac{1}{2} = 1 \end{cases}$

$\boxed{\text{Jump discontinuity at: } x = 1}$

Not cont.

9) $f(x) = \frac{x+2}{2x^2 + 2x + 1}$ $= 0$?

a b c

Continuous

discriminant $b^2 - 4ac \rightarrow 2^2 - 4(2)(1)$
(from quadratic formula)

$\rightarrow 4 - 8 = -4$, so the solutions are imaginary $\rightarrow$ never cross x-axis, so never zero!

Find all vertical and horizontal asymptotes of each function.

10) $h(x) = \frac{x^2 - 4}{\sqrt{2x^4 + 4}}$

always "+" whether ∞ or $-\infty$ since x^2

4 always +

No vertical and horizontal at $y = \frac{\sqrt{2}}{2}$

$\sqrt{2x^4} \rightarrow \pm\sqrt{2} \cdot x^2$

$\frac{x^2}{\sqrt{2} \cdot x^2} \rightarrow \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{2}$

$(\sqrt{2x^4 + 4})^2 = (0)^2$? $\rightarrow$ so no V.A.

$2x^4 + 4 = 0 \rightarrow 2x^4 = -4$ no!

11) $h(x) = \frac{3x - 21}{x^2 + 4x - 6}$

Bottom Heavy $y = 0$ H.A.

Vertical at $x = -2 \pm \sqrt{10}$ and horizontal at $y = 0$

$x^2 + 4x - 6 = 0$?

a b c

$\frac{-4 \pm \sqrt{4^2 - 4(1)(-6)}}{2(1)}$

$\frac{-4 \pm \sqrt{16 + 24}}{2} \rightarrow \frac{-4 \pm \sqrt{40}}{2}$

$\frac{-4 \pm 2\sqrt{10}}{2} \rightarrow -2 \pm \sqrt{10}$

12) Find all the vertical asymptotes for the function

$f(x) = \frac{1}{2\cos^2 x + (2 - \sqrt{3})\cos x - \sqrt{3}}$ where $0 < x \leq 2\pi$

$\frac{\pi}{6}, \frac{11\pi}{6}, \pi$

$(2\cos x - \sqrt{3})(\cos x + 1)$

$2\cos x - \sqrt{3} = 0$ $\cos x + 1 = 0$

Q1, Q4 $\cos x = \frac{\sqrt{3}}{2}$ $\cos x = -1$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$ $x = \pi$

13) Determine the constants a and b that make the following function continuous.

$$f(x)=\begin{cases}2x-4a, & x<-3\\ 3ax+b, & -3\le x<2\\ 4x-5b, & x\ge 2\end{cases}$$

$a=\frac{11}{9}$ $b=\frac{1}{9}$

$2(-3)-4a=-6-4a$

$3a(-3)+b = -9a+b=-6-4a$

$-5a+b=-6$

$4(2)-5b$
$=8-5b$

$3a(2)+b$
$=6a+b$

$8-5b=6a+b$

$(8=6a+6b)\div 2$

$4=3a+3b$

$(-6=-5a+b)3$

$4=3a+3b$

$-(-18=-15a+3b)$

$22=18a$

$a=\frac{22}{18}=11/9$

$-5\left(\frac{11}{9}\right)+b=-6$

$-\frac{55}{9}+b=-6$

$b=-\frac{54}{9}+\frac{55}{9}=1/9$

14) If $f(x)=\dfrac{x-1}{x}$ and $g(x)=1-x$, then $f(g(x))=$

A) -1 B) $\dfrac{1}{1-x}$ C) $\dfrac{1}{x}$

*D) $\dfrac{x}{x-1}$ E) $\dfrac{x}{1-x}$

$f(g(x))=\dfrac{g(x)-1}{g(x)}$

$=\dfrac{1-x-1}{1-x}=\dfrac{-x}{1-x}\to\dfrac{x}{-(1-x)}\to\dfrac{x}{x-1}$

15) What is the point of discontinuity, c for the function $h(x)=\dfrac{2x^2+x-6}{6-4x}$?

A) $-\dfrac{3}{2}$ B) $-\dfrac{2}{3}$ C) $\dfrac{2}{3}$ D) 6 *E) $\dfrac{3}{2}$

$6-4x=0$

$-4x=-6$

$x=\frac{3}{2}\to c$

16) What value should be assigned to $h(c)$ to make the function continuous?

A) $\dfrac{4}{7}$ *B) $-\dfrac{7}{4}$ C) 2 D) $-\dfrac{1}{6}$ E) $\dfrac{3}{2}$

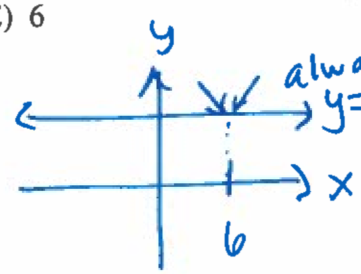
$\dfrac{2x^2+x-6}{-4x+6}=\dfrac{\cancel{(2x-3)}(x+2)}{-2\cancel{(2x-3)}}$

$\dfrac{x+2}{-2}\to\dfrac{\frac{3}{2}+2}{-2}=\dfrac{\frac{7}{2}}{-2}=-7/4$

constant

17) $\lim\limits_{x\to 6} 5=$

A) Does Not Exist *B) 5
C) -5 D) 1
E) 6



always goes to 5

18) $\lim\limits_{x\to 3}\dfrac{x}{x-3}=$ → V.A. @ 3

*A) Does Not Exist B) $-\infty$
C) -3 D) 1
E) ∞

$\lim\limits_{x\to 3^-}$

$\dfrac{x}{x-3}\to\dfrac{2.9}{2.9-3}\to\dfrac{+}{-}\to-\infty$

$\neq$

$\lim\limits_{x\to 3^+}\dfrac{3.1}{3.1-3}\to\dfrac{+}{+}\to\infty$

19) Determine the points of discontinuity of $f(x) = \frac{3}{x} + \frac{x-3}{2x-5}$ → zero on top is not P.O.D :)

x → $x=0$; $2x-5$ → $x = 5/2$

A) $0, \frac{5}{2}, 3$

*B) $0, \frac{5}{2}$

C) No points of discontinuity

D) 0

E) $\frac{5}{2}$

20) Is the function $h(x)$ continuous at $x = 0$? Justify your answer.

$$h(x) = \begin{cases} \frac{\sin 5x}{x}, & x \neq 0 \\ (x-5)(x-1), & x = 0 \end{cases}$$

$\lim_{x \to 0} \frac{\sin 5x}{x} \cdot \frac{5}{5} \to \lim_{x \to 0} \frac{\sin 5x}{5x} \cdot 5 \to 5$

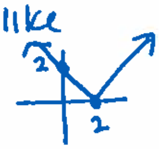
$(0-5)(0-1) = 5$

Yes. LHL= RHL= $h(0) = 5$

Since $\lim_{x \to 0^-} h(x) = \lim_{x \to 0^+} h(x) = \lim_{x \to 0} h(x) = f(0) = 5$, $h(x)$ is cont. @ $x = 0$.

$-(3x^2-5x-2) \to -(3x+1)(x-2)$

21) $\lim_{x \to 2^+} \frac{2+5x-3x^2}{|2-x|}$ → $y = |2-x|$ looks like



So $-x+2$ if $x<2$

but $x-2$ if $x>2$

-7 $x>2$

$\frac{-(3x+1)\cancel{(x-2)}}{\cancel{x-2}}$

$-(3(2)+1)$

-7

22) $\lim_{x \to 2^-} \frac{2+5x-3x^2}{|2-x|}$

7 $x<2$

$\frac{-(3x+1)(x-2)}{-x+2}$

$\frac{-(3x+1)\cancel{(x-2)}}{-\cancel{(x-2)}}$

$\frac{-(3(2)+1)}{-1} = \frac{-7}{-1} = 7$

23) $\lim_{x \to 2} \frac{2+5x-3x^2}{|2-x|}$

DNE

LHL ≠ RHL

24) 1,2,3 Without graphing, provide a written explanation using the IVT to explain why the function $f(x)=x^2-2x-8$ has a zero in the interval [2,5]. Then, state the value of the zero that is guarenteed. 4

① $f(x)$ is continous on closed interval $[2,5]$

② $f(2)=2^2-2(2)-8=-8$

$f(5)=5^2-2(5)-8=7$

③ since $-8<0<7$, by IVT there is a "c" in $(2,5)$ such that $f(c)=0$.

④ $x^2-2x-8=0$

$(x-4)(x+2)=0$

$x=4,-2 \rightarrow$ not in $[2,5]$

$x=4$

25) $f(x)=\begin{cases}\dfrac{2x-3}{x^2}, & x>4\\ 3^x-1, & x\le 4\end{cases}$

DNE a.) $\lim\limits_{x\to 4} f(x)$ $\qquad \dfrac{2(4)-3}{4^2}=\dfrac{5}{16} \qquad 3^4-1=80 \qquad$ LHL ≠ RHL

8 b.) $\lim\limits_{x\to 2} f(x)$ ↳ <4 ☺ $\qquad 3^2-1=8$

0 c.) $\lim\limits_{x\to\infty} f(x)$ EBM $\quad x>4 \quad$ Bottom Heavy

-1 d.) $\lim\limits_{x\to-\infty} f(x)$ EBM $\quad x<4 \qquad 3^{-\infty}-1 \rightarrow \dfrac{1}{3^\infty}-1 \rightarrow 0-1=-1$

↳ Bottom Heavy

Evaluate each limit.

26) $\lim\limits_{x\to 3} -\dfrac{2}{x^2-6x+9} \rightarrow \dfrac{-2}{(x-3)^2}$ $\quad x=3$ V.A.

$-\infty$

$3^- \rightarrow 2.9$

$3^+ \rightarrow 3.1$ → Both make $(x-3)^2$ +

So $\dfrac{-2}{+} \rightarrow -\infty$

27) $\lim\limits_{x\to\infty} \dfrac{x^2-2x^3+10}{5+3x^2}$ EBM $\qquad \dfrac{-2x^3}{3x^2}=\dfrac{-2x}{3}$

$-\infty$

↳ Linear slope is negative

$-\infty$

28) $\lim\limits_{x\to 2} \dfrac{\dfrac{(x-4)}{(x-4)}\dfrac{1}{x-3}-\dfrac{2}{x-4}\dfrac{(x-3)}{(x-3)}}{x-2}$

$-\dfrac{1}{2}$

$x-4-2(x-3)$

$x-4-2x+6$

$\dfrac{\dfrac{-x+2}{(x-4)(x-3)}}{x-2} \rightarrow \dfrac{-\cancel{(x-2)}}{(x-4)(x-3)\cancel{x-2}} \rightarrow \dfrac{-1}{(x-4)(x-3)}$

$\downarrow$

$(-1/2) \leftarrow \dfrac{-1}{(-2)(-1)} \Leftarrow \dfrac{-1}{(2-4)(2-3)}$

29) $\lim\limits_{x\to 0} \dfrac{\sin 2x}{4x^2+8x}$ $\quad 2x(2x+4)$ $\qquad$ can't change

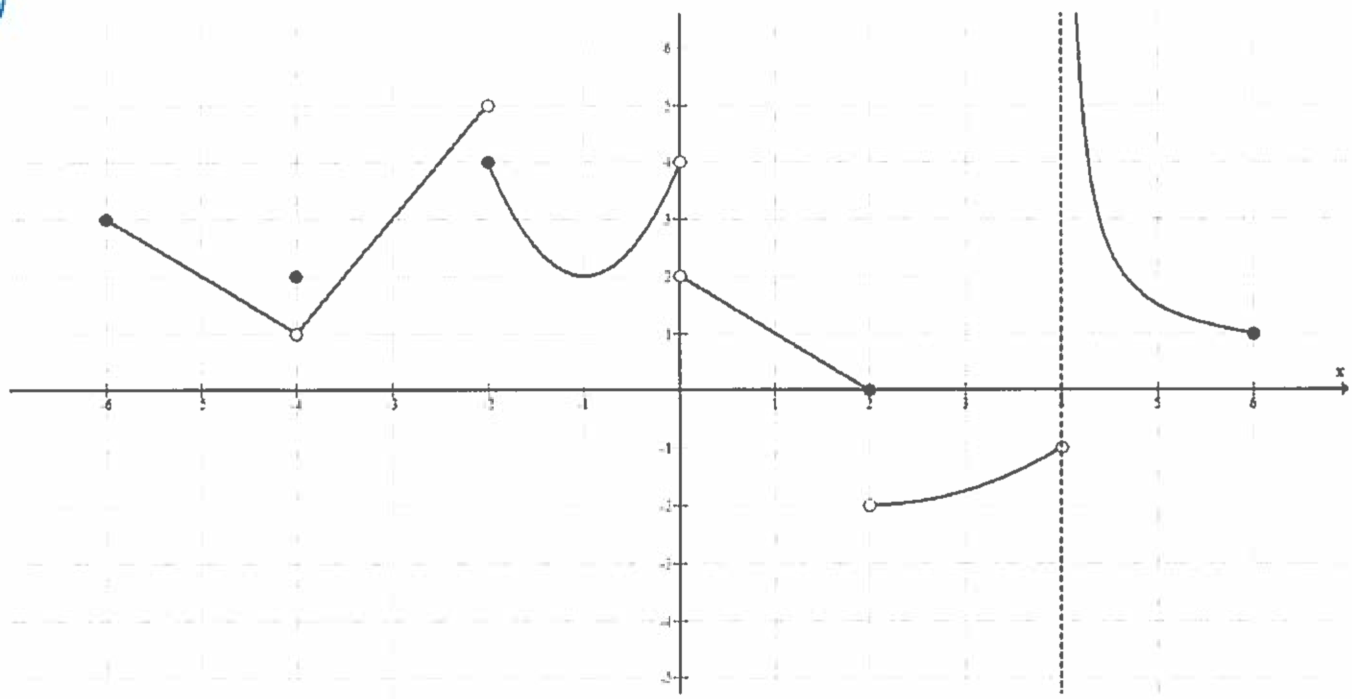
$\dfrac{1}{4}$

$\lim\limits_{x\to 0} \dfrac{\sin \cancel{2x}}{\cancel{2x}} \cdot \dfrac{1}{2x+4}$

$\lim\limits_{x\to 0} \dfrac{1}{2x+4} \rightarrow \dfrac{1}{2(0)+4}$

$\rightarrow 1/4$

30.)



The graph of a function f is drawn above, answer the questions:

a $f(-4) =?$	b $\lim_{x\to-4^-} f(x) = ?$	c $\lim_{x\to-4^+} f(x) =?$	d $\lim_{x\to-4} f(x) =?$
2	1	1	1
e $f(-2) =?$	f $\lim_{x\to-2^-} f(x) = ?$	g $\lim_{x\to-2^+} f(x) =?$	h $\lim_{x\to-2} f(x) =?$
4	5	4	DNE
i $f(0) =?$	j $\lim_{x\to0^-} f(x) = ?$	k $\lim_{x\to0^+} f(x) =?$	l $\lim_{x\to0} f(x) =?$
undefined	4	2	DNE
m $f(2) =?$	n $\lim_{x\to2^-} f(x) = ?$	o $\lim_{x\to2^+} f(x) =?$	p $\lim_{x\to2} f(x) =?$
0	0	-2	DNE
q $f(4) =?$	r $\lim_{x\to4^-} f(x) = ?$	s $\lim_{x\to4^+} f(x) =?$	t $\lim_{x\to4} f(x) =?$
Undefined	-1	∞	DNE