PreCalculus 4B Test 2 Review

Name: Key Date: _____

1. What is the period of the function $f(x) = 2\sin\frac{1}{2}x$?

A. $\frac{1}{2}$ B. π C. 2π D. 4π

$\frac{2\pi}{1/2} = 4\pi$

2. Determine the period of the function:

$y = \frac{1}{2}\sin\left(\frac{x}{3} - \pi\right)$

A. $\frac{1}{2}$ B. $\frac{2\pi}{3}$ C. 6π D. 9π

$\frac{1}{2}\sin\left(\frac{1}{3}x - \pi\right)$

$\frac{2\pi}{1/3} = 6\pi$

3. What is the period for the function $y = 4\sec 2\pi x$?

A. 2π B. 4π C. 4 D. 1

$\frac{2\pi}{2\pi} = 1$

4. Determine the the period of the function $y = 3\tan 7x$.

A. $\frac{\pi}{7}$ B. $\frac{\pi}{3}$ C. $\frac{2\pi}{3}$ D. 6π

$\frac{\pi}{7}$

5. As x increases from 0 to π radians, $\sin x$:

A. increases throughout the interval
B. decreases throughout the interval
C. increases, then decreases
D. decreases, then increases

y↓ y↑

6. For $y = 17 + 8\cos 6(2\theta - 32)$ state the (1) amplitude, (2) period, (3) vertical shift (positive or negative), and (4) phase displacement.

$17 + 8\cos 6\cdot 2(\theta - 16)$

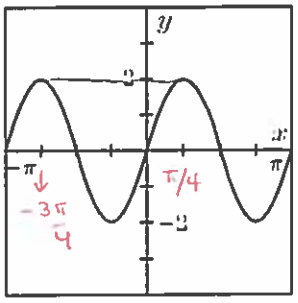
$17 + 8\cos 12(\theta - 16$

A. (1) 8, (2) $\frac{\pi}{6}$, (3) 17, (4) 16
B. (1) 8, (2) $\frac{6}{\pi}$, (3) 17, (4) 16
C. (1) 17, (2) $\frac{\pi}{6}$, (3) 17, (4) 32
D. (1) 8, (2) $\frac{6}{\pi}$, (3) 16, (4) 17

$\frac{2\pi}{12} = \frac{\pi}{6}$

7. Which of the following equations describes the graph?

A. $y = 2\sin\frac{\theta}{2}$
B. $y = \frac{1}{2}\sin\theta$
C. $y = \sin\theta$
D. $y = 2\sin 2\theta$



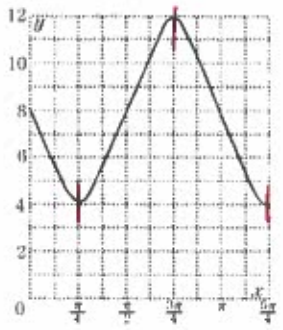
$\sin k\theta$

$P = \frac{2\pi}{K} \rightarrow \pi = \frac{2\pi}{k}$

$k\pi = 2k$

$k = 2$

8. Given the graph of this trigonometric function, over which domain(s) could an inverse function be constructed?



I. $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ ✓

II. $\left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$ ✓

III. $\left[0, \frac{5\pi}{4}\right]$ X repeats

A. I only

B. I and II

C. II and III

D. I, II and III

9. Which of the following is the graph of $f(x) = \frac{1}{2}\sec(x - \frac{\pi}{2})$?

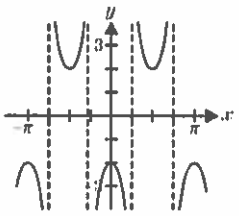
↳ right $\frac{\pi}{2}$

$\sec\theta = \frac{1}{\cos\theta}$

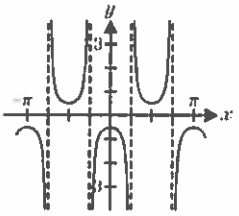
$\cos\theta = 0$ @ $\frac{\pi}{2}$

so asymptotes @ $\frac{\pi}{2}$'s are now @ π's

A.



B.



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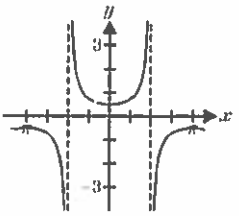

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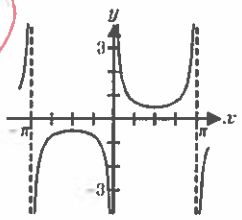
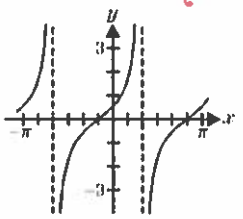


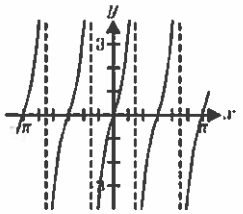
10. Which of the following is the graph of $f(x) = 2\cot(x + \frac{\pi}{3})$?

A.

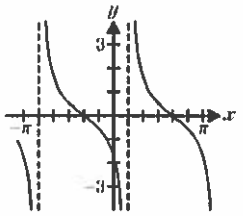
B.



C.



D.



$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sin\theta = 0$

@ $0, \pi$, etc.

So asymptotes

@ $0, \pi$, etc.

But

shift

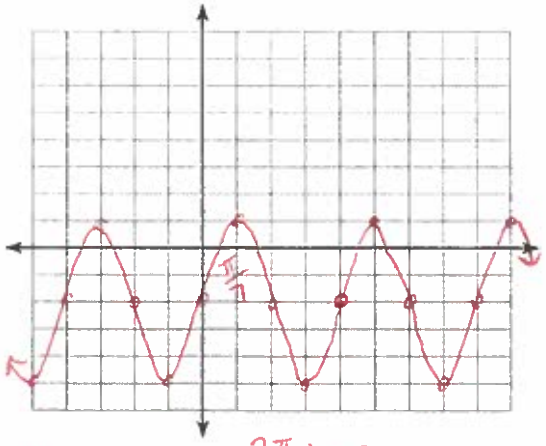
$\frac{\pi}{3}$ left

so $-\frac{\pi}{3}, \frac{2\pi}{3}$, etc.

Fill out the given table. Sketch the graph.

$\frac{1}{4} \div \frac{1}{3} = \frac{1}{4} \cdot \frac{3}{1} = \frac{3}{4}$

11) $y = 3\sin(3.5x - 2\pi) - 2$



$3\sin 3.5\left(x - \frac{2\pi}{3.5}\right) - 2$

$2 \div 3.5? \rightarrow 2 \div \frac{7}{2} \rightarrow 2 \cdot \frac{2}{7} = \frac{4}{7}$

Period	$2\pi/3.5 = 4\pi/7$
Amplitude	3
Midline	−2
Max	−5
Min	1
Phase Shift	$4\pi/7$ Right
Count By #	$\pi/7$
X-axis Scale	$\pi/7$
Asymptotes	n/a

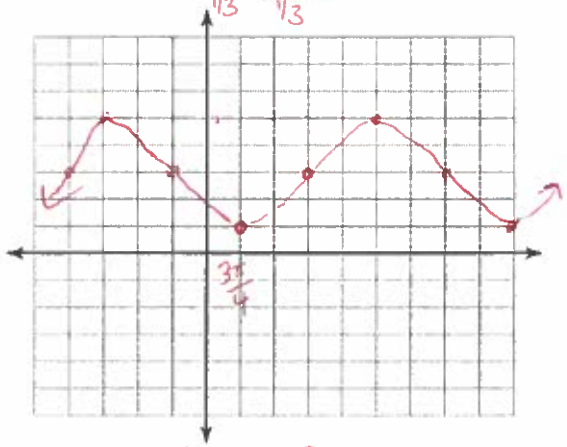
mid
max
mid
min
mid

$3\sin 3.5\left(x - \frac{4\pi}{7}\right) - 2$

$\frac{P}{4} = \frac{4\pi/7}{4} = \pi/7$

12) $y = -2\cos\left(\frac{1}{3}x - \frac{\pi}{4}\right) + 3$

1/3 1/3



$-2\cos\frac{1}{3}\left(x - \frac{3\pi}{4}\right) + 3$

Period	$2\pi/(1/3) = 6\pi$
Amplitude	2
Midline	3
Max	5
Min	1
Phase Shift	$3\pi/4$ Right
Count By #	$3\pi/2$ (2 @ a time)
X-axis Scale	$3\pi/4$ → otherwise, P.S. will be in between
Asymptotes	n/a

min
mid
max
mid
min

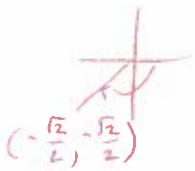
$\frac{6\pi}{4} = \frac{3\pi}{2}$

13) The graphs of sec θ and tan θ have asymptotes at $\frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}$, etc. because both have cos θ in the denominator and cos $\frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}$ is equal to zero.

Find the exact value of each trigonometric function.

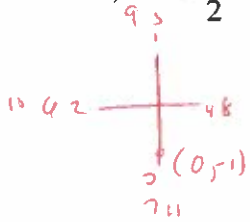
14) $\sin -\frac{17\pi}{6}$ $-\frac{1}{2}$

15) $\sec -135°$ $\frac{2}{-\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$

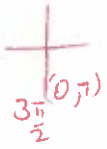


16) $\cos \frac{23\pi}{6}$ $\frac{\sqrt{3}}{2}$

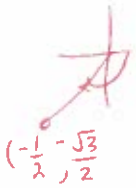
17) $\tan \frac{11\pi}{2}$ $\frac{-1}{0}$ undefined



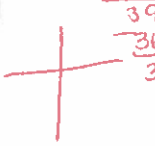
18) $\cot \frac{3\pi}{2}$ 0



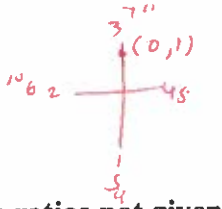
19) $\sec -480°$ $\frac{1}{-1/2} = -2$



20) $\tan 750°$ $\frac{1}{2} \div \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$



21) $\csc -\frac{11\pi}{2}$ $\frac{1}{1} = 1$

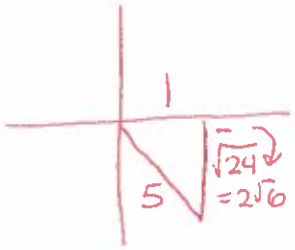


Find the exact values of the five trigonometric ratios not given.

22) $\sec \theta = 5$ and $\sin \theta < 0$ IV

I, IV III, IV

$\sin\theta = \frac{-2\sqrt{6}}{5}$ $\csc\theta = \frac{5}{-2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = -\frac{5\sqrt{6}}{12}$



$\cos\theta = \frac{1}{5}$

$\tan\theta = -2\sqrt{6}$

$\cot\theta = \frac{1}{-2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = -\frac{\sqrt{6}}{12}$

Find the exact value of each expression.

23) $\tan^{-1}\sqrt{3}$ $\frac{\pi}{3}$

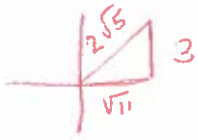
24) $\sin^{-1}-\frac{\sqrt{2}}{2}$ $-\frac{\pi}{4}$

25) $\sec^{-1}-\frac{2\sqrt{3}}{3}$ $\frac{5\pi}{6}$

26) $\cos^{-1}\frac{3\pi}{4}$ Undefined

27) $\cot^{-1}(\cos\pi)$ -1 $\frac{3\pi}{4}$

28) $\sin\sec^{-1}\frac{2\sqrt{55}}{11}$ $\frac{2\sqrt{55}}{11}\cdot\frac{\sqrt{11}}{\sqrt{11}}=\frac{2\cdot\sqrt{11}\sqrt{5}}{11\sqrt{11}}\to\frac{2\sqrt{5}}{\sqrt{11}}$



$(2\sqrt{5})^2=(\sqrt{11})^2+y^2$

$20=11+y^2$

$y=3$

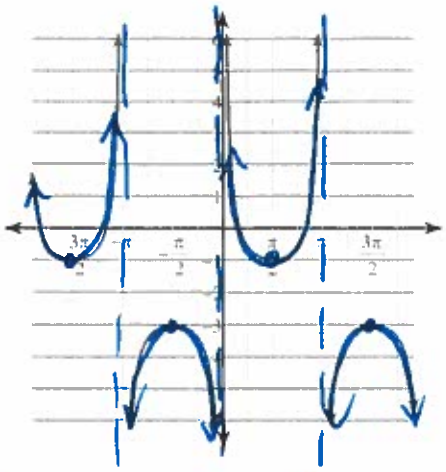
$\frac{3}{2\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{3\sqrt{5}}{10}$

29) $\sec^{-1}\left(\tan-\frac{\pi}{4}\right)$ -1 π

30) $\cos^{-1}\left(\csc\frac{\pi}{2}\right)$ $\cos^{-1}1=0$

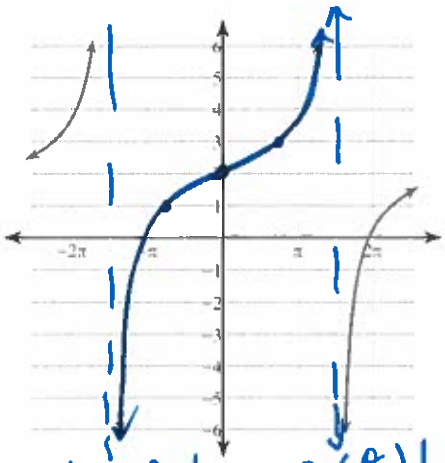
Graph each function using radians.

31) $y = \csc\theta - 2$



θ	$\sin\theta$	$\csc\theta$	$\csc\theta - 2$
0	0	asy	asy
$\pi/2$	1	1	−1
π	0	asy	asy
$3\pi/2$	−1	−1	−3
2π	0	asy	asy

32) $y = \tan\frac{\theta}{3} + 2$

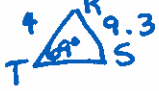


θ	$\tan\theta$	new θ $(\frac{\theta}{3})$	$\tan(\frac{\theta}{3})+2$
0	0	0	2
$\pi/4$	1	$3\pi/4$	3
$\pi/2$	asy	$3\pi/2$	asy
$3\pi/4$	−1	$9\pi/4$	1
π	0	3π	2

State the number of possible triangles that can be formed using the given measurements.

33) In $\triangle TRS$, $m\angle T = 69°$, $s = 4$ yd, $t = 9.3$ yd

One triangle



$\frac{\sin 69°}{9.3} = \frac{\sin S}{4}$ ✓

$\angle S = 23.7°$ or ~~$180 - 23.7$~~ too big w/ $\angle T = 69°$

(1)

34) In $\triangle HPK$, $m\angle H = 38°$, $k = 35$ m, $h = 33$ m

Two triangles

$\frac{\sin 38°}{33} = \frac{\sin K}{35}$

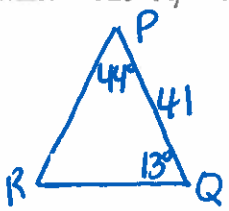
$\angle K = 40.8°$ ✓ or $180 - 40.8 = 139.2$ ✓ OK w/ $\angle H = 38°$

(2)

Solve each triangle. Round your answers to the nearest tenth.

35) In $\triangle RPQ$, $m\angle P = 44°$, $m\angle Q = 13°$, $r = 41$ m

$m\angle R = 123°$, $q = 11$ m, $p = 34$ m



$\frac{\sin 123°}{41} = \frac{\sin 13°}{q}$

$q = \frac{41\sin 13°}{\sin 123°}$

$q = 11$ m

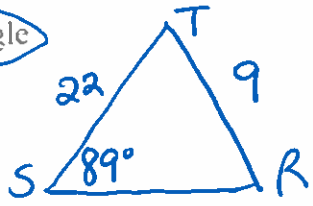
$\angle R = 123°$

$\frac{\sin 44°}{p} = \frac{\sin 123°}{41}$

$p = 34$ m

36) In $\triangle STR$, $m\angle S = 89°$, $r = 22$ ft, $s = 9$ ft

Not a triangle



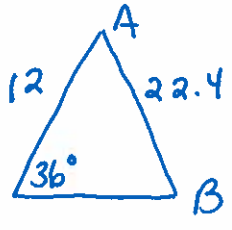
$\frac{\sin 89°}{9} = \frac{\sin R}{22}$

$\sin^{-1}\left(\frac{22\sin 89°}{9}\right) = R$

NO solution

37) In $\triangle CAB$, $m\angle C = 36°$, $b = 12$ in, $c = 22.4$ in

$m\angle A = 125.6°$, $m\angle B = 18.4°$, $a = 31$ in



2nd ↓ $\frac{\sin 125.6}{a} = \frac{\sin 36}{22.4}$

$a = 31$in

1st ↓ $\frac{\sin 36°}{22.4} = \frac{\sin B}{12}$

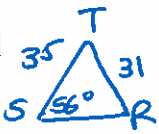
$\angle B = 18.4°$ or ~~161.6° too big w/ 36°~~

$\angle A = 125.6°$

38) In $\triangle STR$, $m\angle S = 56°$, $r = 35$ yd, $s = 31$ yd

$m\angle T = 54.6°$, $m\angle R = 69.4°$, $t = 30.5$ yd

Or $m\angle T = 13.4°$, $m\angle R = 110.6°$, $t = 8.7$ yd



$\frac{\sin 56°}{31} = \frac{\sin R}{35}$

$\angle R = 69.4°$ or $110.6°$

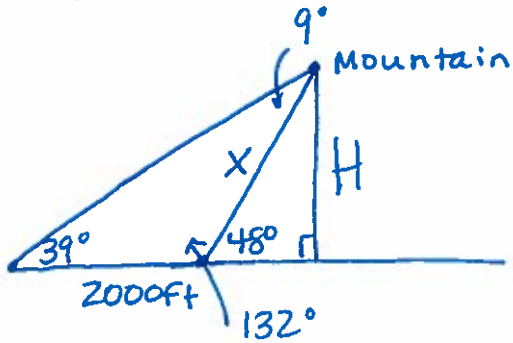
$\angle T = 54.6°$ or $13.4°$

$\frac{\sin 56°}{31} = \frac{\sin 54.6°}{t}$ or $\frac{\sin 56°}{31} = \frac{\sin 13.}{t}$

$t = 30.5$ yd or 8.7 yd

39) To estimate the height of a mountain above a level plain, the angle of elevation to the top of the mountain is measured to be 39°. Two thousand feet closer to the mountain along the plain, it is found that the angle of elevation is 48°. Estimate the height of the mountain.

5979 ft



$\frac{\sin 9°}{2000} = \frac{\sin 39°}{x}$

$x = 8045.8$

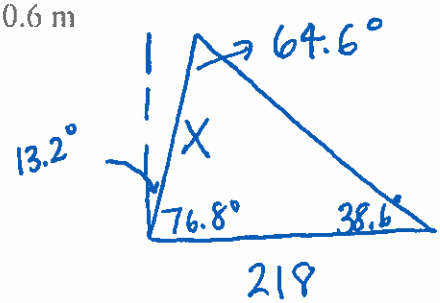
$\sin 48° = \frac{H}{x \to 8045.8}$

$180° - (132° + 39°) =$

$H = 5,979$ ft

40) The bell tower of the cathedral in Pisa, Italy, leans 13.2° from the vertical. A tourist stands 218 m from its base, with the tower leaning directly toward her. She measures the angle of elevation to the top of the tower to be 38.6°. Find the length of the tower to the nearest tenth of a meter.

150.6 m



$\frac{\sin 64.6°}{218} = \frac{\sin 38.6°}{x}$

$x = 150.6$ m