Test 2 Practice

Name

1. What is the average rate of change of the function *f* given by $f(x) = x^4 - 5x$ on the closed interval [0, 3]?

$$\frac{f(3)-f(0)}{3-0}$$

$$\frac{66-0}{3}$$

$f(3) = 3^4 - 5(3)$
$= 81 - 15$
$= 66$
$f(0) = 0$

(A) 8.5

(B) 8.7

(C) 22

(D) 33

(E) 66

2. $\lim_{h \to 0} \frac{\ln(4+h) - \ln(4)}{h}$ is

$f(x+h) - f(x)$

derivative of ln x when x = 4

Hint * $\frac{d}{dx} \ln x = \frac{1}{x}$

(A) 0

(B) $\frac{1}{4}$

(C) 1

(D) *e*

(E) nonexistent

3. At $x = 3$, the function given by

$$f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geq 3 \end{cases} \text{ is}$$

$2x$

6

$2x = 6$ if $x = 3$ ✓

$3^2 = 9$

$6(3) - 9 = 18 - 9 = 9$

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(A) ~~undefined~~

(B) continuous but not differentiable

never the answer

(C) ~~differentiable but not continuous~~

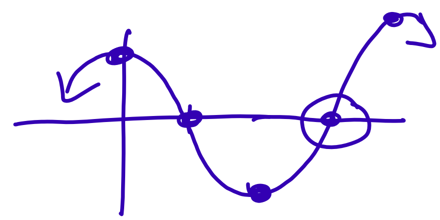
(D) ~~neither continuous nor differentiable~~

(E) both continuous and differentiable

$f(x+h)-f(x)$

4. What is $\lim\limits_{h\to 0}\frac{\cos(\frac{3\pi}{2}+h)-\cos(\frac{3\pi}{2})}{h}$?

what is the derivative of cos x at $x=\frac{3\pi}{2}$?



(A) 1

(B) $\frac{\sqrt{2}}{2}$

(C) 0

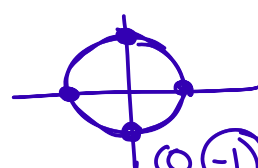
(D) -1

(E) The limit does not exist.

$\frac{d}{dx}\cos x = -\sin x$

$-\sin\left(\frac{3\pi}{2}\right)$

$-\,-1$



(0, -1)

(cos, sin)

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5. Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x = -1$?

$f'(x) = 12x^2 - 5$

$f'(-1) = 12(-1)^2 - 5 = 7$

$f(-1) = 4(-1)^3 - 5(-1) + 3$

$4(-1) + 5 + 3$

$-4 + 5 + 3$

4

$\boxed{y - 4 = 7(x+1)}$

$y - 4 = 7x + 7 \longrightarrow y = 7x + 11$

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(A) y = 7x - 3

(B) y = 7x + 7

(C) y = 7x + 11

(D) y = -5x - 1

(E) y = -5x - 5

$-2+-1=k$

6. In the xy-plane, the line $x + y = k$, where k is a constant, $\boxed{\text{is tangent}}$ to the graph of $y = x^2 + 3x + 1$. $\boxed{\text{What is the value of } k\text{?}}$

$y = -1x + k$ ✓ slope-intercept

slope is -1 ✓

(A) −3

(B) −2

$\frac{dy}{dx} = 2x + 3$

$2x + 3 = -1$

$2x = -4$

$x = -2$

(C) −1

$y = (-2)^2 + 3(-2) + 1$

$= 4 - 6 + 1$

$y = -1$

(D) 0

(E) 1

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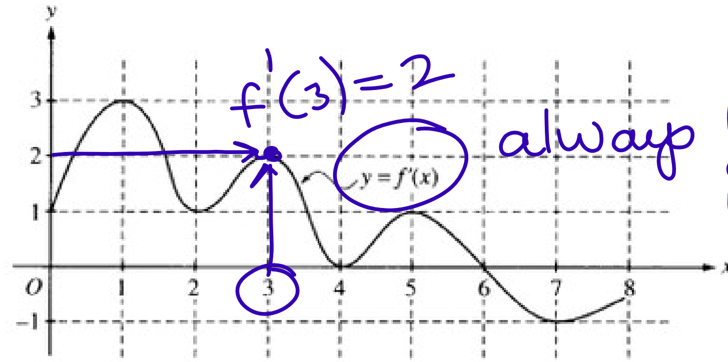
7. Refer to the graph and the information below.



$f'(3)=2$

always look at what graph is

The function *f* is defined on the closed interval [0,8]. The graph of its derivative f' is shown above.

The point (3,5) is on the graph of *y=f(x)* . An equation of the line tangent to the graph of *f* at (3, 5) is

$y-5=2(x-3)$

(A) *y*=2

(B) *y*=5

(C) *y*-5=2(*x*-3)

(D) *y*+5=2(*x*-3)

(E) *y*+5=2(*x*+3)

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8. $f(x)=\begin{cases}\frac{x^2-4}{x-2} & \text{if } x\neq 2\\ 1 & \text{if } x=2\end{cases}$

$\frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$ $2+2=4$

Let *f* be the function defined above. Which of the following statements about *f* are true?

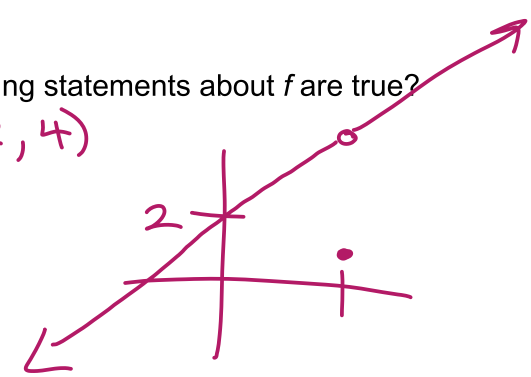
~~I.~~ *f* has a limit at x=2. (it's 4)

~~II.~~ *f* is continuous at x=2.

~~III.~~ *f* is differentiable at x=2.

(you can take the derivative)

Hole at (2, 4)



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(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

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9. $f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$

$f'(x) = c$

$f'(x) = 2x - c$

Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

(A) −4

(B) −2

(C) 0

(D) 2

(E) 4

$2c + d = 2^2 - 2c$

$2c + d = 4 - 2c$

$\boxed{4c + d = 4}$

$4(2) + d = 4$

$8 + d = 4$

$d = -4$

$c = 2(2) - c$

$c = 4 - c$

$2c = 4$

$c = 2$

$2 + (-4)$

$= -2$

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$f'(x)=1$

$f'(x)=4$

10. $f(x) = \begin{cases} x + 2 \text{ if } x \leq 3 \\ 4x - 7 \text{ if } x > 3 \end{cases}$ one has an equal

Let *f* be the function given above. Which of the following statements are true about *f* ?

I. $\lim_{x \to 3} f(x)$ exists.

II. *f* is continuous at *x* = 3.

III. *f* is differentiable at *x* = 3.

$3+2=4(3)-7$

$5=5$

$1 \neq 4$

(A) None

(B) I only

(C) II only

(D) I and II only

(E) I, II, and III

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either $2+x-3$ OR $2-x+3$

11. If *f(x)* = *2* + | *x-3* | for all *x*, then the value of the derivative *f′(x)* at *x* = *3* is

$(x-1)$ $-x+5$

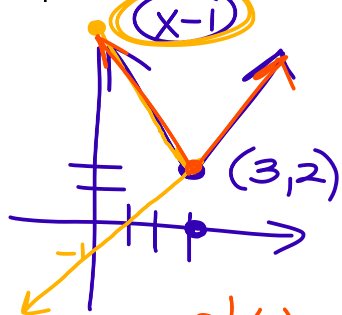
(A) -1

(B) 0

(C) 1

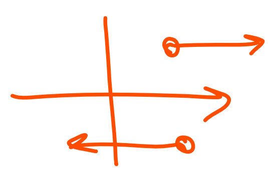
(D) 2

(E) Nonexistent



$f'(3)$

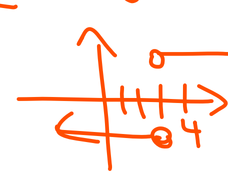
$f'(x)=$



$f(x)=\frac{x-3}{|x-3|}$

$\frac{x-3}{x-3}$ OR $\frac{x-3}{-x+3}$

1 or -1



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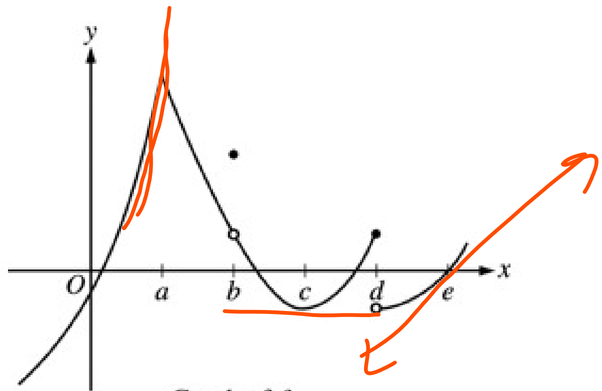
12.



Graph of f

The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a

(B) b

(C) c

(D) d

(E) e

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13. The $\lim\limits_{h \to 0} \frac{\overset{f(x+h)}{\tan 3(x+h)} - \overset{f(x)}{\tan(3x)}}{h}$ is

what is the derivative of tan (3x)?

$\frac{d}{dx} \tan x = \sec^2(x)$

$\frac{d}{dx} \tan(3x) = \sec^2(3x) \cdot 3$

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(A) 0

(B) $3\sec^2(3x)$

(C) $\sec^2(2x)$

(D) $3\cot(3x)$

(E) nonexistent

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \frac{x^2-2}{x}$ for all $x \neq 0$.

$y+3 = \frac{7}{2}(x-4)$

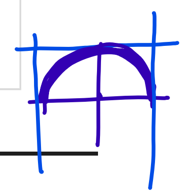
14. Write an equation for the line tangent to the graph of h at $x = 4$.

$x=4$ $\quad h'(4) = \frac{4^2-2}{4} = \frac{14}{4} = \frac{7}{2}$

$y=-3$

Please respond on separate paper, following directions from your teacher.

$y = \sqrt{25-x^2}$ $\quad y^2 = 25 - x^2$ $\quad x^2+y^2=25$



$f(-3) = \sqrt{25-(-3)^2} = 4$

The function f is defined by $f(x) = \sqrt{25-x^2}$ for $-5 \leq x \leq 5$.

$f(x) = (25-x^2)^{\frac{1}{2}}$ $\quad f'(x) = \frac{1}{2}(25-x^2)^{-\frac{1}{2}} \cdot (-2x)$

15. Write an equation for the line tangent to the graph of f at $x = -3$.

$f'(x) = \frac{-x}{\sqrt{25-x^2}}$

$f'(-3) = \frac{1}{2}(25-(-3)^2)^{-\frac{1}{2}}$

$y-4 = \frac{3}{4}(x+3)$

Let f be the function given by $f(x)=x^3-7x+6$.

$x=-1$ $\quad f(-1)=12$

$f'(x) = 3x^2-7$

$f'(-1) = -4$

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16. Write an equation of the line tangent to the graph of *f* at $x = -1$.

$y - 12 = -4(x + 1)$

Let $f(x) = 12 - x^2$ for $x \geq 0$ and $f(x) \geq 0$.

17. The line tangent to the graph of *f* at the point $(k, f(k))$ intercepts the *x*-axis at $x = 4$. What is the value of *k*?

y=0 here

$f'(x) = -2x$

$y - f(k) = -2k(x - k)$

$0 - f(k) = -2k(4 - k)$

$-12 + k^2 = -2k(4 - k)$

$-12 + k^2 = -8k + 2k^2$

$+12 - k^2 \quad +12 \quad -k^2$

$0 = k^2 - 8k + 12$

$0 = (k - 6)(k - 2)$

$k = \cancel{6}, 2$

$k = 2$

Let *f* be the function defined by $f(x) = 3x^5 - 5x^3 + 2$.

18. Write the equation of each horizontal tangent line to the graph of *f*.

$f'(x) = 0$

$15x^4 - 15x^2 = 0$

$15x^2(x^2 - 1) = 0$

$15x^2(x - 1)(x + 1) = 0$

$x = 0, 1, -1$

$y = 2 \quad y = 0 \quad y = 4$

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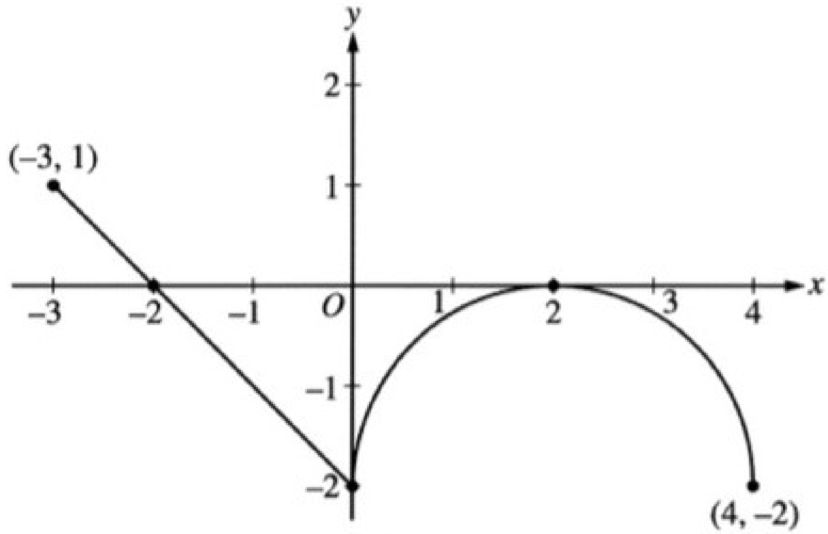

Graph of f'

Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

19. Find an equation for the line tangent to the graph of f at the point (0, 3).

$f'(0) = -2$ (according to graph of $f'(x)$)

$\boxed{y - 3 = -2(x - 0)}$

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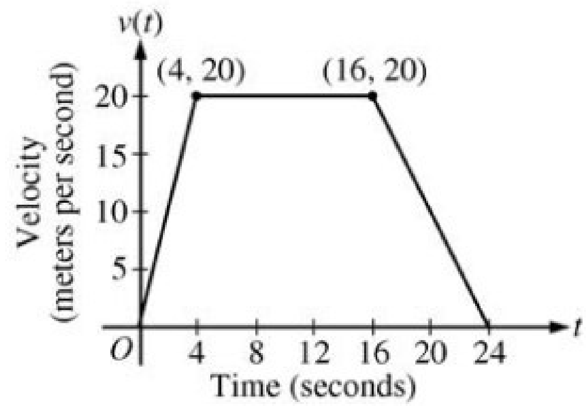
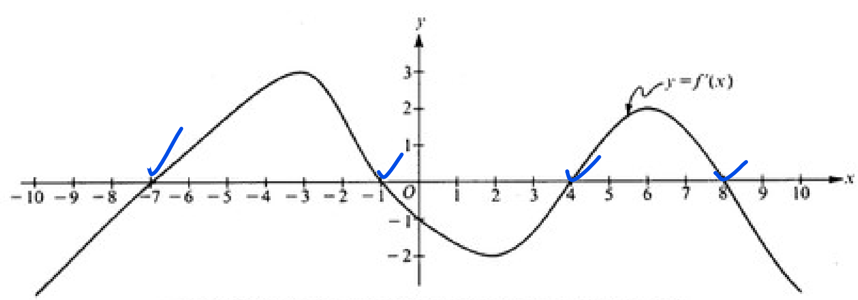


A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity *v(t)*, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

20. For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

$\lim_{x\to 4^-} v'(t) = \frac{20-0}{4-0} = 5$ $\quad v'(4)$ Does not exist $\quad v'(20) = \frac{0-20}{24-16} = \frac{-20}{8} = \frac{-5}{2}$

$\lim_{x\to 4^+} v'(t) = 0$



Note: This is the graph of the derivative of *f*, not the graph of *f*.

The figure above shows the graph of f' , the derivative of a function *f*. The domain of *f* is the set of all real numbers *x* such that $-10 \leq x \leq 10$.

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21. For what values of x does the graph of f have a horizontal tangent?

f has horizontal tangent if $f'(x)=0$

$x = -7, -1, 4, 8$