AP Calculus

Name Key

Test 2 *No Calculators*

Date________ Period____

Evaluate each limit.

1) $\lim_{x \to \infty} \left(\frac{\sin x}{x} - 1 \right)$

-1

$\frac{-1 \leftrightarrow 1}{\infty} - 1$

$0 - 1 = -1$

2) $\lim_{x \to \frac{3\pi}{4}} -2\sin(2x)$

2

$-2\sin\left(2 \cdot \frac{3\pi}{4}\right)$

$-2\sin\frac{3\pi}{2}$

$-2(-1) = 2$

Find all vertical and horizontal asymptotes of each function.

3) $h(x) = \frac{\sqrt{6x^2 - 9}}{x + 9}$

$\frac{x\sqrt{6}}{x} \rightarrow \pm\sqrt{6}$ b/c $\frac{+}{+}$ or $\frac{+}{-}$

Vertical at $x = -9$

Horizontal at $y = \pm\sqrt{6}$

$x = -9$

$y = \pm\sqrt{6}$

4) Determine the constant a that make the following function continuous.

$$f(x) = \begin{cases} 2x - 4a, & x < -2 \\ 4x - 5, & x \geq -2 \end{cases}$$

$a = \frac{9}{4}$

$2(-2) - 4a = 4(-2) - 5$

$-4 - 4a = -13$

$-4a = -9$

$a = 9/4$

Use the definition of the derivative (limit definition) to find the derivative of each function with respect to *x*.

5) $f(x) = \sqrt{4x + 5}$

$f'(x) = \frac{2}{\sqrt{4x + 5}}$

$\lim_{h \to 0} \frac{\sqrt{4(x+h)+5} - \sqrt{4x+5}}{h} \cdot \frac{\sqrt{4(x+h)+5} + \sqrt{4x+5}}{\sqrt{4(x+h)+5} + \sqrt{4x+5}}$

$\frac{4x + 4h + 5 - 4x - 5}{\cdots} \rightarrow \frac{4}{\sqrt{4(x+0)+5} + \sqrt{4x+5}}$

$= \frac{4}{2 \cdots} \rightarrow \frac{2}{\sqrt{4x+5}}$

Find the equation of the line NORMAL to the function at the given point. Use the ALTERNATE METHOD OF THE DERIVATIVE to find the slope of the tangent line.

6) $y = -x^2 + 8x - 16$ at $x = 2$

$y = -4$

$$\lim_{x \to 2} \frac{-x^2 + 8x - 16 + 4}{x - 2} \to \frac{-x^2 + 8x - 12}{x - 2}$$

$$\to \frac{-(x^2 - 8x + 12)}{x - 2} \to \frac{-(x-6)(x-2)}{x-2} = -x + 6$$

$\to -2 + 6$

$\to 4$

$y = -\frac{1}{4}x - \frac{7}{2}$

$y + 4 = -\frac{1}{4}(x - 2)$

$4 \perp -\frac{1}{4}$

$$\boxed{y + 4 = -\frac{1}{4}(x - 2)}$$

A particle's position function is $s(t)$. Find the average velocity of the particle over $[0,\pi]$

7) $s(t) = 2\sin\frac{t}{2} + \cos t + \frac{2t}{\pi} - 3$

$\frac{2}{\pi}$

$$\frac{\left(2\sin\left(\frac{\pi}{2}\right) + \cos\pi + \frac{2\pi}{\pi} - 3\right) - \left(2\sin 0 + \cos 0 + 0 - 3\right)}{\pi - 0}$$

$$\frac{(2 \cdot 1 + -1 + 2 - 3) - (1 - 3)}{\pi} = \frac{0 + 2}{\pi} = \frac{2}{\pi}$$

For each problem, find the points (x, y) where the tangent line to the function is horizontal. You can use any method you please to answer this question.

8) $y = x^3 - 3x^2 + 5$

(0, 5), (2, 1)

$3x^2 - 6x = 0$ $\quad$ 0, 2

$3x(x - 2) = 0$

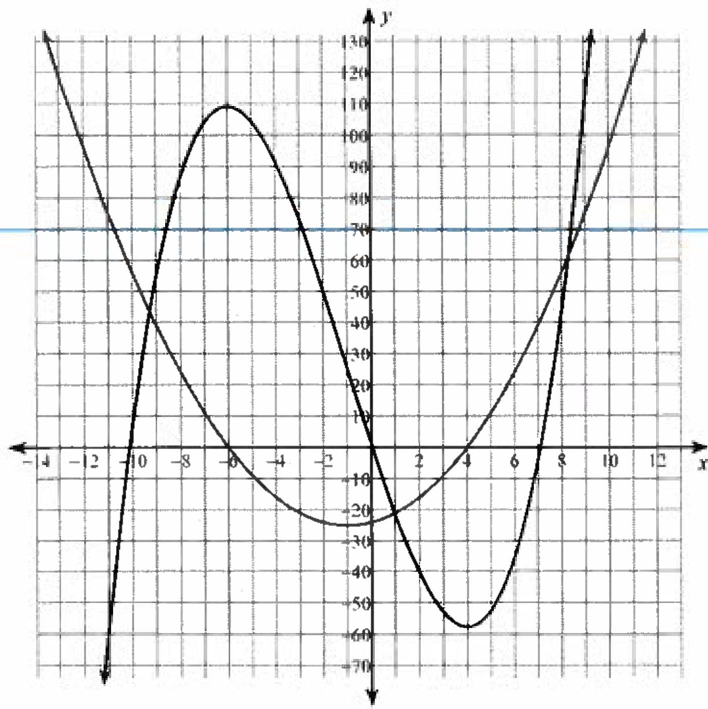
$$\boxed{(0,5),\ (2,1)}$$

9) $\lim_{h\to 0} \dfrac{\tan\left(2\left(\frac{\pi}{8}+h\right)\right)-\tan\frac{\pi}{4}}{h}$ represents the slope of the tangent line of the function tan 2x when $x =$ π/8.

slope, tangent, tan $2x$, $\frac{\pi}{8}$

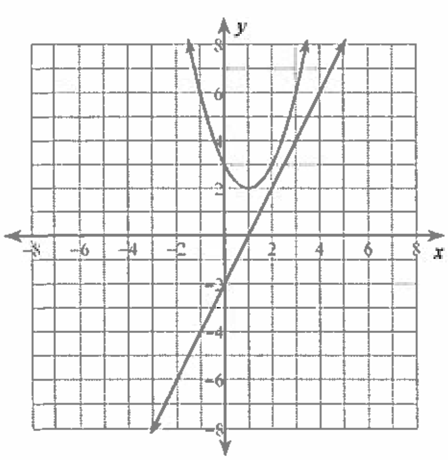
Given the graph of $f(x)$, sketch an approximate graph of $f'(x)$ given $f'(-1) = -25$ and $f'(11) = 119$

10)



Given the graph of $f'(x)$, sketch a possible graph of $f(x)$ given $f(1) = 2$ and $f(3) = 6$.

11)



Given the following function, answer the following questions.

$$f(x) = \begin{cases} 4 - x, & x \le 0 \\ -4x + 5, & 0 < x \le 3 \\ 3x^2 - 22x + 32, & x > 3 \end{cases}$$

12) $\lim_{x \to 0} f(x)$

DNE

DNE

13) $\lim_{x \to \infty} f(x)$

∞

$3(\infty)^2$

∞

14) $\lim_{x \to -\infty} f(x)$

∞

$4+\infty$

∞

15) $\lim_{x \to 3} f(x)$

-7

$-4(3)+5 = 3(3)^2-22(3)+3$

$-7=-7$

-7

16) $f'(2)$

-4

-4

17) $f'(4)$

2

$6x-22$

$6(4)-22$

2

18) Is the graph of $f(x)$ differentiable at $x = 0$. Explain your answer.

No, because it is not continuous at $x = 0$ since the limit DNE

NO, Not continuous @ $x=0$

19) Is the graph of $f(x)$ differentiable at $x = 3$. Explain your answer.

Yes. because it is continuous at $x = 3$ and the derivative from the left is -4 and the right is -4 at $x = 3$

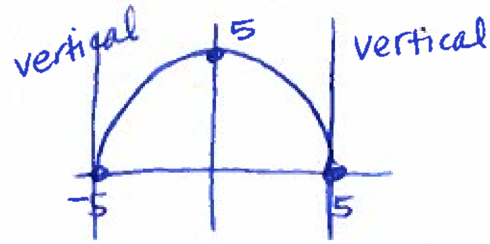
yes, continuous @ $x = 3$ $f'(3^-)=f'(3^+)=-4$

For the following function, find the interval for which the function is differentiable.

20) $f(x) = \sqrt{25 - x^2}$

$(-5, 5)$

$(-5, 5)$



↓
semicircle
so vertical tangents at starting points

For the following function $f(x) = |2x - 5|$,

21) Find the interval for which the function is continuous

$(-\infty, \infty)$

$(-\infty, \infty)$

22) Find the interval for which the function is differentiable

$(-\infty, \frac{5}{2}), (\frac{5}{2}, \infty)$

$(-\infty, \frac{5}{2}), (\frac{5}{2}, \infty)$

23) Find $f'(1)$

-2

$f'(1) = -2$

24. The limit $\lim_{\Delta x \to 0} \dfrac{\dfrac{4}{2-(-1+\Delta x)^2}-4}{\Delta x}$ represents an $f'(c)$ for a function $f(x)$ and a number c. Find f and c.

a. $f(x)=2-x^2$; $c=4$ b. $f(x)=\dfrac{4}{2-x^2}$; $c=4$ c. $f(x)=\dfrac{4}{2-x^2}$; $c=-1$

d. $f(x)=2-x^2$; $c=-1$ e. $f(x)=\dfrac{4}{2-x^2}$; $c=2$

25. At $x=2$, the function given by $f(x)=\begin{cases} x^2+1, & x \le 2 \\ 4x-3, & x>2 \end{cases}$ is

a. undefined b. continuous, but not differentiable c. differentiable, but not continuous

d. neither continuous nor differentiable e. both continuous and differentiable

26. Find $f''(x)$ if $f(x)=3x^5-6x^2$.

$15x^4-12x$

$60x^3-12$

a. $f''(x)=15x^4-12x$ b. $f''(x)=60x^3-12$

c. $f''(x)=180x^2$ d. $f''(x)=3x^5-12x^2$

e. None of these

27. If $f(2)=-1$ and $f'(2)=3$, find an equation of the tangent line when $x=2$.

a. $y-3=2(x+1)$ b. $y-2=3(x+1)$

c. $y+1=3(x-2)$ d. $y+1=2(x-2)$

e. None of these

28. If $f(x)=\dfrac{2x}{x^2+1}$, which of the following will calculate the derivative of $f(x)$.

a. $f'(x)=\dfrac{\dfrac{2x+\Delta x}{x^2+\Delta x+1}-\dfrac{2x}{x^2+1}}{\Delta x}$ b. $f'(x)=\lim_{\Delta x \to 0}\dfrac{\dfrac{2x+\Delta x}{x^2+\Delta x+1}-\dfrac{2x}{x^2+1}}{\Delta x}$

c. $f'(x)=\lim_{\Delta x \to 0}\dfrac{\dfrac{2(x+\Delta x)}{(x+\Delta x)^2+1}-\dfrac{2x}{x^2+1}}{\Delta x}$ d. $f'(x)=\dfrac{\dfrac{2(x+\Delta x)}{(x+\Delta x)^2+1}-\dfrac{2x}{x^2+1}}{\Delta x}$