AP Calculus AB **Name** ______________________ **Date** ________

1. Suppose that $g(x) = h(f(x))$, $f(3) = 6$, $h(3) = 2$, $h'(6) = 4$, $h'(3) = 15$ and $f'(3) = 8$. Find $g'(3)$.

(A) 32 **(B)** 8
(C) 120 **(D)** 60

$g'(x) = h'(f(x))\ f'(x)$
$g'(3) = h'(f(3))\ f'(3)$
$= h'(6)\quad 8$
$= 4\ 8$

2. Suppose that $g(x) = \dfrac{h(x)}{f(x)}$, $f(3) = 6$, $h(3) = 2$, $h'(6) = 4$, $h'(3) = 15$ and $f'(3) = 9$. Find $g'(3)$.

(A) $\frac{1}{3}$ **(B)** 3
(C) 2 **(D)** $\frac{5}{3}$

$$g'(x) = \frac{f(x)\,h'(x) - h(x)\,f'(x)}{(f(x))^2}$$

$$g'(3) = \frac{6\ 15 - 2\ 9}{6^2} = \frac{90-18}{36} = \frac{72}{36} = 2$$

3. The limit $\displaystyle\lim_{\Delta x \to 0} \frac{\dfrac{4}{2-(-1+\Delta x)^2} - 4}{\Delta x}$ represents an $f'(c)$ for a function $f(x)$ and a number c. Find f and c.

h h h

~~**(A)**~~ $f(x) = 2 - x^2$; $c = 4$ ~~**(B)**~~ $f(x) = \dfrac{4}{2-x^2}$; $c = 4$
(C) $f(x) = \dfrac{4}{2-x^2}$; $c = -1$ **(D)** $f(x) = 2 - x^2$; $c = -1$

$$\boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)}$$

4. At $x = 2$, the function given by $f(x) = \begin{cases} x^2 + 1, & x \le 2 \\ 4x - 3, & x > 2 \end{cases}$ is

$2^2 + 1 = 4(2) - 3$
$5 = 5$

(A) both continuous and differentiable **(B)** continuous, but not differentiable
~~**(C)**~~ differentiable, but not continuous ~~**(D)**~~ neither continuous nor differentiable

$x^2 + 1 \to 2x \quad 2(2) = 4$
$4x - 3 \to 4$

5. A counter records people entering a fast food restaurant. The data below shows the number of customers that enter the restaurant for each two-hour period starting at 8:00 am. Estimate the rate at which the customers are entering the store at 1:00 p.m.

↓
table

x	Hours after 8:00 am	0	2	4	6	8	10	12
y	Number of Customers	0	21	42	36	14	58	23

$$\frac{36-42}{6-4} = \frac{-6}{2} = -3$$

(A) 6 customers/h **(B)** -3 customers/hr

(C) 39 customers/hr **(D)** -6 customers/hr

6. Consider the function $f(x) = x^2 - x - 6$.

$$f(6) = 6^2 - 6 - 6 = 36 - 6 - 6 = 24$$

$$f(0) = -6$$

a. Find the average rate of change of $f(x)$ on $[0,6]$.

$$= \frac{f(6)-f(0)}{6-0} = \frac{24+6}{6} = \frac{30}{6} = \boxed{5}$$

b. Find the instantaneous rate of change of $f(x)$ when $x = 4$.

$$f'(x) = 2x - 1$$

$$f'(4) = 2(4) - 1 = \boxed{7}$$

c. Write an equation of the line tangent to f at the point where $x = 4$.

$$f(4) = 4^2 - 4 - 6 = 16 - 4 - 6 = 6$$

$$\boxed{y - 6 = 7(x-4)}$$

d. Still using $f(x)$ above, if $g(x) = [f(x)]^2$, find all values of x where the tangent lines to g are horizontal.

$$g'(x) = 2(f(x))' \cdot f'(x)$$

$$0 = 2(x^2 - x - 6)(2x-1)$$

$$0 = 2(x+2)(x-3)(2x-1)$$

$$\boxed{x = 3, -2, \frac{1}{2}}$$

7. Find $f'(x)$ if $f(x) = \sin^5 3x$.

$(\sin(3x))^5$

$t^5 \to 5t^4$

$\sin t \to \cos t$

$3x \to 3$

~~**(A)**~~ $f'(x) = 5\sin^4 3x \cos 3x$ **(B)** $f'(x) = 60\sin^4 3x \cos^3 3x$

~~**(C)**~~ $f'(x) = 15\cos^4 3x$ **(D)** $f'(x) = 15\sin^4 3x \cos 3x$

$f'(x) = 5(\sin(3x))^4 \cdot \cos(3x) \cdot 3$

8. If $y = x^2(1-2x)^3$, find y'.

(A) $y' = -2x(2x-1)^2(5x-1)$ **(B)** $y' = -6x^2(2x-1)^2$

(C) $y' = 3x^2(2x-1)^2$ **(D)** $y' = -x(2x-1)^2(x-2)$

$y' = 2x(1-2x)^3 + x^2\, 3(1-2x)^2 \, -2$

$(1-2x)^2\, 2x\left((1-2x) - 3x\right)$

$-(1-2x)^2\, 2x(-1+5x)$

9. Find $f''(x)$ if $f(x) = 3x^5 - 6x^2$.

(A) $f''(x) = 15x^4 - 12x$ **(B)** $f''(x) = 60x^3 - 12$

(C) $f''(x) = 180x^2$ **(D)** $f''(x) = 3x^5 - 12x^2$

$f'(x) = 15x^4 - 12x$

$f''(x) = 60x^3 - 12$

10. If $y = 5x^3 \tan x$, then $\dfrac{dy}{dx} =$

$$\frac{dy}{dx} = 15x^2 \tan x + 5x^3 \sec^2 x$$

11. If $f(2) = -1$ and $f'(2) = 3$, find an equation of the tangent line when $x = 2$.

(A) $y - 3 = 2(x+1)$ **(B)** $y - 2 = 3(x+1)$

(C) $y + 1 = 3(x-2)$ **(D)** $y + 1 = 2(x-2)$

$y_1 \quad m \quad x_1$

f(x+h)

12. If $f(x)=\dfrac{2x}{x^2+1}$, which of the following will calculate the derivative of $f(x)$.

~~**(A)**~~ $f'(x)=\dfrac{\dfrac{2x+\Delta x}{x^2+\Delta x+1}-\dfrac{2x}{x^2+1}}{\Delta x}$

(B) $f'(x)=\lim\limits_{\Delta x\to 0}\dfrac{\dfrac{2x+\cancel{\Delta x}\,h}{x^2+\Delta x+1}-\dfrac{2x}{x^2+1}}{\Delta x}$

(C) $f'(x)=\lim\limits_{\cancel{\Delta x}\,h\to 0}\dfrac{\dfrac{2(x+\cancel{\Delta x}\,h)}{(x+\cancel{\Delta x}\,h)^2+1}-\dfrac{2x}{x^2+1}}{\cancel{\Delta x}\,h}$

~~**(D)**~~ $f'(x)=\dfrac{\dfrac{2(x+\Delta x)}{(x+\Delta x)^2+1}-\dfrac{2x}{x^2+1}}{\Delta x}$

13. Find $f'(\frac{\pi}{6})$ if $f(t)=\sin(3t)$

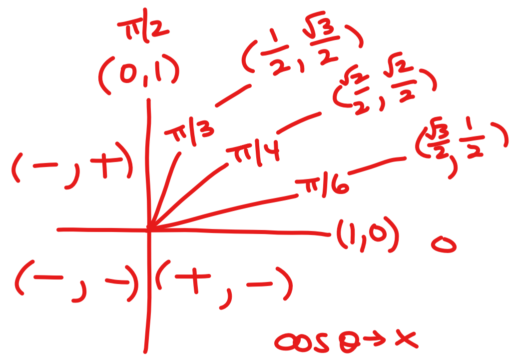
$f'(t)=\cos(3t)\cdot 3$

$f'(\frac{\pi}{6})=\cos(3\,\frac{\pi}{6})\;3$

$=\cos(\frac{\pi}{2})\;3$

$=0\;3$

$=0$

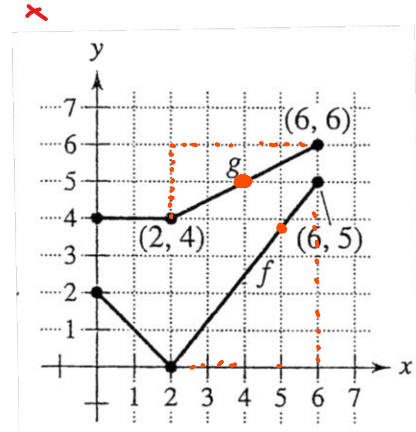


$\cos\theta \to x$

$\sin\theta \to y$

$\tan\theta \to \dfrac{\sin\theta}{\cos\theta}=\dfrac{y}{x}$

14. Given $h(x)=f(g(x))$, use the graph to the right to find $h'(4)$.



(A) $\frac{1}{2}$ **(B)** $\frac{5}{4}$

(C) $\frac{5}{8}$ **(D)** 5

$h'(x)=f'(g(x))\cdot g'(x)$

$h'(4)=f'(g(4))\;g'(4)$

$f'(5)\cdot\left(\frac{2}{4}\right) \longrightarrow \left(\frac{5}{4}\right)\left(\frac{2}{4}\right)=\frac{10}{16}=\frac{5}{8}$

15. If $f(x) = \frac{4x+5}{(2x+1)^2}$, then $f'(x) =$ $\dfrac{(2x+1)^2 4 - (4x+5)2(2x+1)^1 2}{(2x+1)^4}$

Unsimplified:

$$\frac{(2x+1)^1 4 - (4x+5)\, 2 \cdot 2}{(2x+1)^3}$$

Simplified:

$8x+4-16x-20$

$-8x-16 \rightarrow \boxed{\dfrac{-8(x+2)}{(2x+1)^3}}$

f'(1/2) $f'\left(\frac{1}{2}\right) = \dfrac{-8\left(\frac{1}{2}+2\right)}{\left(2\left(\frac{1}{2}\right)+1\right)^3} = -1(2.5)$

$\boxed{= -2.5 \text{ or } \frac{-5}{2}}$

16. Consider the following information in the chart.

x	*f*(*x*)	*g*(*x*)	*f*′(*x*)	*g*′(*x*)	*f*″(*x*)	*g*″(*x*)
1	2	-5	-4	2	16	2
2	$\frac{1}{2}$	-2	$-\frac{1}{2}$	4	1	2

a. If $h(x) = f(x)g(x)$, find $h'(1)$.

$h'(x) = f(x)\,g'(x) + g(x)\,f'(x)$

$h'(1) = f(1)\,g'(1) + g(1)\,f'(1)$

$= (2)(2) + (-5)(-4)$

$= 4 + 20 = 24$

b. If $h(x) = x \cdot g(f(x))$, find $h'(1)$.

$h'(x) = x\,g'(f(x))\,f'(x) + 1 \cdot g(f(x))$

$h'(1) = 1\,g'(f(1))\,f'(1) + g(f(1))$

$= g'(2)\,(-4) + g(2) \rightarrow (4)(-4) + (-2) \boxed{= -18}$

17. $y = \csc(5x^2)$

$\frac{dy}{dx} = -\csc(5x^2)\cot(5x^2) \cdot 10x$

D