

ABCALC Differentiation Review Session Problems

Key

Find the derivative of each function

$$1. y = \frac{2}{3}x^4 + 5x - x^{-3}$$

$$\frac{dy}{dx} = \frac{8}{3}x^3 + 5 + \frac{3}{x^4}$$

$$2. y = \frac{2}{3}\sqrt[3]{x^2} - \frac{3}{4}x^{\frac{3}{5}}$$

$$\frac{dy}{dx} = \frac{4}{9}x^{-\frac{1}{3}} - \frac{9}{20}x^{-\frac{2}{5}}$$

Find the point where  $f(x) = x^2 - 6x + 11$  has a horizontal tangent line.

$$f'(x) = 2x - 6$$

$$0 = 2x - 6$$

$$x = 3$$

$$f(3) = 3^2 - 6(3) + 11$$

$$9 - 18 + 11$$

$$f(3) = 2$$

$$(3, 2)$$

Find the derivative of each function

$$3) y = (2x^4 - 3)(x^2 + 1)$$

$$y = 2x^6 + 2x^4 - 3x^2 - 3$$

$$\frac{dy}{dx} = 12x^5 + 8x^3 - 6x$$

$$4) y = \frac{4x^5 + x^2 + 4}{5x^2 - 2}$$

$$\frac{dy}{dx} = \frac{(5x^2 - 2)(20x^4 + 2x) - (4x^5 + x^2 + 4)(10x)}{(5x^2 - 2)^2}$$

$$(5x^2 - 2)^2$$

6. Evaluate  $\lim_{h \rightarrow 0} \frac{(x+h)^8 - x^8}{h} \rightarrow \frac{dy}{dx} x^8 = 8x^7$

**Free Response Practice 1 (no calculator)**

Consider the curve given by  $xy^2 - x^3y = 6$ .

- (a) Show that  $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$ .
- (b) Find all points on the curve whose  $x$ -coordinate is 1, and write an equation for the tangent line at each of these points.
- (c) Find the  $x$ -coordinate of each point on the curve where the tangent line is vertical.

(a)  $(1)(y^2) + (x)(2y)\left(\frac{dy}{dx}\right) - \left(\left(x^3\frac{dy}{dx} + 3x^2y\right)\right) = 0$   
 $y^2 + 2xy\frac{dy}{dx} - x^3\frac{dy}{dx} - 3x^2y = 0$   
 $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$

(b)  $(1)y^2 - (1)y = 6$       $\frac{dy}{dx} \Big|_{(1,3)} = \frac{3(1)^2(3) - (3)^2}{2(1)(3) - (1)^3} = 0$   
 $y^2 - y - 6 = 0$   
 $(y-3)(y+2) = 0$       $\frac{dy}{dx} \Big|_{(1,-2)} = \frac{3(1)^2(-2) - (-2)^2}{2(1)(-2) - (1)^3} = \frac{-10}{-5} = 2$   
 $y = 3$   
 $y = -2$       $y - 3 = 0(x - 1)$  and  $y + 2 = 2(x - 1)$

(c)  $2xy - x^3 = 0$       $x^5 - 2x^5 = 24$   
 $y = \frac{x^3}{2x} = \frac{x^2}{2}$       $-x^5 = 24$   
 $x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$       $x^5 = -24$   
 $\frac{x^5}{4} - \frac{x^5}{2} = 6$       $x = \sqrt[5]{-24}$

ABCALC Differentiation Review Session Problems

Find the derivative of each function

$(4x^2+2)^{\frac{1}{2}}$   $(\sin(x^3))^2$   $x^2 \rightarrow 2x$   
 $\sin x \rightarrow \cos x$   
 $x^3 \rightarrow 3x^2$

a)  $f(x) = \sqrt{4x^2+2}$

$f'(x) = \frac{1}{2\sqrt{4x^2+2}}(8x)$

b)  $f(x) = \sin^2(x^3)$

$f'(x) = 2\sin(x^3) \cdot \cos(x^3) \cdot 3x^2$

c)  $f(x) = \frac{\sec(5x)}{2} \rightarrow \frac{1}{2}\sec(5x)$

$f'(x) = \frac{1}{2}\sec(5x)\tan(5x) \cdot 5$

d)  $f(x) = \tan^2(\sqrt{x^3+5x})$

$x^2 \rightarrow 2x$   
 $\tan x \rightarrow \sec^2 x$   
 $\sqrt{x} \rightarrow \frac{1}{2\sqrt{x}}$   
 $x^3+5x \rightarrow 3x^2+5$

$f'(x) = 2\tan\sqrt{x^3+5x} \cdot \sec^2\sqrt{x^3+5x} \cdot \frac{1}{2\sqrt{x^3+5x}} \cdot (3x^2+5)$

Find the equation of the tangent line to  $y = \sin^2 x$  at  $x = \frac{\pi}{3}$

$y = (\sin x)^2$

$y(\frac{\pi}{3}) = (\sin \frac{\pi}{3})^2$   
 $= (\frac{\sqrt{3}}{2})^2 = \frac{3}{4}$

$\frac{dy}{dx} = 2\sin x \cos x$

$\frac{dy}{dx} \Big|_{x=\frac{\pi}{3}} = 2\sin \frac{\pi}{3} \cos \frac{\pi}{3} = 2(\frac{\sqrt{3}}{2})(\frac{1}{2}) = \frac{\sqrt{3}}{2}$

$y - \frac{3}{4} = \frac{\sqrt{3}}{2} (x - \frac{\pi}{3})$

Find the equation of the tangent line at the pt(s) where  $y=1$ .

$$\frac{d}{dx} x^8 + 8y^5 = 41y - 32$$

$$x^8 + 8 = 41 - 32$$

$$x^8 = 9 - 8 \quad x^8 = 1 \quad x = \pm 1$$

$$8x^7 + 40y^4 \frac{dy}{dx} = 41 \frac{dy}{dx}$$

$$8(1) + 40 \frac{dy}{dx} = 41 \frac{dy}{dx} \rightarrow \frac{dy}{dx} = 8 \quad y=1 = 8(x-1)$$

$$8(-1) + 40 \frac{dy}{dx} = 41 \frac{dy}{dx} \rightarrow \frac{dy}{dx} = -8 \quad y=-1 = -8(x+1)$$

Find the derivative

a)  $x^3 + (3x^2)y + (2xy)^2 = 12$

$$3x^2 + (-6x)(y) + (-3x^2) \frac{dy}{dx} +$$

$$(2)(y^2) + (2x)(2y) \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-3x^2 + 6xy - 2y^2}{-3x^2 + 4xy}$$

b)  $\sin x + 2\cos 2y = 1$

$$\cos x + 2(-\sin(2y))(2 \frac{dy}{dx}) = 0$$

$$\frac{dy}{dx} = \frac{+\cos x}{+4\sin 2y}$$

ABCALC Differentiation Review Session Problems

Use the table to find the derivatives of the following at  $x=4$ .

|       | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|-------|--------|---------|--------|---------|
| $x=4$ | 3      | -2      | 7      | 1       |
| $x=7$ | 5      | -1      | 4      | 6       |

a)  $f(x)g(x)$   
 $f'(4)g(4) + f(4)g'(4) =$   
 $(-2)(7) + (3)(1) = -11$

c)  $3f(x) + 4g(x)^2$   
 $3f'(4) + 4 \cdot 2 \cdot g(4) \cdot g'(4)$   
 $3(-2) + 8(7)(1)$   
 $-6 + 56$   
 $50$

b)  $\frac{f(x)}{g(x)}$   $\frac{g(4)f'(4) - f(4)g'(4)}{(g(4))^2}$   
 $\frac{(7)(-2) - (3)(1)}{(7)^2} = \frac{-17}{49}$

d)  $f(g(x))$   
 $f'(g(4)) \cdot g'(4)$   
 $f'(7) \cdot (1)$   
 $(-1) \cdot (1)$   
 $-1$

Given  $s(t) = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t + 5$  is the position function for a particle moving along the x-axis, find the following.

a) displacement from  $t=1$  to  $t=4$   $s(4) - s(1) = -1.5$

b) times when the particle changes direction  $t = 2, 5$   
 $s'(t) = t^2 - 7t + 10 = (t-2)(t-5)$   $\frac{+}{-} \frac{-}{+}$  b/c  $s'(t)$  changes signs

c) avg velocity from  $t=1$  to  $t=4$

$\frac{s(4) - s(1)}{4-1} = -0.5$

d) instantaneous velocity at  $t=3$   $v(3) = s'(3) = 9 - 21 + 10 = -2$

e) value of  $t$  when the acceleration is 0.

$s''(t) = 2t - 7$

$0 = 2t - 7$

$t = \frac{7}{2}$

ABCALC Differentiation Review Session Problems

The graph shows the velocity  $v = f(t)$  of a particle moving along a horizontal coordinate axis.

(a) When does the particle reverse direction?  $t = 1, 4.2$   $v(t)$  changes signs

(b) When is the particle moving at a constant speed?  $(5, 6)$

(c) When is the particle moving at its greatest speed?  $t = 3$

(d) Graph the acceleration (where defined).

On a test it will explicitly be one or the other

Kinda ambiguous, so I interpreted it 2 ways

Find the derivative

a)  $y = \sin^2(\cot(5x))$

- $x^2 \rightarrow 2x$
- $\sin x \rightarrow \cos x$
- $\cot x \rightarrow -\csc^2 x$
- $5x \rightarrow 5$

$$\frac{dy}{dx} = 2\sin(\cot(5x)) \cdot \cos(\cot(5x)) \cdot (-\csc^2(5x)) \cdot 5$$

$$\frac{dy}{dx} = \frac{1 - y \sec(xy) \tan(xy)}{x \sec(xy) \tan(xy)}$$

~~b)  $\frac{d}{dx} \sec(xy) = x$~~

~~$\frac{d}{dx} \sec(xy) = \sec(xy) \tan(xy) (1 \cdot y + x \frac{dy}{dx}) = 1$~~

~~$\frac{dy}{dx} = \frac{1 - y \sec(xy) \tan(xy)}{x \sec(xy) \tan(xy)}$~~

2 or  $\frac{d}{dx} \sec(xy) = x$

$\sec(xy) \tan(xy) (1 \cdot y + x \frac{dy}{dx}) = 1$

$\frac{dy}{dx} = \frac{1 - y \sec(xy) \tan(xy)}{x \sec(xy) \tan(xy)}$