1.) $\dfrac{\sec x}{\cot x + \tan x} \rightarrow \dfrac{\frac{1}{\cos x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}} \rightarrow \dfrac{\frac{1}{\cos x}}{\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}} \rightarrow \dfrac{\frac{1}{\cos x}}{\frac{1}{\sin x \cos x}} \rightarrow \dfrac{1}{\cos x} \cdot \sin x \cos x$

$\rightarrow \underline{\sin x}$

2) $\dfrac{\cos x - \csc x}{\sin x - \sec x} \rightarrow \dfrac{\cos x - \frac{1}{\sin x}}{\sin x - \frac{1}{\cos x}} \rightarrow \dfrac{\frac{\sin x \cos x - 1}{\sin x}}{\frac{\sin x \cos x - 1}{\cos x}}$

$\rightarrow \dfrac{\sin x \cos x - 1}{\sin x} \cdot \dfrac{\cos x}{\sin x \cos x - 1} \rightarrow \dfrac{\cos x}{\sin x} \rightarrow \underline{\cot x}$

3.) $\dfrac{\sin x + \tan x}{\cos x + 1} \rightarrow \dfrac{\sin x + \frac{\sin x}{\cos x}}{\cos x + 1} \rightarrow \dfrac{\frac{\sin x \cos x + \sin x}{\cos x}}{\cos x + 1}$

$\rightarrow \dfrac{\frac{\sin x(\cos x + 1)}{\cos x}}{\cos x + 1} \rightarrow \dfrac{\sin x(\cos x + 1)}{\cos x} \cdot \dfrac{1}{\cos x + 1} \rightarrow \dfrac{\sin x}{\cos x} \rightarrow \underline{\tan x}$

4) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$

$\sin^2 x + 2\sin x \cos x + \cos^2 x + \sin^2 x - 2\sin x \cos x + \cos^2 x$

$= 1 + 1 = \underline{2}$ ✓

5) $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x}$

$\sin^2 x + \cos^2 x = \underline{1}$

6) $\dfrac{\cos x}{\sec x + \tan x} \cdot \dfrac{(\sec x - \tan x)}{(\sec x - \tan x)} \rightarrow \dfrac{1 - \sin x}{\sec^2 x - \tan^2 x} \rightarrow \underline{1 - \sin x}$

7) $\dfrac{\sin^4 x - \cos^4 x}{\sin^3 x + \cos^3 x} \rightarrow \dfrac{(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)}{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}$

$\rightarrow \dfrac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x + \cos x)(1 - \sin x \cos x)} \rightarrow \dfrac{\sin x - \cos x}{1 - \sin x \cos x} = RHS$

8) $\csc x[\csc x + \underbrace{\sin(-x)}_{\text{odd}}] = \cot^2 x$

$\csc^2 x + \csc x(-\sin x)$

$\csc^2 x - 1 = \cot^2 x = RHS$

9.) $\dfrac{\sin 2x}{1+\cos 2x} = \dfrac{2\sin x \cos x}{1+2\cos^2 x - 1} \rightarrow \dfrac{2\sin x \cos x}{2\cos x \cos x}$

$\rightarrow \dfrac{\sin x}{\cos x} \rightarrow \tan x = RHS$

10.) $\tan \frac{x}{2} = \csc x - \cot x$

$\tan \frac{x}{2} = \dfrac{\sqrt{1-\cos x}}{\sqrt{1+\cos x}} \cdot \dfrac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} \rightarrow \dfrac{\sqrt{1-\cos^2 x}}{1+\cos x} \rightarrow \dfrac{\sqrt{\sin^2 x}}{1+\cos x}$

$\rightarrow \dfrac{\sin x}{1+\cos x} \cdot \dfrac{1-\cos x}{1-\cos x} \rightarrow \dfrac{\sin x(1-\cos x)}{1-\cos^2 x}$

$\rightarrow \dfrac{\sin x(1-\cos x)}{\sin^2 x} \rightarrow \dfrac{1-\cos x}{\sin x} \rightarrow \dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x} \rightarrow \csc x - \cot x = RHS$

11.) $\dfrac{\sin x \cos y + \sin y \cos x + \sin x \cos y - \sin y \cos x}{\cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y}$

$\rightarrow \dfrac{2\sin x \cos y}{2\cos x \cos y} \rightarrow \dfrac{\sin x}{\cos x} \rightarrow \tan x = RHS$

Pre-Calculus   Name Key

Test 3 Review

Date________ Period____

Use the angle sum identity to find the exact value of each.

12) cos 165°

$\frac{-\sqrt{6}-\sqrt{2}}{4}$

$\cos(120+45)$

$\cos 120 \cos 45 - \sin 120 \sin 45$

$\left(\frac{-1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{-\sqrt{2}-\sqrt{6}}{4}$

13) sin 75° $\frac{\sqrt{6}+\sqrt{2}}{4}$

$\sin(45+30)$

$\sin 45 \cos 30 + \sin 30 \cos 45$

$\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

Use the angle difference identity to find the exact value of each.

14) $\tan -\frac{\pi}{12}$

$\sqrt{3}-2$

$-\frac{1}{12} = \frac{-3}{3}\frac{x}{4} + \frac{4}{3}\frac{y}{4}$

$\tan\left(\frac{2\pi}{3} - \frac{3\pi}{4}\right)$

$\frac{\tan\frac{2\pi}{3} - \tan\frac{3\pi}{4}}{1+\tan\frac{2\pi}{3}\tan\frac{3\pi}{4}} \rightarrow \frac{-\sqrt{3}+1}{1+(-\sqrt{3})(-\sqrt{1})}$

$\frac{-\sqrt{3}+1}{1+\sqrt{3}}\cdot\frac{1-\sqrt{3}}{1-\sqrt{3}} \rightarrow \frac{-\sqrt{3}+3+1-\sqrt{3}}{1-3} \rightarrow \frac{-2\sqrt{3}+4}{-2} \rightarrow \sqrt{3}-2$

$\hookrightarrow \frac{(1-\sqrt{3})^2}{-2}$ fine too

15) $\cos\frac{7\pi}{12}$ $\frac{\sqrt{2}-\sqrt{6}}{4}$

$\frac{7}{12} = \frac{3}{3}\frac{x}{4} - \frac{4}{3}\frac{4}{4}$

$\cos\left(\frac{5\pi}{4} - \frac{2\pi}{3}\right)$

$\cos\frac{5\pi}{4}\cos\frac{2\pi}{3} + \sin\frac{5\pi}{4}\sin\frac{2\pi}{3}$

$\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right)+\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{2}-\sqrt{6}}{4}$

Use an addition or subtraction formula to write the expression as a trigonometric function of one number, and then find its exact value.

16) cos 24° cos 21° - sin 24° sin 21°

$\frac{\sqrt{2}}{2}$

$\cos(24+21)$

$\cos(45) = \frac{\sqrt{2}}{2}$

17) sin 211° cos 29° + sin 29° cos 211° $-\frac{\sqrt{3}}{2}$

$\sin(211+29)$

$\sin(240) = -\frac{\sqrt{3}}{2}$

18) $\cos\frac{13\pi}{18}\cos\frac{2\pi}{9} + \sin\frac{13\pi}{18}\sin\frac{2\pi}{9}$

0

$\cos\left(\frac{13\pi}{18} - \frac{2\pi}{9}\right)$

$\cos\left(\frac{13\pi}{18} - \frac{4\pi}{18}\right)$

$\cos\left(\frac{9\pi}{18}\right) = \cos\frac{\pi}{2} = 0$

19) $\left(\tan\frac{17\pi}{12} + \tan\frac{\pi}{4}\right) \div \left(1 - \tan\frac{17\pi}{12}\tan\frac{\pi}{4}\right)$

$-\sqrt{3}$

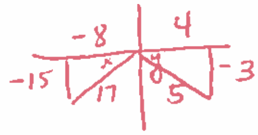
$\tan\left(\frac{17\pi}{12}+\frac{\pi}{4}\right)$

$\tan\left(\frac{17\pi}{12}+\frac{3\pi}{12}\right)$

$\tan\left(\frac{20\pi}{12}\right) \rightarrow \tan\frac{5\pi}{3} \rightarrow -\sqrt{3}$

Assuming $\sin x = -\frac{15}{17}$ **if** $\pi < x < \frac{3\pi}{2}$ **and** $\tan y = -\frac{3}{4}$ **if** $\frac{3\pi}{2} < y < 2\pi$

20) $\sin(x + y)$



$-\frac{36}{85}$

$\left(-\frac{15}{17}\right)\left(\frac{4}{5}\right) + \left(-\frac{8}{17}\right)\left(-\frac{3}{5}\right)$

$\frac{-60 + 24}{85} = \frac{-36}{85}$

21) $\cos(x + y)$ $-\frac{77}{85}$

$\left(-\frac{8}{17}\right)\left(\frac{4}{5}\right) - \left(\frac{-15}{17}\right)\left(\frac{-3}{5}\right)$

$\frac{-32 - 45}{85} = \frac{-77}{85}$

22) $\tan(x - y)$ $\frac{\tan x - \tan y}{1 + \tan x \tan y}$

$-\frac{84}{13}$

$\frac{\frac{15}{8} + \frac{3}{4}}{1 + \frac{15}{8}\left(-\frac{3}{4}\right)}$ $\frac{\frac{15+6}{8}}{\frac{32-45}{32}} \rightarrow \frac{21}{8} \cdot \frac{32}{-13}$

$= -\frac{84}{13}$

23) $\sec(x - y)$ $\frac{85}{13}$

$= \frac{1}{\cos(x-y)}$ just denominator

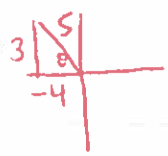
$= \left(-\frac{8}{17}\right)\left(\frac{4}{5}\right) + \left(\frac{-15}{17}\right)\left(\frac{-3}{5}\right)$

$\frac{-32 + 45}{85} = \frac{13}{85}$

$\sec(x-y) = \frac{85}{13}$

Use a double-angle identity to find the exact value of each expression.

24) $\tan \theta = -\frac{3}{4}$ and $90° < \theta < 180°$



Find $\cos 2\theta$

$\frac{7}{25}$

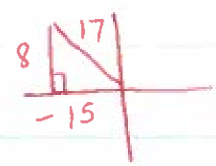
$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\left(\frac{-4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$\frac{16 - 9}{25} = \frac{7}{25}$

25) $\cos \theta = -\frac{15}{17}$ and $90° < \theta < 180°$ $-\frac{240}{289}$

Find $\sin 2\theta$



$2\left(\frac{8}{17}\right)\left(\frac{-15}{17}\right) = \frac{-240}{289}$

26) $\csc\theta = \frac{5}{3}$ and $90° < \theta < 180°$

Find $\tan 2\theta$ $\quad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$-\frac{24}{7}$

$$\frac{2\left(\frac{3}{-4}\right)}{1-\left(\frac{-3}{4}\right)^2} = \frac{-\frac{3}{2}\cdot\frac{8}{8}}{\frac{16}{16}\cdot 1-\frac{9}{16}} = -\frac{24}{7}$$

27) $\sin\theta = \frac{\sqrt{23}}{23}$ and $90° < \theta < 180°$ $\quad \to \frac{1}{\sqrt{23}}$ $\quad \frac{21}{23}$

Find $\cos 2\theta$ $\quad \cos 2\theta = \cos^2\theta - \sin^2\theta$

$$\cos 2\theta = \left(-\frac{\sqrt{22}}{\sqrt{23}}\right)^2 - \left(\frac{1}{\sqrt{23}}\right)^2$$

$$= \frac{22}{23} - \frac{1}{23} = \frac{21}{23}$$

Use the half-angle identities to find the exact value of each.

28) $\tan\frac{11\pi}{8}$ $\quad \tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$

Q3 $\swarrow$

$1+\sqrt{2}$ $\quad \tan^2\left(\frac{11\pi}{8}\right) = \frac{1-\cos\frac{11\pi}{4}}{1+\cos\frac{11\pi}{4}}$

$$= \frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{\frac{2}{2}-\frac{\sqrt{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}$$

Fine or

$$\frac{2+\sqrt{2}}{\sqrt{2}}\cdot\frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{(2+\sqrt{2})^2}{4-2} = \sqrt{\frac{(2+\sqrt{2})^2}{2}} \to \frac{2+\sqrt{2}}{\sqrt{2}}, \quad \frac{2\sqrt{2}+2}{2} = \sqrt{2}+1$$

29) $\cos 247.5°$ $\quad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$-\frac{\sqrt{2-\sqrt{2}}}{2}$ $\quad \cos^2 247.5 = \frac{1}{2}(1+\cos 495°)$

$$= \frac{1}{2}\left(1-\frac{\sqrt{2}}{2}\right)$$

$$\Rightarrow \frac{1}{2}-\frac{\sqrt{2}}{4} \Rightarrow \frac{2-\sqrt{2}}{4}$$

$$= -\frac{\sqrt{2-\sqrt{2}}}{2} \text{ or } -\sqrt{\frac{1}{2}\left(1-\frac{\sqrt{2}}{2}\right)}$$

Find the exact value of each.

30) $\tan\theta = \frac{12}{5}$ where $0 \le \theta < \frac{\pi}{2}$

Find $\cos\frac{\theta}{2}$

$\frac{3\sqrt{13}}{13}$ $\quad \cos\left(\frac{\theta}{2}\right) = \frac{1}{2}(1+\cos\theta)$

$$\cos\left(\frac{\theta}{2}\right) = \frac{1}{2}\left(1+\frac{5}{13}\right)$$

$$\Rightarrow \frac{1}{2}+\frac{5}{26} \to \frac{13}{26}+\frac{5}{26}$$

$$\Rightarrow \frac{18}{26} \to \sqrt{\frac{9}{13}} \quad \frac{3}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}} \quad \frac{3\sqrt{13}}{3}$$

31) $\cos\theta = -\frac{1}{8}$ where $3\pi \le \theta < \frac{7\pi}{2}$

Find $\cos\frac{\theta}{2}$

$\frac{\sqrt{7}}{4}$ $\quad \cos\left(\frac{\theta}{2}\right) = \frac{1}{2}(1+\cos\theta)$

$$= \frac{1}{2}\left(1-\frac{1}{8}\right)$$

$\frac{3\pi}{2} < \frac{\theta}{2} < \frac{7\pi}{4}$

cos + Q4

$$= \frac{1}{2}\left(\frac{7}{8}\right)$$

$$= \sqrt{\frac{7}{16}} \to \frac{\sqrt{7}}{4}$$

32) $\cos\theta = -\frac{3}{5}$ where $450 \le \theta < 540$

$225 < \frac{\theta}{2} < 270$

Q3 sin −

Find $\sin\frac{\theta}{2}$

$-\frac{2\sqrt{5}}{5}$

$$\sin\left(\frac{\theta}{2}\right) = \frac{1}{2}\left(1+\frac{3}{5}\right)$$

$$= \frac{1}{2}+\frac{3}{10} \to \frac{8}{10} \to \sqrt{\frac{4}{5}}$$

$$\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$