Precalculus Name________________________________

The Algebra of Matrices Notes

Date________________ Period____

R x C

Given $A = \begin{bmatrix} 5 & -2 \\ -3 & 7 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 7 \\ 4 & -2 \end{bmatrix}$ $C = \begin{bmatrix} 6 & 3 & -9 \\ 3 & 7 & -5 \end{bmatrix}$

2x2

$D = \begin{bmatrix} 6 & -2 & 2 \\ -3 & 7 & -4 \\ 1 & 3 & -8 \end{bmatrix}$ $E = \begin{bmatrix} 2 & -5 \\ -4 & 1 \\ 3 & -8 \end{bmatrix}$ $F = \begin{bmatrix} -4 & 11 & 2 \\ 6 & 1 & 0 \\ 2 & 3 & -9 \end{bmatrix}$

3x3 3x2

$G = \begin{bmatrix} 2 & -12 & -1 \\ 5 & -4 & 6 \end{bmatrix}$ $H = \begin{bmatrix} -10 & 8 \\ 0 & 1 \\ 7 & -3 \end{bmatrix}$ $I = \begin{bmatrix} 2 & -1 \\ 3 & 7 \end{bmatrix}$

2x3

Carry out the indicated algebraic operation, or explain why it cannot be performed.

(2x2)(2x3) → (2x3)

1) *BG*

$\begin{bmatrix} -1 & 7 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 2 & -12 & -1 \\ 5 & -4 & 6 \end{bmatrix}$

$(-1)(2)+(7)(5)$ $(-1)(-12)+(7)(-4)$ $(-1)(-1)+(7)(6)$
$-2+35$ $12-28$ $1+42$

$8-10$ $-48+8$ $-4-12$
$(4)(2)+(-2)(5)$ $(4)(-12)+(-2)(-4)$ $(4)(-1)+(-2)(6)$

$\begin{bmatrix} 33 & -16 & 43 \\ -2 & -40 & -16 \end{bmatrix}$

(3x3)(3x2) = (3x2) (3x2)

2) *DHE*

Not possible

3) C^2

$\begin{bmatrix} \cdot & \end{bmatrix}^2$

N/P

(2x3)(2x3)

4) $A^3 = A \cdot A \cdot A$

$\begin{bmatrix} 5 & -2 \\ -3 & 7 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -3 & 7 \end{bmatrix}$

$\begin{bmatrix} 5\cdot 5+-2\cdot -3 & -10-14 \\ -15-21 & 6+49 \end{bmatrix} \rightarrow \begin{bmatrix} 31 & -24 \\ -36 & 55 \end{bmatrix}$

$\begin{bmatrix} 31 & -24 \\ -36 & 55 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -3 & 7 \end{bmatrix} = \begin{bmatrix} 155+72 & -62-168 \\ -180-165 & 72+385 \end{bmatrix}$

$\begin{bmatrix} 227 & -230 \\ -345 & 457 \end{bmatrix}$

Solve for *x* and *y*.

5) $3\begin{bmatrix} x & y \\ y & x \end{bmatrix} = \begin{bmatrix} 6 & -9 \\ -9 & 6 \end{bmatrix}$

ok to divide by a scalar

$\frac{}{3} \quad \frac{}{3}$

$\begin{bmatrix} x & y \\ y & x \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix}$

$x = 2$ $\quad y = -3$

6) $\begin{bmatrix} x & y \\ -y & x \end{bmatrix} - \begin{bmatrix} y & x \\ x & -y \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -6 & 6 \end{bmatrix}$

$x - y = 4$

$+ \quad -y - x = -6$

$-2y = -2$

$y = 1$

$x = 5$

$y - x = -4$

$+ \quad x + y = 6$

$2y = 2$

$y = 1$

Write the system of equations as a matrix equation.

7) $2x - 5y = 6$
$4x + 2y = 3$

$$\begin{bmatrix} 2 & -5 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

coefficient — variable — solution

8) $6x - 5z = -1$
$-x + 2y = 7$

$$\begin{bmatrix} 6 & 0 & -5 \\ -1 & 2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$$

Let $A = \begin{bmatrix} \mathbf{4} & \mathbf{7} & \mathbf{1} & \mathbf{3} \\ \mathbf{1} & \mathbf{-2} & \mathbf{-7} & \mathbf{2} \end{bmatrix}$ (2x4) $B = \begin{bmatrix} \mathbf{-2} & \mathbf{8} & \mathbf{3} & \mathbf{-1} \end{bmatrix}$ (1x4) $C = \begin{bmatrix} \mathbf{-3} \\ \mathbf{1} \\ \mathbf{5} \\ \mathbf{6} \end{bmatrix}$ (4x1)

9) Determine which of the following products are defined, and calculate the ones that are:

~~*ABC*~~ *ACB* (2x4 4x1; 2x1 1x4) ~~*BAC*~~ ~~*BCA*~~ ~~*CAB*~~ ~~*CBA*~~

Final answer (2x4)

$$\begin{bmatrix} 4 & 7 & 1 & 3 \\ 1 & -2 & -7 & 2 \end{bmatrix}\begin{bmatrix} -3 \\ 1 \\ 5 \\ 6 \end{bmatrix}$$

1st, 2nd

$$= \begin{bmatrix} -12+7+5+18 \\ -3-2-35+12 \end{bmatrix} = \begin{bmatrix} -12+30 \\ -38+10 \end{bmatrix} = \begin{bmatrix} 18 \\ -28 \end{bmatrix}\begin{bmatrix} -2 & 8 & 3 & -1 \end{bmatrix} = \begin{bmatrix} -36 & 144 & 54 & -18 \\ +56 & -224 & -84 & 28 \end{bmatrix}$$

Solve each equation or state if there is no unique solution.

10) Find the elements of C if: $A = \begin{bmatrix} -3 & -4 \\ 8 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -1 \\ 2 & -4 \end{bmatrix}$ and $3A - 4B + 6C = \begin{bmatrix} 13 & 22 \\ 10 & 4 \end{bmatrix}$

$$\begin{bmatrix} -9 & -12 \\ 24 & 18 \end{bmatrix} + \begin{bmatrix} -20 & 4 \\ -8 & 16 \end{bmatrix} + 6C = \begin{bmatrix} 13 & 22 \\ 10 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -29 & -8 \\ 16 & 34 \end{bmatrix} + 6C = \begin{bmatrix} 13 & 22 \\ 10 & 4 \end{bmatrix} + \begin{bmatrix} +29 & +8 \\ -16 & -34 \end{bmatrix}$$

$$\frac{6C}{6} = \begin{bmatrix} 42 & 30 \\ -6 & -30 \end{bmatrix} \div 6 \rightarrow C = \begin{bmatrix} 7 & 5 \\ -1 & -5 \end{bmatrix}$$

11) Find the missing values in $\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 11 \\ 20 & 29 \end{bmatrix}$

$3(4a + 2b = 10)$
$-4(3a + 5b = 11)$

$12a + 6b = 30$
$-12a - 20b = -44$

$-14b = -14$
$b = 1$ ✓

$4a + 2(1) = 10$
$4a + 2 = 10$
$4a = 8$
$a = 2$ ✓

$4c + 2d = 20$
$3c + 5d = 29$
$(2c + d = 10)5$

$3c + 5d = 29$
$-10c - 5d = -50$

$\frac{-7c}{-7} = \frac{-21}{-7}$
$c = 3$ ✓

$2(3) + d = 10$
$6 + d = 10$
$d = 4$ ✓

12) A small fast-food chain with restaurants in Santa Monica, Long Beach, and Anaheim sells only hamburgers, hot dogs, and milk shakes. On a certain day, sales were distributed according to the following matrix.

Number of items sold

	Santa Monica	Long Beach	Anaheim
Hamburgers	4000	1000	3500
Hot Dogs	400	300	200
Milk Shakes	700	500	9000

$= A$

The price of each item is given by the following matrix.

Hamburger	Hot Dog	Milk Shake
.90	.80	1.10

$= B$

(i) Calculate the product *BA* $(1 \times 3)(3 \times 3) = (1 \times 3)$

$$\begin{bmatrix} 9(4000) \to 3600 & 9(1000) \to 900 & 9(3500) \to 3150 \\ +\ 8(400) \to 320 & 8(300) \to 240 & 8(200) \to 160 \\ +\ 11(700) \to 770 & 11(500) \to 550 & 11(9000) \to 9900 \end{bmatrix} = \begin{bmatrix} 4{,}690 & 1{,}690 & 13{,}210 \end{bmatrix}$$

(ii) Interpret the entries in the product matrix *BA*

Revenue from each location

Determine if each statement is sometimes, always or never true for matrices A and B. Explain.

13) If $A + B$ exists, then $A - B$ exists.

ALWAYS

14) If kA exists and kB exists, then $kA + kB$ exists.

Sometimes

15) If k is a real number, then kA and kB exist.

ALWAYS

16) If $A - B$ does not exist, then $B - A$ does not exist.

ALWAYS

17) If A and B have the same number of elements, then $A + B$ exists.

Sometimes

18) If AB exists, then BA exists.

Sometimes

Precalculus Name________________________________

The Algebra of Matrices Notes

Date________________ Period____

Given $A = \begin{bmatrix} 5 & -2 \\ -3 & 7 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 7 \\ 4 & -2 \end{bmatrix}$ $C = \begin{bmatrix} 6 & 3 & -9 \\ 3 & 7 & -5 \end{bmatrix}$

$D = \begin{bmatrix} 6 & -2 & 2 \\ -3 & 7 & -4 \\ 1 & 3 & -8 \end{bmatrix}$ $E = \begin{bmatrix} 2 & -5 \\ -4 & 1 \\ 3 & -8 \end{bmatrix}$ $F = \begin{bmatrix} -4 & 11 & 2 \\ 6 & 1 & 0 \\ 2 & 3 & -9 \end{bmatrix}$

$G = \begin{bmatrix} 2 & -12 & -1 \\ 5 & -4 & 6 \end{bmatrix}$ $H = \begin{bmatrix} -10 & 8 \\ 0 & 1 \\ 7 & -3 \end{bmatrix}$ $I = \begin{bmatrix} 2 & -1 \\ 3 & 7 \end{bmatrix}$

Carry out the indicated algebraic operation, or explain why it cannot be performed.

1) BG

2) DHE

3) C^2

4) A^3

Solve for *x* and *y*.

5) $3\begin{bmatrix} x & y \\ y & x \end{bmatrix} = \begin{bmatrix} 6 & -9 \\ -9 & 6 \end{bmatrix}$

6) $\begin{bmatrix} x & y \\ -y & x \end{bmatrix} - \begin{bmatrix} y & x \\ x & -y \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -6 & 6 \end{bmatrix}$

Write the system of equations as a matrix equation.

7) $2x - 5y = 6$
$4x + 2y = 3$

8) $6x - 5z = -1$
$-x + 2y = 7$

Let $A = \begin{bmatrix} 4 & 7 & 1 & 3 \\ 1 & -2 & -7 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -2 & 8 & 3 & -1 \end{bmatrix}$ $C = \begin{bmatrix} -3 \\ 1 \\ 5 \\ 6 \end{bmatrix}$

9) Determine which of the following products are defined, and calculate the ones that are:

ABC *ACB* *BAC* *BCA* *CAB* *CBA*

Solve each equation or state if there is no unique solution.

10) Find the elements of C if: $A = \begin{bmatrix} -3 & -4 \\ 8 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -1 \\ 2 & -4 \end{bmatrix}$ and $3A - 4B + 6C = \begin{bmatrix} 13 & 22 \\ 10 & 4 \end{bmatrix}$

$$\begin{bmatrix} 7 & 5 \\ -1 & -5 \end{bmatrix}$$

11) Find the missing values in $\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 11 \\ 20 & 29 \end{bmatrix}$

$a = 2$, $b = 1$, $c = 3$, $d = 4$

12) A small fast-food chain with restaurants in Santa Monica, Long Beach, and Anaheim sells only hamburgers, hot dogs, and milk shakes. On a certain day, sales were distributed according to the following matrix.

Number of items sold

	Santa Monica	Long Beach	Anaheim
Hamburgers			
Hot Dogs			
Milk Shakes			

$1 = A$

The price of each item is given by the following matrix.

Hamburger	Hot Dog	Milk Shake

$1 = B$

(i) Calculate the product BA

(ii) Interpret the entries in the product matrix BA

Determine if each statement is sometimes, always or never true for matrices A and B. Explain.

13) If $A + B$ exists, then $A - B$ exists.

Always

14) If kA exists and kB exists, then $kA + kB$ exists.

Sometimes

15) If k is a real number, then kA and kB exist.

Always

16) If $A - B$ does not exist, then $B - A$ does not exist.

Always

17) If A and B have the same number of elements, then $A + B$ exists.

Sometimes

18) If AB exists, then BA exists.

Sometimes