Precalculus Name_____

The Algebra of Matrices Notes

Date_____ Period____

Given $A = \begin{bmatrix} 5 & -2 \\ -3 & 7 \end{bmatrix}$ (2 x 2)

$B = \begin{bmatrix} -1 & 7 \\ 4 & -2 \end{bmatrix}$ (2 x 2)

$C = \begin{bmatrix} 6 & 3 & -9 \\ 3 & 7 & -5 \end{bmatrix}$ (2 x 3)

$D = \begin{bmatrix} 6 & -2 & 2 \\ -3 & 7 & -4 \\ 1 & 3 & -8 \end{bmatrix}$ (3 x 3)

$E = \begin{bmatrix} 2 & -5 \\ -4 & 1 \\ 3 & -8 \end{bmatrix}$ (3 x 2)

$F = \begin{bmatrix} -4 & 11 & 2 \\ 6 & 1 & 0 \\ 2 & 3 & -9 \end{bmatrix}$ (3 x 3)

$G = \begin{bmatrix} 2 & -12 & -1 \\ 5 & -4 & 6 \end{bmatrix}$ (2 x 3)

$H = \begin{bmatrix} -10 & 8 \\ 0 & 1 \\ 7 & -3 \end{bmatrix}$ (3 x 2)

$I = \begin{bmatrix} 2 & -1 \\ 3 & 7 \end{bmatrix}$ (2 x 2)

Carry out the indicated algebraic operation, or explain why it cannot be performed.

1) *BG*

(2 x 2)(2 x 3) same

→ 2 x 3

$\begin{bmatrix} -1 & 7 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 2 & -12 & -1 \\ 5 & -4 & 6 \end{bmatrix}$

$= \begin{bmatrix} (-1)(2)+(7)(5) & (-1)(-12)+(7)(-4) & (-1)(-1)+(7)(6) \\ (4)(2)+(-2)(5) & (4)(-12)+(-2)(-4) & (4)(-1)+(-2)(6) \end{bmatrix}$

$= \begin{bmatrix} 33 & -16 & 43 \\ -2 & -40 & -16 \end{bmatrix}$

2) *DHE*

DH → 3x3 3x2 → 3x2

3x2 3x2

Different

~~No solution~~

undefined

  Made with Infinite Precalculus.

you can only raise square matrices to powers

3) C^2

$2 \times \cancel{2}$ $\cancel{2} \times 3$

Different

Undefined

4) $A^3 \rightarrow A\ A\ A$

$$\begin{bmatrix} 5 & -2 \\ -3 & 7 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -3 & 7 \end{bmatrix} = \begin{bmatrix} 25+6 & -10-14 \\ -15-21 & 6+49 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 31 & -24 \\ -36 & 55 \end{bmatrix} \rightarrow A^2 A$$

$$\begin{bmatrix} 31 & -24 \\ -36 & 55 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -3 & 7 \end{bmatrix} \rightarrow$$

$72 + 385$

$$\begin{bmatrix} 31\,5 - 24 - 3 & 31\,{-2} \\ & +{-24}\,7 \\ -36\,5 + & 1\,{-36}\cdot{-2} \\ 55\,{-3} & +55\cdot 7 \end{bmatrix} \rightarrow \begin{bmatrix} 227 & -230 \\ -345 & 457 \end{bmatrix}$$

Solve for x and y.

5) $3\begin{bmatrix} x & y \\ y & x \end{bmatrix} = \begin{bmatrix} 6 & -9 \\ -9 & 6 \end{bmatrix}$

$x = 2$

$y = -3$

$3x = 6 \quad 3y = -9$

$x = 2 \quad y = -3$

6) $\begin{bmatrix} x & y \\ -y & x \end{bmatrix} - \begin{bmatrix} y & x \\ x & -y \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -6 & 6 \end{bmatrix}$ ✓

$1 - 5 = -4$

$x - y = 4 \quad y - x = -4$

$+\ x + y = 6 \rightarrow 5 + y = 6$

$2x = 10$

$x = 5 \quad y = 1$

Write the system of equations as a matrix equation.

7) $2x - 5y = 6$
$4x + 2y = 3$

coefficient matrix

solution

$$\begin{bmatrix} 2 & -5 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

variable matrix

8) $6x - 5z = -1$
$-x + 2y = 7$

$$\begin{bmatrix} 6 & 0 & -5 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$$

2×3 $\quad 3 \times 1$ $\quad 2 \times 1$

Let $A = \begin{bmatrix} 4 & 7 & 1 & 3 \\ 1 & -2 & -7 & 2 \end{bmatrix}$ $\quad B = \begin{bmatrix} -2 & 8 & 3 & -1 \end{bmatrix}$ $\quad C = \begin{bmatrix} -3 \\ 1 \\ 5 \\ 6 \end{bmatrix}$

2×4 $\quad 1 \times 4$ $\quad 4 \times 1$

9) Determine which of the following products are defined, and calculate the ones that are:

~~ABC~~ $\quad ACB$ (2×4) $\quad$ ~~BAC~~ $\quad$ ~~BCA~~ $\quad$ ~~CAB~~ $\quad$ ~~CBA~~

AB not defined

AC → 2×1

$2 \times 1 \quad 1 \times 4$

$$\begin{bmatrix} 4 & 7 & 1 & 3 \\ 1 & -2 & -7 & 2 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} -12+7+5+18 \\ -3-2-35+12 \end{bmatrix} = \begin{bmatrix} 18 \\ -28 \end{bmatrix}$$

$$\begin{bmatrix} 18 \\ -28 \end{bmatrix} \begin{bmatrix} -2 & 8 & 3 & -1 \end{bmatrix} = \begin{bmatrix} -36 & 144 & 54 & -18 \\ 56 & -224 & -84 & 28 \end{bmatrix}$$

$2 \times 1 \quad 1 \times 4$

Solve each equation or state if there is no unique solution.

10) Find the elements of C if: $A = \begin{bmatrix} -3 & -4 \\ 8 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -1 \\ 2 & -4 \end{bmatrix}$ and $3A - 4B + 6C = \begin{bmatrix} 13 & 22 \\ 10 & 4 \end{bmatrix}$

$$\begin{bmatrix} -9 & -12 \\ 24 & 18 \end{bmatrix} + \begin{bmatrix} -20 & 4 \\ -8 & 16 \end{bmatrix} + 6C = \begin{bmatrix} 13 & 22 \\ 10 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -29 & -8 \\ 16 & 34 \end{bmatrix} + 6C = \begin{bmatrix} 13 & 22 \\ 10 & 4 \end{bmatrix} - \begin{bmatrix} -29 & -8 \\ 16 & 34 \end{bmatrix}$$

$$- \begin{bmatrix} -29 & -8 \\ 16 & 34 \end{bmatrix}$$

$$\frac{6C}{6} = \begin{bmatrix} 42 & 30 \\ -6 & -30 \end{bmatrix} \div 6 \rightarrow C = \begin{bmatrix} 7 & 5 \\ -1 & -5 \end{bmatrix}$$

11) Find the missing values in $\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 11 \\ 20 & 29 \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$4a + 2b = 10$

$3a + 5b = 11$

$(2a + b = 5) \cdot -5$

$3a + 5b = 11$

$-10a - 5b = -25$

$-7a = -14$

$a = 2$

$4(2) + 2b = 10$

$2b = 10 - 8$

$2b = 2$

$b = 1$

$-2(4c + 2d = 20)$

$3c + 5d = 29$

$-5(2c + d = 10)$

$3c + 5d = 29$

$-10c - 5d = -50$

$-7c = -21$

$c = 3$

$4(3) + 2d = 20$

$2d = 20 - 12$

$2d = 8$

$d = 4$

12) A small fast-food chain with restaurants in Santa Monica, Long Beach, and Anaheim sells only hamburgers, hot dogs, and milk shakes. On a certain day, sales were distributed according to the following matrix.

Number of items sold

	Santa Monica	Long Beach	Anaheim
Hamburgers	4000	1000	3500
Hot Dogs	400	300	200
Milk Shakes	700	500	9000

$= A$

The price of each item is given by the following matrix.

Hamburger	Hot Dog	Milk Shake
$0.90	80	1.10

$= B$

(i) Calculate the product BA

BA

$$\begin{bmatrix} 9(4000)+ & 9(1000)+ & 9(3500) \\ 8(400)+ & 8(300) & .8(200) \\ 1.1(700) & 11(500)+ & 11(9000) \end{bmatrix} \rightarrow \begin{bmatrix} \$4,690 & \$1,690 & \$13,210 \end{bmatrix}$$

(ii) Interpret the entries in the product matrix BA

Revenue in SM, LB, Anaheim

Determine if each statement is sometimes, always or never true for matrices *A* and *B*. Explain.

13) If $A + B$ exists, then $A - B$ exists.

ALWAYS

14) If kA exists and kB exists, then $kA + kB$ exists.

sometimes

15) If k is a real number, then kA and kB exist.

always

16) If $A - B$ does not exist, then $B - A$ does not exist.

always

17) If A and B have the same number of elements, then $A + B$ exists.

1x4 has 4

2x2 has 4

Sometimes

18) If AB exists, then BA exists.

Sometimes

2x2 2x3 ☺

2x3 2x2 ☹