1.) $f'(x) = \lim_{h \to 0} \frac{2(x+h) - 3 - (2x-3)}{h} \to \frac{2x+2h-3-2x+3}{h} \to \frac{2h}{h} \to 2$

$f'(x) = 2 \qquad f'(0) = 2$

2.) $f'(x) = \lim_{h \to 0} \frac{\sqrt{1+2(x+h)} - \sqrt{1+2x}}{h} \cdot \frac{\sqrt{1+2(x+h)} + \sqrt{1+2x}}{\sqrt{1+2(x+h)} + \sqrt{1+2x}} \to$

$\lim_{h \to 0} \frac{1+2(x+h) - (1+2x)}{h(\sqrt{1+2(x+h)} + \sqrt{1+2x})} \to \frac{1+2x+2h-1-2x}{h(\sqrt{1+2(x+h)} + \sqrt{1+2x})} \to$

$\lim_{h \to 0} \frac{2h}{h(\sqrt{1+2(x+h)} + \sqrt{1+2x})} \to \frac{2}{\sqrt{1+2(x+h)} + \sqrt{1+2x}} \to \frac{2}{2\sqrt{1+2x}}$

$f'(x) = \frac{1}{\sqrt{1+2x}} \qquad f'(4) = \frac{1}{\sqrt{1+8}} = \frac{1}{3}$

3.) $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - (x+h) - (x^2 - x)}{h} \to \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h}$

$\lim_{h \to 0} \frac{2xh + h^2 - h}{h} \to 2x + h - 1$

$f'(x) = 2x - 1 \qquad f'(1) = 1$

4.) $f'(x) = \lim_{h \to 0} \frac{\frac{x}{x} \cdot \frac{1}{x+h} - \frac{1}{x} \cdot \frac{x+h}{x+h}}{h} \to \frac{x - (x+h)}{h(x(x+h))} \to \frac{x - x - h}{h(x(x+h))}$

$\lim_{h \to 0} \frac{-h}{h(x(x+h))} \to \frac{-1}{x(x+h)}$

$f'(x) = \frac{-1}{x^2} \qquad f'(2) = -\frac{1}{4}$

5) $g'(x) = 6x - 2$

6) $k(x) = x + 2x^{2/3} + 4x^{\frac{1}{3}} - 2x^{2/3} - 4x^{1/3} - 8$

$k(x) = x - 8$

$k'(x) = 1$

7.) $p'(x) = 3(x-1)^2(1) = 3(x-1)^2$

8.) $y'(x) = 2x - 3 + 5x^{-2} - 14x^{-3}$

9.) $w'(x) = 2(3x^2+4)'(6x) = 12x(3x^2+4)$

10.) $G(x) = 6x^2 + 15x - 2x - 5 = 6x^2 + 13x - 5$

$G'(x) = 12x + 13$

11.) $J(x) = \frac{1}{4}x^3 - \frac{1}{6}x^2 + \frac{1}{2}$

$J'(x) = \frac{3}{4}x^2 - \frac{1}{3}x$

12.) $s(x) = x^{\frac{1}{2}} + 17x^{\frac{2}{3}}$

$s'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{34}{3}x^{-\frac{1}{3}}$

13.) $t(x) = \frac{5}{2}x^{-3} - \frac{3}{5}x^{-4}$

$t'(x) = \frac{-15}{2}x^{-4} + \frac{12}{5}x^{-5}$

14.) $v'(x) = 2\pi x^2 + 20\pi x$

15) 0

16.) m

17.) nx^{n-1}

18) $x^3 + 3x^2 - x$ for example

19.) $y = 7x^2 - 3$

$y' = 14x$ $\quad y'(1) = 14(1) = 14$

$\frac{dy}{dx}$ is same as y' $\quad \left.\frac{dy}{dx}\right|_{x=2} = 14(2) = 28$

so $= 14x$

20.) 

$$f(x)=\begin{cases} x-1 & x\ge 1 \\ -x+1 & x<1 \end{cases}$$

$$f'(x)=\begin{cases} 1 & x>1 \\ -1 & x<1 \end{cases}$$

Not differentiable at $x=1$

$f'(1^-)\neq f'(1^+)$

21) $f'(x)=\begin{cases} 1 & x<1 \\ 2x & x>1 \end{cases}$ Not differentiable @ $x=1$

$1\neq 2(1)$

* Don't forget to check continuity 1st though $1=1^2$ ✓

22) $f(x)=(1-x^2)^{\frac{1}{2}}$

$f'(x)=\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) = \dfrac{-2x}{2\sqrt{1-x^2}}$

$f'(1)=\dfrac{-2(1)}{2\sqrt{1-1^2}} \rightarrow \dfrac{-1}{0}$ undefined Not differentiable @ $x=1$

23) Check continuity $2^2\neq 4(2)-2$

not continuous so not differentiable

24) $(1-1)^3=(1-1)^2 \rightarrow$ continuous ✓

$f'(x)=\begin{cases} 3(x-1)^2(1) & x<1 \\ 2(x-1)^1(1) & x>1 \end{cases} \rightarrow 3(1-1)^2=2(1-1)^1$

$0=0$ ✓

Differentiable :)

25) $\frac{1}{2}(1)\neq\sqrt{1}-1$

not continuous so not differentiable

1.) $\frac{dy}{dx} = -2x$ $\quad \frac{dy}{dx}\Big|_{x=-1} = -2(-1) = 2$ $\quad y = 4-(-1)^2$

$= 3$

T: $y-3 = 2(x+1)$

2.) $y = 2x^{\frac{1}{2}}$ $\quad \frac{dy}{dx} = 2(\tfrac{1}{2})x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}} \rightarrow \frac{dy}{dx}\Big|_{x=1} = \frac{1}{\sqrt{1}} = 1$

$y = 2\sqrt{1} = 2$ $\quad$ T: $y-2 = 1(x-1)$

3.) $\frac{dy}{dx} = 1-4x$ $\quad 1-4(1) = -3$ $\quad y = 1-2(1)^2 = -1$

T: $y+1 = -3(x-1)$

4.) $\frac{dy}{dx} = -3x^{-4}$ $\quad \frac{-3}{(-2)^4} = \frac{-3}{16}$ $\quad y = (-2)^{-3} = -\frac{1}{8}$

T: $y + \frac{1}{8} = \frac{-3}{16}(x+2)$

5.) $\frac{dy}{dx} = 3x^2+3$ $\quad 3(1)^2+3 = 6$ $\quad y = 1^3+3(1) = 4$

T: $y-4 = 6(x-1)$

6.) $\frac{dy}{dx} = 2x+4$ $\quad 2x+4=0$ $\quad y = (-2)^2+4(-2)-1$

$x = -2$ $\quad 4-8-1$

-5

$\boxed{(-2,-5)}$

7.) $y = x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \nearrow \frac{1}{2\sqrt{x}} = \frac{1}{4}$ $\quad x = 4$

$y = \sqrt{4} = 2$ $\quad y - 2 = \frac{1}{4}(x-4)$

8.) $A = \pi r^2$

$\frac{dA}{dr} = 2\pi r \Big|_{r=3} \rightarrow 6\pi$ cm

9.) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2 \Big|_{r=2} \rightarrow 16\pi \text{ cm}^2$

10.) $\lim_{x \to 0^+} x^2 \sin\left(\frac{1}{x}\right) \rightarrow 0$ $\quad \lim_{x \to 0^-} x^2 \sin\left(\frac{1}{x}\right) \rightarrow 0$

So since $\lim_{x \to 0} f(x) = 0 = f(0)$ it is continuous

and $x^2 \sin\left(\frac{1}{x}\right)$ is a smooth curve so yes there is a tangent at the origin.

11.) (a) $\frac{dy}{dx} = 3x^2 - 4 \Big|_{x=2} \rightarrow 3(2)^2 - 4 = 8$ $\quad$ T: $y - 1 = 8(x-2)$

(b) $3x^2 - 4$ has vertex @ $(0,-4)$ so range is $[-4, \infty)$

(c) $3x^2 - 4 = 8 \rightarrow 3x^2 = 12 \rightarrow x^2 = 4 \rightarrow x = \pm 2$ $\quad y = 2^3 - 4(2) + 1 = 1$

$y = (-2)^3 - 4(-2) + 1 = 1$

T: $y - 1 = 8(x-2)$ or $y - 1 = 8(x+2)$

12.) $\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} \rightarrow \frac{1}{3\sqrt[3]{x^2}} \rightarrow \frac{1}{3 \cdot 0}$ not differentiable @ $x=0$

13.) $\frac{dy}{dx} = \frac{1}{4}x^{-\frac{3}{4}} \rightarrow \frac{1}{4(0)^{3/4}}$ not differentiable @ $x=0$

14.) $\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} \rightarrow \frac{2}{3(0)^{\frac{1}{3}}}$ not differentiable @ $x=0$

15.) $\frac{dy}{dx} = \frac{5}{4}x^{\frac{1}{4}} \rightarrow \frac{5}{4}(0)^{\frac{1}{4}} = 0$ differentiable @ $x=0$

16.) $\frac{dy}{dx} = \frac{4}{3}x^{\frac{1}{3}} \rightarrow \frac{4}{3}(0)^{\frac{1}{3}} = 0$ differentiable @ $x=0$

17.) $\frac{dy}{dx} = \frac{1}{5}x^{-\frac{4}{5}} \rightarrow \frac{1}{5(0)^{4/5}} \rightarrow$ not differentiable @ $x=0$

18.) $\frac{dy}{dx} = \frac{5}{3}x^{\frac{2}{3}} \rightarrow \frac{5}{3}(0)^{2/3} \rightarrow$ differentiable @ $x=0$

19.) $\frac{dy}{dx} = \frac{2}{5}x^{-\frac{3}{5}} \rightarrow \frac{2}{5(0)^{3/5}} \rightarrow$ not differentiable @ $x=0$

20.) $x^{\frac{\text{even}}{\text{odd}}} \rightarrow 14, 16, 19 \rightarrow$ differentiable @ all values except 0 unless exponent ≥ 1, then differentiable @ 0 too.

$x^{\frac{\text{odd}}{\text{odd}}} \rightarrow 12, 17, 18 \rightarrow$ same answer as $\uparrow$

$x^{\frac{\text{odd}}{\text{even}}} \rightarrow 13, 15 \rightarrow$ differentiable if $x > 0$
only differentiable @ $x=0$ if exponent > 1.