

Simplify the trigonometric expression.

$$1.) \cos^3 x + \sin^2 x \cos x$$

$$\cos x (\cos^2 x + \sin^2 x)$$

$$\cos x (1) = \boxed{\cos x}$$

$$5.) \tan x \cos x \csc x$$

$$\frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} \cdot \frac{1}{\sin x} = \boxed{1}$$

$$2.) \frac{\tan x}{\sec(-x)} = \frac{\tan x}{\sec x}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= \sin x$$

$$6.) \frac{1 + \cot A}{\csc A}$$

$$\frac{\frac{\sin A}{\sin A} + \frac{\cos A}{\sin A}}{\frac{1}{\sin A}} = \boxed{\sin A + \cos A}$$

$$* \frac{1}{\csc A} + \frac{\cot A}{\csc A} \rightarrow \boxed{\sin A + \cos A}$$

$$3.) \frac{\sec x - \cos x}{\tan x}$$

$$\frac{\frac{1}{\cos x} - \frac{\cos x}{1} \cdot \frac{\cos x}{\cos x}}{\frac{\sin x}{\cos x}}$$

$$\frac{1 - \cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \boxed{\sin x}$$

$$7.) \frac{\cos x}{\sec x + \tan x}$$

$$\frac{\frac{\cos x}{1} \cdot \frac{\cos x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} \rightarrow \frac{\cos^2 x}{1 + \sin x} \rightarrow \frac{1 - \sin^2 x}{1 + \sin x}$$

$$\boxed{1 - \sin x} \leftarrow \frac{(1 + \sin x)(1 - \sin x)}{1 + \sin x}$$

$$4.) \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x}$$

$$\frac{\sin x}{\frac{1}{\sin x}} + \frac{\cos x}{\frac{1}{\cos x}}$$

Verify the identity.

$$\sin^2 x + \cos^2 x = \boxed{1}$$

$$8.) \cot x \tan x = 1$$

$$\frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos x}$$

$$1 = 1 \checkmark$$

9.) $\csc x \cos x = \cot x$

$$\frac{1}{\sin x} \frac{\cos x}{1}$$

$$\frac{\cos x}{\sin x}$$

$\cot x = \cot x \checkmark$

13.) $\frac{\tan^2 x \cos x}{2 \sec x} = \frac{1}{2} \sin^2 x$

$$\frac{\frac{1}{2} \frac{\cancel{\cos x}}{1} \frac{\sin^2 x}{\cancel{\cos^2 x}} \frac{\cancel{\cos x}}{1}}{\frac{\cancel{2}}{1} \frac{1}{\cancel{\cos x}}}$$

$$\frac{1}{1} \frac{1}{\cos x}$$

$\frac{1}{2} \sin^2 x = \text{RHS} \checkmark$

10.) $\frac{\sin x}{\tan x} = \cos x$

$$\frac{\cos x}{\sin x} \frac{\sin x}{1}$$

$$\frac{\cancel{\sin x}}{\cancel{\cos x}}$$

$\cos x = \cos x \checkmark$

14.) $\sec x - \sin x = \frac{1 - \sin x \cos x}{\cos x}$

$$\frac{1}{\cos x} - \frac{\sin x \cancel{\cos x}}{\cancel{\cos x}}$$

$\checkmark \text{LHS} = \sec x - \sin x$

11.) $\frac{\tan x}{\csc x} = \frac{\sin^2 x}{\cos x}$

$$\frac{\sin x}{1} \frac{\sin x}{\cos x}$$

$$\frac{\sin^2 x}{\cos x} = \frac{\sin^2 x}{\cos x} \checkmark$$

15.) $\sec^2 x + \cot x = \frac{\sin x + \cos^3 x}{\cos^2 x \sin x}$

$$\frac{\sin x}{\cos^2 x \sin x} + \frac{\cos^3 x}{\cos^2 x \sin x}$$

$$\frac{1}{\cos^2 x} + \frac{\cos x}{\sin x}$$

$\checkmark \text{LHS} = \sec^2 x + \cot x$

12.) $\frac{\sec x \csc x}{\cot x} = \sec^2 x$

$$\frac{\frac{1}{\cos x} \frac{1}{\cancel{\sin x}} \frac{\cancel{\sin x}}{\cos x}}{\frac{\cancel{\cos x}}{\cancel{\sin x}}}$$

$$\frac{1}{\cos^2 x}$$

$$\frac{1}{\cos^2 x}$$

$\sec^2 x = \text{RHS} \checkmark$

(right hand side)

16.) $\csc^2 x - \tan x = \frac{\cos x - \sin^3 x}{\sin^2 x \cos x}$

$$\frac{\cos x}{\sin^2 x \cos x} - \frac{\sin^3 x}{\sin^2 x \cdot \cos x}$$

$$\frac{1}{\sin^2 x} - \frac{\sin x}{\cos x}$$

$\checkmark \text{LHS} = \csc^2 x - \tan x$

$$17.) \csc x + 1 = \frac{1 + \sin x}{\sin x}$$

$$\frac{1}{\sin x} + \frac{\sin x}{\sin x}$$

$$\checkmark \text{ LHS} = \csc x + 1$$

$$18.) \sec x - \tan x \sin x = \cos x$$

$$\frac{1}{\cos x} - \frac{\sin x}{\cos x} \sin x$$

$$\frac{1 - \sin^2 x}{\cos x}$$

$$\frac{\cos^2 x}{\cos x} = \cos x = \text{RHS} \checkmark$$

$$19.) \tan x + \cot x = \sec x \csc x$$

$$\frac{\sin x}{\sin x \cos x} + \frac{\cos x}{\sin x} \frac{\cos x}{\cos x}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\cos x \sin x} = \sec x \cdot \csc x$$

RHS \checkmark

$$20.) \sec x - \cos x = \tan x \sin x$$

$$\frac{1}{\cos x} - \frac{\cos x}{1} \cdot \frac{\cos x}{\cos x}$$

$$\frac{1 - \cos^2 x}{\cos x}$$

$$\frac{\sin^2 x}{\cos x} \rightarrow \frac{\sin x}{\cos x} \frac{\sin x}{1}$$

$$\rightarrow \tan x \sec x = \text{RHS} \checkmark$$

$$21.) 1 + \tan^2 B = \sec^2 B$$

$$\sec^2 B = \sec^2 B$$

$$22.) \sec^2 x - 1 = \sin^2 x \sec^2 x$$

$$\tan^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} \rightarrow \sin^2 x \frac{1}{\cos^2 x}$$

$$\rightarrow \sin^2 x \sec^2 x \checkmark \text{RHS}$$

$$23.) 1 - \csc^2 x = -\cot^2 x$$

$$-1(\csc^2 x - 1)$$

$$-1(\cot^2 x) = \text{RHS} \checkmark$$

$$24.) \csc x - \sin x = \cos x \cot x$$

$$\frac{1}{\sin x} - \frac{\sin x}{1} \frac{\sin x}{\sin x}$$

$$\frac{1 - \sin^2 x}{\sin x} \rightarrow \frac{\cos^2 x}{\sin x}$$

$$\rightarrow \cos x \frac{\cos x}{\sin x}$$

$$\rightarrow \cos x \cdot \cot x = \text{RHS} \checkmark$$