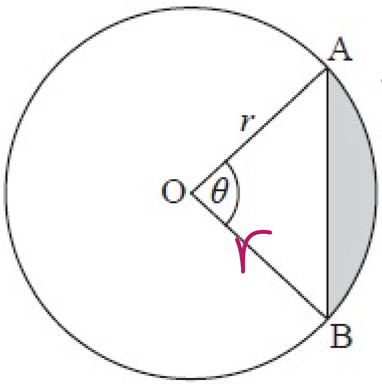
Trig Notes Calculator                    Name:____________________

**1a.** *[3 marks]*

$A_{circle} = \pi r^2$    $C = 2\pi r$

A circle centre O and radius $r$ is shown below. The chord [AB] divides the area of the circle into two parts. Angle AOB is $\theta$ .

$\frac{\theta}{2\pi}$ is the fraction of the circlein question when $\theta \to$ central angle




Area $\pi r^2 \cdot \frac{\theta}{2\pi} = \frac{\pi r^2 \theta}{2\pi} = \frac{r^2\theta}{2} = \boxed{\frac{1}{2}\theta r^2}$

Arc Length $2\pi r \cdot \frac{\theta}{2\pi} = \frac{2\pi r\theta}{2\pi} = \boxed{\theta r}$

→ probably will have variable in it

Find an expression for the area of the shaded region.

Area of sector − △

$$\frac{1}{2}\theta r^2 - \frac{1}{2}r^2\sin\theta$$

**1b.** *[5 marks]*

The chord [AB] divides the area of the circle in the ratio 1:7. Find the value of $\theta$ .

shaded region is $\frac{1}{8}$ of the entire circle

$$\frac{1}{2}\theta r^2 - \frac{1}{2}r^2\sin\theta = \frac{1}{8}\pi r^2$$

$$\frac{1}{2}\theta - \frac{1}{2}\sin\theta = \frac{1}{8}\pi$$

$y_1 = \frac{1}{8}\pi$

$y_2 = \frac{1}{2}x - \frac{1}{2}\sin x$

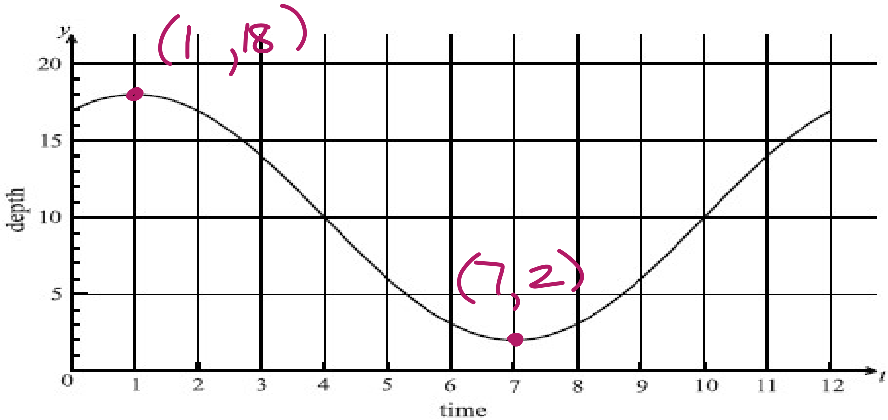
Window $0 \leftrightarrow 2\pi$

ZOOM FIT

$\theta = 177$

**2a.** *[3 marks]*

The following graph shows the depth of water, $y$ metres , at a point P, during one day. The time $t$ is given in hours, from midnight to noon.




Use the graph to write down an estimate of the value of $t$ when

(i) the depth of water is minimum;

$t = 7$

(ii) the depth of water is maximum;

$t = 1$

(iii) the depth of the water is increasing most rapidly.

$t = 10$

**2b.** *[6 marks]*

The depth of water can be modelled by the function $y = A\cos B(t - 1) + C$ .

(i) Show that $A = 8$ .

A is amplitude $\frac{18-2}{2} = \frac{16}{2} = 8$

$18 - 10 = 8$

(ii) Write down the value of $C$.

10

(iii) Find the value of $B$.

Period $= (7-1)(2) = 12$

$\frac{2\pi}{B} = 12$

$2\pi = 12B$

$\frac{2\pi}{12} = B$ $B = \frac{\pi}{6}$

**2c.** *[2 marks]*

A sailor knows that he cannot sail past P when the depth of the water is less than 12 m . Calculate the values of *t* between which he cannot sail past P.

$y = 8\cos\frac{\pi}{6}(t-1)+10 \qquad y = 12$

$t = 3\ 52$

$t = 10\ 5$

**3a.** *[3 marks]*

Let $f(x) = \frac{3x}{2} + 1$ , $g(x) = 4\cos\left(\frac{x}{3}\right) - 1$ . Let $h(x) = (g \circ f)(x)$ .

Find an expression for $h(x)$ .

$h(x) = 4\cos\left(\frac{f(x)}{3}\right) - 1 = 4\cos\left(\frac{\frac{3x}{2}+1}{3}\right) - 1$

$\downarrow$

$4\cos\left(\frac{1}{2}x + \frac{1}{3}\right) - 1$

**3b.** *[1 mark]*

Write down the period of $h$ .

$\frac{2\pi}{1/2} = 4\pi$

**3c.** *[2 marks]*

Write down the range of $h$ .

$4(1) - 1 = 3$

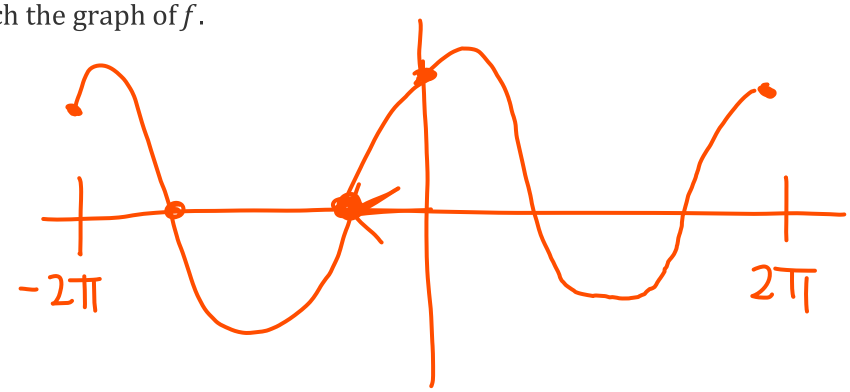
$4(-1) - 1 = -5$

$[-5, 3]$

**4a.** *[3 marks]*

Let $f(x) = 3\sin x + 4\cos x$ , for $-2\pi \leq x \leq 2\pi$ .

Sketch the graph of *f* .



**4b.** *[3 marks]*

Write down

(i) the amplitude; 5

(ii) the period; 6.28

(iii) the *x*-intercept that lies between $-\frac{\pi}{2}$ and 0. $-0927$

**4c.** *[3 marks]*

Hence write $f(x)$ in the form $p\sin(qx+r)$ .

use what you just did

$5\sin(x+927)$

$\frac{2\pi}{b}=2\pi$

**4d.** *[2 marks]*

Write down one value of *x* such that $f'(x)=0$ .

$f'(x) = 5\cos(x+927) = 0$

$x = 644$

**4e.** *[2 marks]*

Write down the two values of *k* for which the equation $f(x)=k$ has exactly two solutions.

5 , $-5$

**4f.** *[5 marks]*

Let $g(x) = \ln(x+1)$ , for $0 \leq x \leq \pi$ . There is a value of *x*, between $0$ and $1$, for which the gradient of *f* is equal to the gradient of *g*. Find this value of *x*.

gradient = slope = derivative

$\boxed{x = .511}$

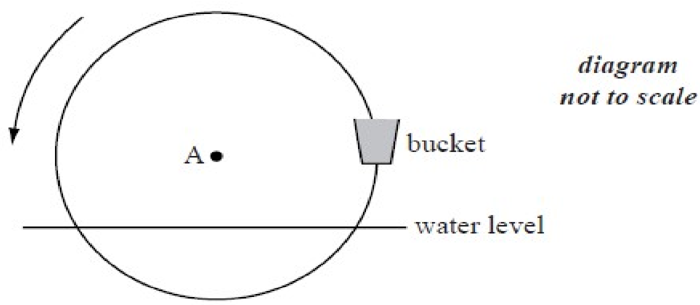
$f'(x) = 5\cos(x + .927)$

$g'(x) = \frac{1}{x+1}$

$5\cos(x + .927) = \frac{1}{x+1}$

**5a.** *[2 marks]*

The following diagram shows a waterwheel with a bucket. The wheel rotates at a constant rate in an anticlockwise (counter-clockwise) direction.




The diameter of the wheel is 8 metres. The centre of the wheel, A, is 2 metres above the water level. After *t* seconds, the height of the bucket above the water level is given by $h = a \sin bt + 2$ .

Show that $a = 4$ .

$d = 8$, so $r = 4$ and a represent amplitude $a = 4$

**5b.** *[2 marks]*

The wheel turns at a rate of one rotation every 30 seconds.

Show that $b = \frac{\pi}{15}$ .

$\frac{2\pi}{b} = 30 \rightarrow \frac{2\pi}{30} = \frac{\pi}{15} = b$

$\frac{2\pi}{30} = \frac{\pi}{15}$

$h = 4\sin\left(\frac{\pi}{15}t\right) + 2$

**5c.** *[6 marks]*

$\frac{m}{s}$

In the first rotation, there are two values of $t$ when the bucket is **descending** at a rate of $0.5\ \mathrm{ms}^{-1}$.

Find these values of $t$.

$h'(t) = 4\cos\left(\frac{\pi}{15}t\right)\frac{\pi}{15}$

$-5 = 4\cos\left(\frac{\pi}{15}t\right)\frac{\pi}{15}$

$t = 10.6$

$t = 19.4$

**5d.** *[4 marks]*

In the first rotation, there are two values of $t$ when the bucket is **descending** at a rate of $0.5\ \mathrm{ms}^{-1}$.

Determine whether the bucket is underwater at the second value of $t$.

$h(19.4) = -1.19$

don't round off

yes underwater

**6a.** *[3 marks]*

$2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$

**Note: In this question, distance is in millimetres.**

Let $f(x) = x + a\sin\left(x - \frac{\pi}{2}\right) + a$, for $x \geqslant 0$.

Show that $f(2\pi) = 2\pi$.

$f(2\pi) = 2\pi + a\sin\left(2\pi - \frac{\pi}{2}\right) + a$

$= 2\pi + a\sin\left(\frac{3\pi}{2}\right) + a$

$= 2\pi + a(-1) + a$

$f(2\pi) = 2\pi$

**6b.** *[3 marks]*

The graph of $f$ passes through the origin. Let $\mathrm{P}_k$ be any point on the graph of $f$ with $x$-coordinate $2k\pi$, where $k \in \mathbb{N}$. A straight line $L$ passes through all the points $\mathrm{P}_k$.

Find the coordinates of $\mathrm{P}_0$ and of $\mathrm{P}_1$.

$2(0)\pi \to 0$

$2(1)\pi \to 2\pi$

$(0,0)$

$(2\pi, 2\pi)$

**6c.** *[3 marks]*

Find the equation of $L$.

$y = x$

$$\frac{2\pi - 0}{2\pi - 0} = 1$$

**6d.** *[2 marks]*

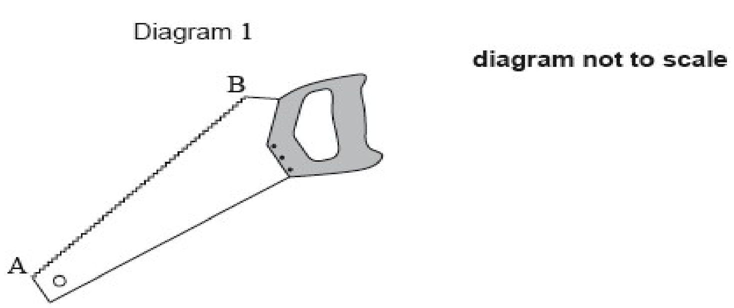
Show that the distance between the $x$-coordinates of $\mathrm{P}_k$ and $\mathrm{P}_{k+1}$ is $2\pi$.

$2(k+1)\pi - 2k\pi$

$2k\pi + 2\pi - 2k\pi = 2\pi$

**6e.** *[6 marks]*

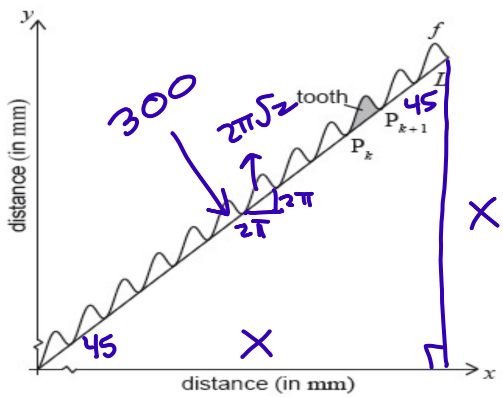
Diagram 1 shows a saw. The length of the toothed edge is the distance AB.




The toothed edge of the saw can be modelled using the graph of $f$ and the line $L$. Diagram 2 represents this model.

Diagram 2

**diagram not to scale**




$f = x + a\sin(x - \frac{\pi}{2}) + a$

$y = x$ is Line L

The shaded part on the graph is called a tooth. A tooth is represented by the region enclosed by the graph of $f$ and the line $L$, between $P_k$ and $P_{k+1}$.

A saw has a toothed edge which is 300 mm long. Find the number of complete teeth on this saw.

$x^2 + x^2 = 300^2$

$2x^2 = 300^2$

$2x^2 = 90000$

$x^2 = 45{,}000$

$x = \sqrt{45{,}000}$

$x = 212\ 13$

Now $\frac{212\ 13}{2\pi}$

$\rightarrow 33\ 76$

33

**7a.** *[3 marks]*

Let $f(x) = 2\sin(3x) + 4$ for $x \in \mathbb{R}$.

The range of $f$ is $k \le f(x) \le m$. Find $k$ and $m$.

$2(1) + 4 = 6 = m$

$2(-1) + 4 = 2 = k$

**7b.** *[2 marks]*

Let $g(x) = 5f(2x)$.

Find the range of $g$.

$5(2\sin(3 \cdot 2x) + 4)$

$5\ 2\sin(6x) + 20$

$[10, 30]$ $\boxed{10 \le y \le 30}$

**7c.** *[3 marks]*

The function $g$ can be written in the form $g\left(x\right) = 10\sin\left(bx\right) + c$.

Find the value of $b$ and of $c$.

$b = 6$

$c = 20$

**7d.** *[2 marks]*

Find the period of $g$.

$$\frac{2\pi}{6} = \frac{\pi}{3}$$

**7e.** *[3 marks]*

The equation $g\left(x\right) = 12$ has two solutions where $\pi \le x \le \frac{4\pi}{3}$. Find both solutions.

$x = 3.82$ $x = 4.03$

**8a.** *[2 marks]*

$A = \pi r^2$ $\quad C = 2\pi r$

The diagram below shows a circle with centre O and radius 8 cm.

$A_{sector} = \pi r^2 \frac{\theta}{2\pi} = \frac{1}{2}r^2\theta$ $\quad$ $\text{Length}_{arc} = 2\pi r \frac{\theta}{2\pi} = r\theta$

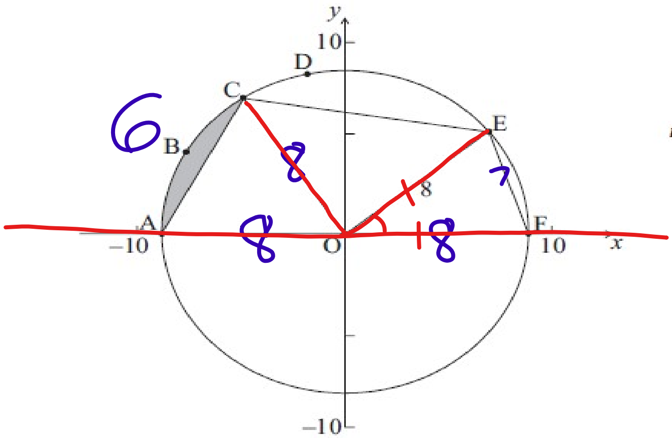



*diagram not to scale*

The points A, B, C, D, E and F are on the circle, and [AF] is a diameter. The length of arc ABC is 6 cm.

Find the size of angle AOC .

$6 = r \cdot \theta$ $\quad$ $\boxed{\theta = \frac{6}{8} \text{ or } \frac{3}{4}}$

**8b.** *[6 marks]*

Hence find the area of the shaded region.

Area sector − Area of △

$\frac{1}{2}(8)^2 \frac{3}{4} - \frac{1}{2}(8)(8)\sin\left(\frac{3}{4}\right) = \boxed{219 \text{ cm}^2}$

**8c.** *[2 marks]*

The area of sector OCDE is $45 \text{ cm}^2$.

Find the size of angle COE .

$\frac{1}{2}r^2\theta = 45$

$\frac{1}{2}(8)^2\theta = 45 \rightarrow \boxed{\theta = \frac{45}{32}}$

**8d.** *[5 marks]*

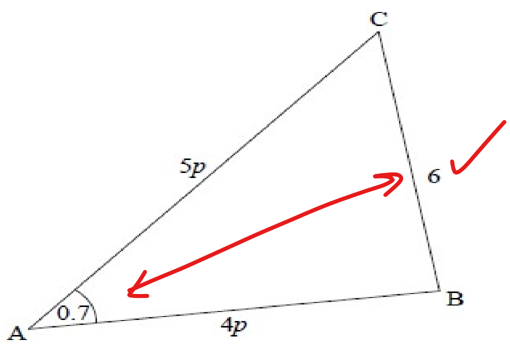
Find EF. L O C

$$EF^2 = 8^2 + 8^2 - 2(8)(8)\cos\left(\pi - \frac{45}{32} - \frac{3}{4}\right)$$

$$EF^2 = 57.27..$$ $\boxed{EF = 7.57 \text{ cm}}$

**9a.** *[4 marks]*

The following diagram shows a triangle ABC.




$BC = 6$, $C\hat{A}B = 0.7$ radians, $AB = 4p$, $AC = 5p$, where $p > 0$.

(i) Show that $p^2(41 - 40\cos 0.7) = 36$. $c^2 = a^2 + b^2 - 2ab\cos C$

$$6^2 = (5p)^2 + (4p)^2 - 2(5p)(4p)\cos(.7)$$

$$36 = 25p^2 + 16p^2 - 40p^2\cos(.7)$$

$$36 = 41p^2 - 40p^2\cos(.7)$$

(ii) Find $p$.

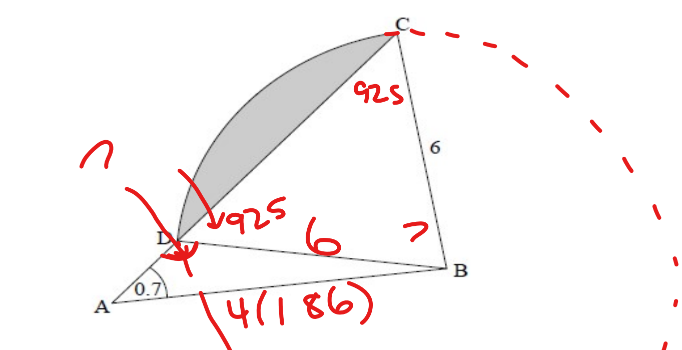
$$36 = p^2(41 - 40\cos(.7))$$

$$\sqrt{p^2} = \sqrt{\frac{36}{41 - 40\cos(.7)}}$$

$p = 1.86$

**9b.** *[1 mark]*

Consider the circle with centre B that passes through the point C. The circle cuts the line CA at D, and $\hat{ADB}$ is obtuse. Part of the circle is shown in the following diagram.




Write down the length of BD. 6

**9c.** *[4 marks]*

Find $\hat{ADB}$.

$$\frac{\sin ADB}{4(1\,86)} = \frac{\sin(7)}{6}$$

$$ADB = \sin^{-1}\left(\frac{4(1\,86)\sin(7)}{6}\right)$$

calculator says

ADB = 925

But ADB is obtuse

$$\boxed{\angle ADB = \pi - .925}$$

**9d.** *[6 marks]*

(i) Show that $\hat{CBD} = 1.29$ radians, correct to 2 decimal places.

$\pi - 925 - 925 = 1.29$

(ii) Hence, find the area of the shaded region.

Sector − △

$\frac{1}{2}r^2\theta - \frac{1}{2}ab\sin C$

$\frac{1}{2}(6)^2(1\,29) - \frac{1}{2}(6)(6)\sin 1\,29 = 5.94$