AB Calculus U-Substitution Day 2 **Name:** ______________________________

1. Evaluate each indefinite integral.

a) $\int 5e^{5x}\,dx$

$u=5x$

$\frac{du}{dx}=5$

$du=5dx$

$\int e^{u}\,du$

$=e^{u}+c$

$\boxed{=e^{5x}+c}$

b) $\int \sqrt{2x-1}\,dx$

$u=2x-1$

$\frac{du}{dx}=2$

$\frac{1}{2}du=dx$

$\frac{1}{2}\int \sqrt{u}\,du$

$=\frac{1}{2}\frac{2}{3}u^{\frac{3}{2}}+c$

$\boxed{=\frac{1}{3}(2x-1)^{3/2}+c}$

c) $\int \sin^3 x \cos x\,dx$

$u=\sin x$

$\frac{du}{dx}=\cos x$

$du=\cos x\,dx$

$\int u^3\,du$

$=\frac{u^4}{4}+c$

$\boxed{=\frac{\sin^4 x}{4}+c}$

d) $\int t\sqrt{t^2+2}\,dt$

$u=t^2+2$

$\frac{du}{dt}=2t$

$\frac{du}{2}=t\,dt$

$=\frac{1}{2}\int \sqrt{u}\,du$

$=\frac{1}{2}\frac{2}{3}u^{3/2}+c$

$\boxed{=\frac{1}{3}(t^2+2)^{3/2}+c}$

e) $\int \tan^2 x \sec^2 x\,dx$

$u=\tan x$

$\frac{du}{dx}=\sec^2 x$

$du=\sec^2 x\,dx$

$\int u^2\,du$

$\frac{u^3}{3}+c$

$\boxed{=\frac{\tan^3 x}{3}+c}$

f) $\int x^2(6-x^3)^4\,dx$

$u=6-x^3$

$\frac{du}{dx}=-3x^2$

$-\frac{1}{3}du=x^2dx$

$-\frac{1}{3}\int u^4\,du$

$-\frac{1}{3}\frac{u^5}{5}+c$

$\boxed{=-\frac{1}{15}(6-x^3)^5+c}$

g) $\int \frac{x^2}{(1+x^3)^2}\,dx$

$u=1+x^3$

$\frac{du}{dx}=3x^2$

$\frac{1}{3}du=x^2dx$

$\frac{1}{3}\int \frac{1}{u^2}\,du$

$\frac{1}{3}\int u^{-2}\,du$

$-\frac{1}{3}u^{-1}+c$

$\boxed{=-\frac{1}{3}(x^3+1)^{-1}+c}$

h) $\int \frac{x^3}{\sqrt{1+x^4}}\,dx$

$u=1+x^4$

$\frac{du}{dx}=4x^3$

$\frac{1}{4}du=x^3dx$

$\frac{1}{4}\int \frac{1}{\sqrt{u}}\,du$

$=\frac{1}{4}u^{-\frac{1}{2}}\,du$

$=\frac{1}{4}\cdot 2u^{\frac{1}{2}}+c \Longrightarrow \boxed{\frac{1}{2}(1+x^4)^{\frac{1}{2}}+c}$

2. Evaluate each of the following definite integrals.

a) $\int_{-1}^{1} x(x^2+1)^3 dx$

$u = 1^2 + 1 = 2$

$u = (-1)^2 + 1 = 2$

$u = x^2 + 1$

$\frac{du}{dx} = 2x$

$\frac{1}{2} du = x dx$

$\frac{1}{2}\int_2^2 u^3 du$

$\boxed{= 0}$

b) $\int_{1}^{2} 2x^2\sqrt{x^3+1}\,dx$

$u = 2^3 + 1 = 9$

$u = 1^3 + 1 = 2$

$u = x^3 + 1$

$\frac{du}{dx} = 3x^2$

$\frac{1}{3} du = x^2 dx$

$\frac{1}{3} \cdot 2\int_2^9 \sqrt{u}\, du$

$\frac{2}{3} \quad \frac{2}{3} u^{3/2} \Big|_2^9 = \frac{4}{9}(9)^{3/2} - \frac{4}{9}(2)^{3/2}$

$\boxed{= \frac{4}{9}(27) - \frac{4}{9}\sqrt{8}}$

c) $\int_{0}^{4} \frac{1}{\sqrt{2x+1}} dx$

$u = 2x + 1$

$\frac{du}{dx} = 2$

$\frac{1}{2} du = dx$

$u = 4(2) + 1 = 9$

$u = 0(2) + 1 = 1$

$\frac{1}{2}\int_1^9 \frac{1}{\sqrt{u}} du$

$= \frac{1}{2}\int_1^9 u^{-\frac{1}{2}} du = \frac{1}{2}\, 2u^{\frac{1}{2}} \Big|_1^9 = \sqrt{9} - \sqrt{1} = 2$

d) $\int_{1}^{9} \frac{1}{\sqrt{x}(1+\sqrt{x})^2} dx$

$u = 1 + \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$2du = \frac{1}{\sqrt{x}} dx$

$u = 1 + \sqrt{1} = 2$

$u = 1 + \sqrt{9} = 4$

$2\int_2^4 \frac{1}{u^2} du$

$2\int_2^4 u^{-2} du = \frac{2}{-1} u^{-1} \Big|_2^4 = -2(4)^{-1} + 2(2)^{-1}$

$= -\frac{2}{4} + \frac{2}{2}$

$= 1/2$

e) $\int_{1}^{2} e^{1-x} dx$

$u = 1 - x$

$\frac{du}{dx} = -1$

$-1 du = dx$

$u = 1 - 2 = -1$

$u = 1 - 1 = 0$

$-1\int_0^{-1} e^u du$

$= -e^u \Big|_0^{-1}$

$= -e^{-1} + e^0 = \boxed{-\frac{1}{e} + 1}$

f) $\int_{1}^{e} \frac{(1+\ln x)^2}{x} dx$

$u = 1 + \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x} dx$

$u = 1 + \ln e = 1 + 1 = 2$

$u = 1 + \ln 1 = 1 + 0 = 1$

$\int_1^2 u^2 du$

$= \frac{u^3}{3} \Big|_1^2 = \frac{2^3}{3} - \frac{1^3}{3} = \frac{8}{3} - \frac{1}{3}$

$= \frac{7}{3}$