AB Calculus U-Substitution Day 1 Homework

Name: Key

1. Evaluate each integral.

a) $\int \frac{2x}{\sqrt{x^2+6}}\,dx$

$u = x^2 + 6$

$du = 2x\,dx$

$\int \frac{du}{\sqrt{u}} \Rightarrow 2u^{\frac{1}{2}} + C$

$2\sqrt{x^2+6} + C$

b) $\int \frac{e^x}{1+2e^x}\,dx$

$u = 1 + 2e^x$

$du = 2e^x\,dx$

$\frac{du}{2} = e^x\,dx$

$\frac{1}{2}\int \frac{du}{u}$

$= \frac{1}{2}\ln|u| + C \rightarrow \frac{1}{2}\ln|1+2e^x| + C$

c) $\int x^2\cos(x^3)\,dx$

$u = x^3$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$\frac{1}{3}\int \cos u\,du$

$\frac{1}{3}\sin u + C \rightarrow \frac{1}{3}\sin x^3 + C$

d) $\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$\int \frac{du}{\sqrt{u}}$

$\hookrightarrow 2u^{\frac{1}{2}} + C = 2\sqrt{\tan x} + C$

e) $\int_1^{\sqrt{2}} x \cdot 2^{-x^2}\,dx$

$u = -(\sqrt{2})^2 = -2$

$u = -(1)^2 = -1$

$u = -x^2$

$du = -2x\,dx$

$-\frac{du}{2} = x\,dx$

$-\frac{1}{2}\int_{-1}^{-2} 2^u\,du$

$\hookrightarrow -\frac{1}{2}\cdot\frac{2^u}{\ln 2}\Big|_{-1}^{-2} \rightarrow \frac{-1}{2\ln 2}(2^{-2} - 2^{-1})$

$\frac{1}{4} - \frac{1}{2}$

$-\frac{1}{4}$

$= \frac{1}{8\ln 2}$

f) $\int_e^{e^2} \frac{1}{x\ln x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$u = \ln e^2 = 2$

$u = \ln e = 1$

$\int_1^2 \frac{du}{u}$

$= \ln|u|\Big|_1^2 = \ln 2 - \ln 1$

$= \ln 2$

g) $\int_0^2 \sqrt{4x+1}\,dx$

$u = 4x+1$

$du = 4\,dx$

$\frac{du}{4} = dx$

$u = 4(2)+1 = 9$

$u = 4(0)+1 = 1$

$\frac{1}{4}\int_1^9 \sqrt{u}\,du$

$\frac{1}{4}\cdot\frac{2}{3}u^{\frac{3}{2}}\Big|_1^9 \rightarrow \frac{1}{6}(9^{\frac{3}{2}} - 1^{\frac{3}{2}})$

$= \frac{1}{6}(27-1)$

$= \frac{26}{6} = \boxed{13/3}$

h) $\int_0^{\pi} \sin\left(\frac{x}{2}\right)dx$

$u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$2\,du = dx$

$u = \frac{\pi}{2}$

$u = \frac{0}{2} = 0$

$2\int_0^{\pi/2} \sin u\,du$

$2(-\cos u)\Big|_0^{\frac{\pi}{2}}$

$2(-\cos\frac{\pi}{2} + \cos 0) \rightarrow 2(-0+1) = 2$

2. Evaluate each of the following.

a) $\frac{d}{dx}\left[\int_x^2 \sec t\, dt\right]$

$\frac{d}{dx} - \int_2^x \sec t\, dt$

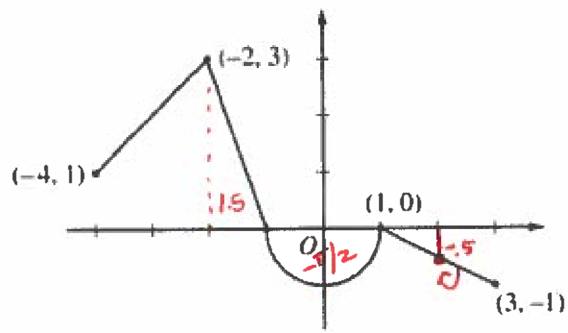
$-\sec x$

b) $\frac{d}{dx}\left[\int_{x^2}^{3x} (e^{-t^2+1} + \sqrt{t})dt\right]$

$3\left(e^{-(3x)^2+1} + \sqrt{3x}\right)$

$-\left(e^{-x^4+1} + x\right)(2x)$

3.



Graph of f

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\, dt$. so $g' = f$

(a) Find the values of $g(2)$ and $g(-2)$. $\int_1^{-2} = -\int_{-2}^{1} = -(1.5 - \frac{\pi}{2}) = \frac{\pi}{2} - 1.5$

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

(a) $g(2) = -\frac{1}{4}$

$g(-2) = \frac{\pi}{2} - 1.5$

(b) $g'(-3) = f(-3) = 2$

$g''(-3) = f'(-3) = 1$

(c) g' or $f = 0$ $x = -1$ rel. max $g'(x)$ changes + to −

$x = 1$ neither $g'(x)$ doesn't change signs

(d) g'' changes signs → f' → $x = -2, x = 0, x = 1$