Unit 10 Progress Check: FRQ Part B

Name

1. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

G.T $\frac{(-1)^n (x^2)^{2n+1}}{(2n+1)!}$

1st 4 terms of $g \to$ $x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!}$ $\frac{(-1)^n x^{4n+2}}{(2n+1)!}$

The Maclaurin series for a function f is given by $f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots$ and converges to $f(x)$ for all x. Let g be the function defined by $g(x) = f(x^2)$.

(a) Find the first four nonzero terms and the general term of the Maclaurin series for g.

(b) Find $\lim_{x\to 0} \frac{x^2 - \frac{x^6}{6} - g(x)}{x^{10}}$.

$\lim_{x\to 0} x^2 - \frac{x^6}{6} - \left(x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \ldots + \frac{(-1)^n x^{4n+2}}{(2n+1)!}\right)$

$\lim_{x\to 0} \frac{-\frac{x^{10}}{5!} + \frac{x^{14}}{7!} + \ldots - \frac{(-1)^n x^{4n+2}}{(2n+1)!}}{x^{10}} = \lim_{x\to 0} \frac{-1}{5!} + \frac{x^4}{7!} + \ldots \frac{(-1) x^{4n-8}}{(2n+1)!}$

$= -\frac{1}{5!}$

(c) Let h be the function defined by $h(x) = \int_0^x g(t)\,dt$. Write the first four nonzero terms and the general term of the Maclaurin series for h. Use the first two terms of your answer to approximate $h(1)$.

dummy variable

$\int_0^x t^2\,dt = \frac{1}{3}t^3 \Big|_0^x = \frac{1}{3}x^3$

do the same for next 3 terms $\to \frac{1}{3}x^3 - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!} - \frac{x^{15}}{15 \cdot 7!}$

G T $\frac{(-1)^n \cdot x^{4n+3}}{(4n+3)(2n+1)!}$

$$h(1) \approx \frac{1}{3}(1)^3 - \frac{(1)^7}{7\,3!}$$

(d) Explain why the approximation found in part (c) differs from the actual value of $h(1)$ by less than $\frac{1}{1000}$.