Vector Notes

1a. *[1 mark]*

A line L passes through points $\mathrm{A}(-3,\ 4,\ 2)$ and $\mathrm{B}(-1,\ 3,\ 3)$.

Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

starts at A and ends at B

$-1 - -3 = 2$
$3 - 4 = -1$
$3 - 2 = 1$

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

1b. *[2 marks]*

Find a vector equation for L. $r = a + tb$

$r = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ / $r = \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

Checking the validity

$\begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$-1 = -3 + 2t$
$3 = 4 - t$ → if $t = 1$
$3 = 2 + t$

1c. *[5 marks]*

The line L also passes through the point $\mathrm{C}(3,\ 1,\ p)$.

Find the value of p.

↑Basically to calculate, you will replace "r" with "C" ☺

$$\begin{pmatrix} 3 \\ 1 \\ p \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$3 = -3 + 2t$ $\quad$ $p = 2 + (3)(1)$

$6 = 2t$

$t = 3$ $\quad$ $p = 5$

1d. *[7 marks]*

The point D has coordinates $(q^2,\ 0,\ q)$. Given that $\overrightarrow{\mathrm{DC}}$ is perpendicular to L, find the possible values of q.

→ so dot product = 0

↳ this is the dot product

$\overrightarrow{DC} = \begin{pmatrix} 3-q^2 \\ 1-0 \\ 5-q \end{pmatrix}$ → dot product with the direction vector → $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$(3-q^2)\,2 + 1 \cdot -1 + (5-q)(1) = 0$

$6 - 2q^2 - 1 + 5 - q = 0$

$0 = 2q^2 + q - 10$ $\quad$ $q = 2$ or $q = -5/2$

$0 = (2q+5)(q-2)$

2a. *[2 marks]*

Let $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$.

Find $\overrightarrow{BC}$ $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$$\begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -8 \\ -1 \\ -1 \end{pmatrix}$$

2b. *[3 marks]*

Find a unit vector in the direction of $\overrightarrow{AB}$

$|\overrightarrow{AB}| = \sqrt{6^2 + (-2)^2 + 3^2} = \sqrt{36+4+9} = \sqrt{49} = 7$

unit vector in direction of $\overrightarrow{AB}$ is $\begin{pmatrix} 6/7 \\ -2/7 \\ 3/7 \end{pmatrix}$

2c. *[3 marks]*

Show that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{AC}$