2a. *[2 marks]*

Let $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$.

Find $\overrightarrow{BC}$

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$\begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -8 \\ -1 \\ -1 \end{pmatrix}$

2b. *[3 marks]*

Find a unit vector in the direction of $\overrightarrow{AB}$

$|\overrightarrow{AB}| = \sqrt{6^2 + (-2)^2 + 3^2} = \sqrt{36+4+9} = \sqrt{49} = 7$

unit vector in direction of $\overrightarrow{AB}$ is $\begin{pmatrix} 6/7 \\ -2/7 \\ 3/7 \end{pmatrix}$

2c. *[3 marks]*

Show that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{AC}$

$\overrightarrow{AB} \; \overrightarrow{AC} = 0$

$(6)(-2) + (-2)(-3) + (3)(2)$

$= 0$

3a. *[3 marks]*

The vertices of the triangle PQR are defined by the position vectors

$$\overrightarrow{OP} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \text{ and } \overrightarrow{OR} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}.$$

Find

(i) $\overrightarrow{PQ}$;

$$\overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{PQ}$$

$$\begin{pmatrix} -4 \\ 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = \overrightarrow{PQ}$$

(ii) $\overrightarrow{PR}$.

$$\overrightarrow{PO} + \overrightarrow{OR} = \overrightarrow{PR}$$

$$\begin{pmatrix} -4 \\ 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix} = \overrightarrow{PR}$$

3b. *[7 marks]* $\angle RPQ$

Show that $\cos R\hat{P}Q = \frac{1}{2}$. → use $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ if P is vertex

$$(-1)(2)+(2)(2)+(1)(4) = |\overrightarrow{PR}||\overrightarrow{PQ}|\cos R\hat{P}Q$$

$$|\overrightarrow{PR}| = \sqrt{2^2+2^2+4^2} = \sqrt{24}$$

$$|\overrightarrow{PQ}| = \sqrt{(-1)^2+2^2+1^2} = \sqrt{6}$$

$$\frac{6}{\sqrt{6}\sqrt{24}} = \frac{\sqrt{24}\sqrt{6}\cos R\hat{P}Q}{\sqrt{24}\sqrt{6}}$$

$$\frac{6}{\sqrt{6}\sqrt{4}\sqrt{6}} = \cos R\hat{P}Q$$

$$\boxed{\frac{1}{2} = \cos R\hat{P}Q}$$

$$\cos^2 R\hat{P}Q + \sin^2 R\hat{P}Q = 1$$

3c. *[6 marks]*

(i) Find $\sin R\widehat{P}Q$.

$$\left(\frac{1}{2}\right)^2 + \sin^2 R\hat{P}Q = 1$$

$$\sin^2 R\hat{P}Q = \frac{3}{4} \qquad \sin R\hat{P}Q = \frac{\sqrt{3}}{2}$$

(ii) Hence, find the area of triangle PQR, giving your answer in the form $a\sqrt{3}$.

$$A_{\Delta PQR} = \frac{1}{2}(|\vec{PR}|)(|\vec{PQ}|)\sin R\hat{P}Q$$

$$= \frac{1}{2}\sqrt{24}\sqrt{6}\,\frac{\sqrt{3}}{2} = 3\sqrt{3}$$

4. *[6 marks]*

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 3 \\ 2p \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} p+1 \\ 8 \end{pmatrix}$.

Find the possible values of p for which $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel.

$$3 = k(p+1) \qquad k = \frac{3}{p+1}$$

$$2p = 8k \qquad k = \frac{2p}{8}$$

$$\frac{3}{p+1} = \frac{2p}{8}$$

$$24 = 2p(p+1)$$

$$24 = 2p^2 + 2p$$

$$0 = 2p^2 + 2p - 24$$

$$0 = 2(p^2 + p - 12)$$

$$0 = 2(p+4)(p-3)$$

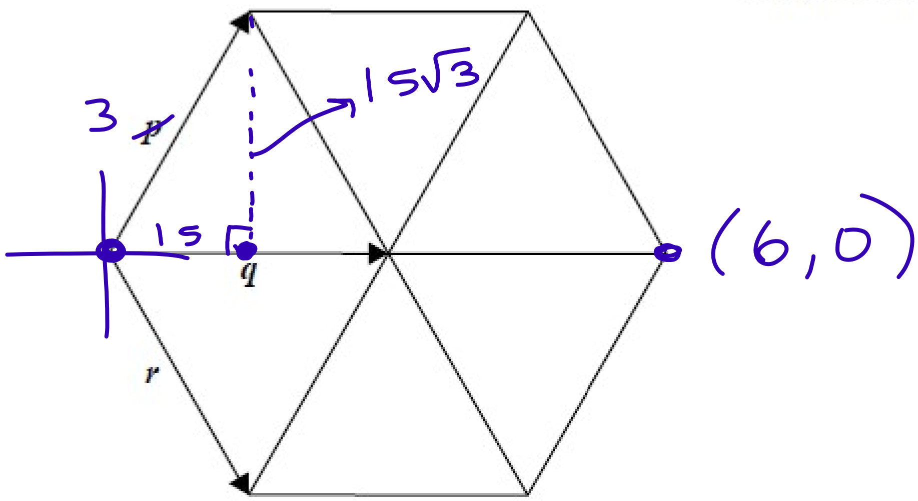
$$\boxed{p = -4, 3}$$

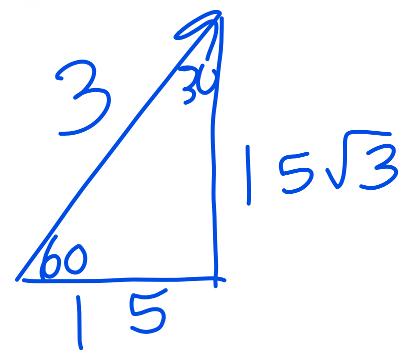
5. *[6 marks]*

Six equilateral triangles, each with side length 3 cm, are arranged to form a hexagon.

This is shown in the following diagram.

diagram not to scale





The vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are shown on the diagram.

Find $\boldsymbol{p}\bullet(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r})$.

$p\ (6,0)$

$(15, 1.5\sqrt{3}) \quad (6,0)$

$= (15)(6) + (15\sqrt{3})(0)$

$= 9$

6a. *[1 mark]*

Point A has coordinates (−4, −12, 1) and point B has coordinates (2, −4, −4).

Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$

$2 - -4 = 6$

$-4 - -12 = 8$

$-4 - 1 = -5$

$\overrightarrow{AB} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$

6b. *[2 marks]*

The line L passes through A and B.

Find a vector equation for L.

$r = \begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$

or

$r = \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$

6c. *[4 marks]*

Point C $(k\,,12\,,-k)$ is on L. Show that $k = 14$.

$$\begin{pmatrix} k \\ 12 \\ -k \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$$

$12 = -4 + 8t$

$16 = 8t$

$t = 2$

$k = 2 + 2(6)$

$k = 14$

6d. *[2 marks]*

Find $\overrightarrow{OB} \bullet \overrightarrow{AB}$.

$$\begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix} \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$$

$(2)(6) + (-4)(8) + (-4)(-5)$

$= 0$

6e. *[1 mark]*

Write down the value of angle OBA.

Since $\overrightarrow{OB}\ \overrightarrow{AB} = 0$ angle OBA $= 90°$

or $\frac{\pi}{2}$

7a. *[2 marks]*

Note: In this question, distance is in metres and time is in seconds.

Two particles P_1 and P_2 start moving from a point A at the same time, along different straight lines.

After t seconds, the position of P_1 is given by $\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

Find the coordinates of A.

coordinates of A are $(4, -1, 3)$

7b. *[3 marks]*

Two seconds after leaving A, P_1 is at point B.

Find $\overrightarrow{\mathrm{AB}}$;

$B = (4 + 2(1), -1 + 2(2), 3 + 2(-2))$

$B = (6, 3, -1)$

$6 - 4 = 2$

$3 - -1 = 4$

$-1 - 3 = -4$

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$

7c. *[2 marks]*

Find $\left|\overrightarrow{\mathrm{AB}}\right|$.

$|\overrightarrow{AB}| = \sqrt{2^2 + 4^2 + (-4)^2} = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$