5a. *[1 mark]*

Two functions, f and g, are defined in the following table.

x	-2	1	3	6
$f(x)$	6	3	1	-2
$g(x)$	-7	-2	5	9

Write down the value of $f(1)$.

3

5b. *[2 marks]* composition

Find the value of $(g \circ f)(1)$. $g(f(x))$

$f(1) = 3$ $\quad g(3) = 5$

5c. *[2 marks]*

Find the value of $g^{-1}(-2)$. → input of g inverse → output of g

1

6a. *[1 mark]*

Let $f(x) = \sqrt{x+2}$ for $x \geq 2$ and $g(x) = 3x - 7$ for $x \in \mathbb{R}$.

Write down $f(14)$.

4

6b. *[2 marks]*

Find $(g \circ f)(14)$.

$f(14) = 4 \qquad g(4) = 12 - 7 = 5$

6c. *[3 marks]*

Find $g^{-1}(x)$.

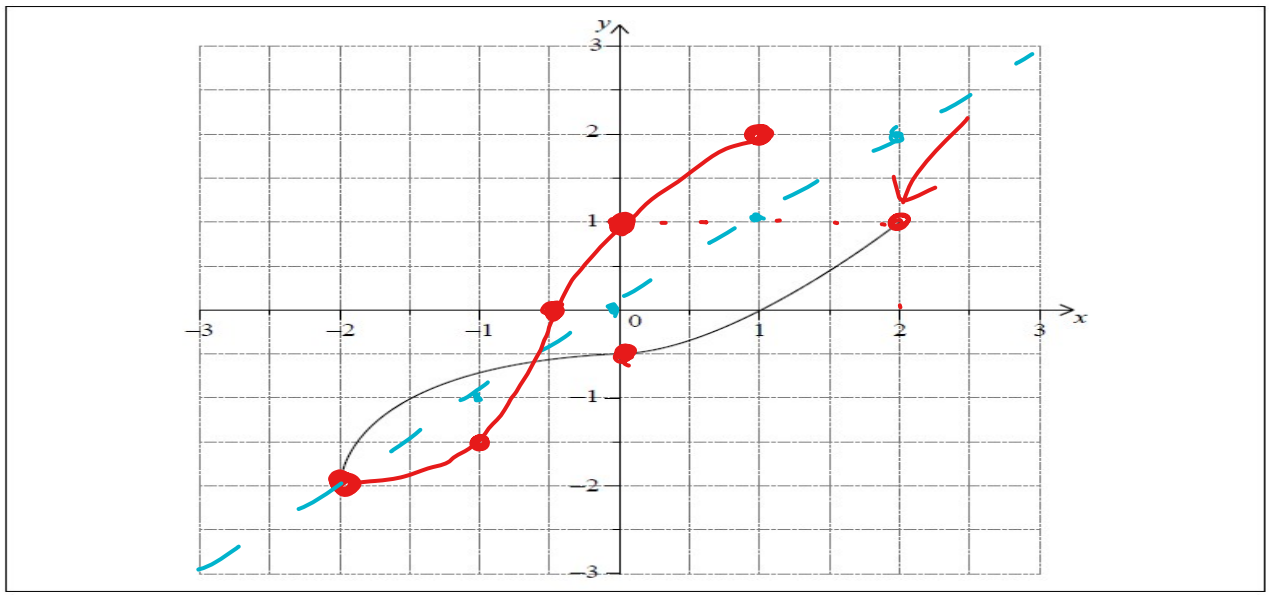
$y = 3x - 7$

$x = 3y - 7$

$\frac{x+7}{3} = g^{-1}(x)$

7a. *[1 mark]*

Consider a function $f(x)$, for $-2 \le x \le 2$. The following diagram shows the graph of f.



Write down the value of $f(0)$.

$-1/2$

1 is input of f^{-1}

so 1 is output of f

7b. *[1 mark]*

Write down the value of $f^{-1}(1)$.

2 $f(2)=1$

7c. *[1 mark]*

Write down the range of f^{-1}.

$-2 \leq x \leq 2$

7d. *[4 marks]*

On the grid above, sketch the graph of f^{-1}.

8a. *[2 marks]*

Consider a function f. The line L_1 with equation $y = 3x + 1$ is a tangent to the graph of f when $x = 2$

Write down $f'(2)$.

3

8b. *[2 marks]*

Find $f(2)$.

$3(2)+1 = 7$

8c. *[5 marks]*

Let $g(x) = f(x^2 + 1)$ and P be the point on the graph of g where $x = 1$.

Show that the graph of g has a gradient of 6 at P.

slope

$g(x) = f(x^2+1)$

$g'(x) = f'(x^2+1) \cdot (2x)$

$g'(1) = f'(2) \cdot (2) = 3 \; 2 = 6$

8d. *[7 marks]*

Let L_2 be the tangent to the graph of g at P. L_1 intersects L_2 at the point Q.

Find the y-coordinate of Q.

$x=1$ at P $\quad g'(1) = 6 \quad L_1 \; y = 3x+1$

$L_2 \quad y - 7 = 6(x-1)$

$g(x) = f(x^2+1)$

$g(1) = f(1^2+1) = f(2) = 7$

$L_1 = L_2$!!!

$y = 3x+1$

$y = 6(x-1)+7$

$3x+1 = 6x-6+7$

$3x+1 = 6x+1$

$3x = 6x \rightarrow \frac{0}{3} = \frac{3x}{3}$

$x = 0$

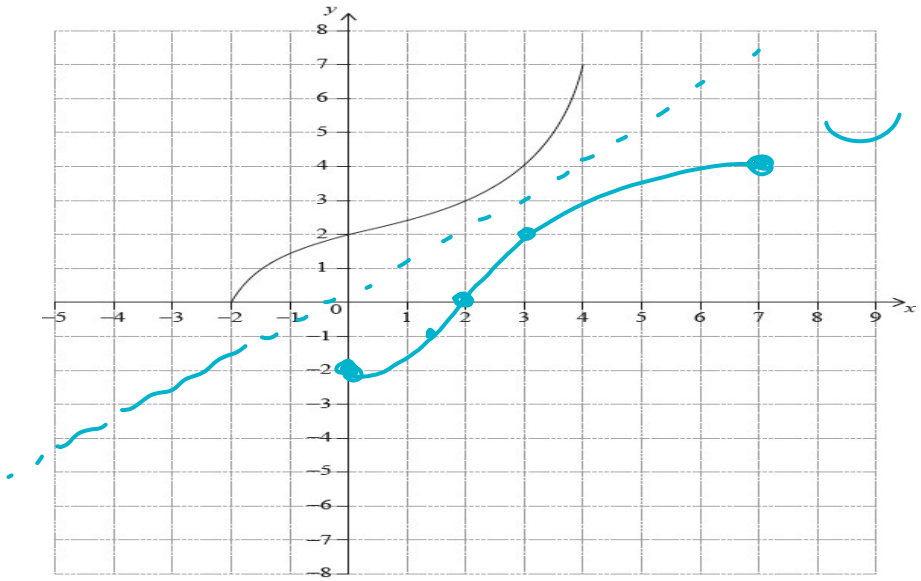
$X = 0$

Q $(0, 1)$

↳ answer

9a. *[1 mark]*

The following diagram shows the graph of a function f, with domain $-2 \leqslant x \leqslant 4$.



The points $(-2,\ 0)$ and $(4,\ 7)$ lie on the graph of f.

Write down the range of f. $[0, 7]$ OR $0 \leq y \leq 7$

9b. *[1 mark]*

Write down $f(2)$;

3

9c. *[1 mark]*

Write down $f^{-1}(2)$.

0

9d. *[3 marks]*

On the grid, sketch the graph of f^{-1}.

10a. *[2 marks]*

Let $f(x) = 1 + e^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.

Find $(g \circ f)(x)$.

↳ composition

$2(1+e^{-x})+b$

10b. *[4 marks]*

Given that $\lim_{x \to +\infty} (g \circ f)(x) = -3$, find the value of b.

end behavior

$\lim_{x \to \infty} 2(1+e^{-x})+b = -3$

$2+b = -3$

$b = -5$

11a. *[1 mark]*

Let $f(x) = \ln x$ and $g(x) = 3 + \ln\left(\frac{x}{2}\right)$, for $x > 0$.

The graph of g can be obtained from the graph of f by two transformations:

a horizontal stretch of scale factor q followed by

a translation of $\begin{pmatrix} h \\ k \end{pmatrix}$.

Write down the value of q;

$q = 2$

11b. *[1 mark]*

Write down the value of h;

$h = 0$

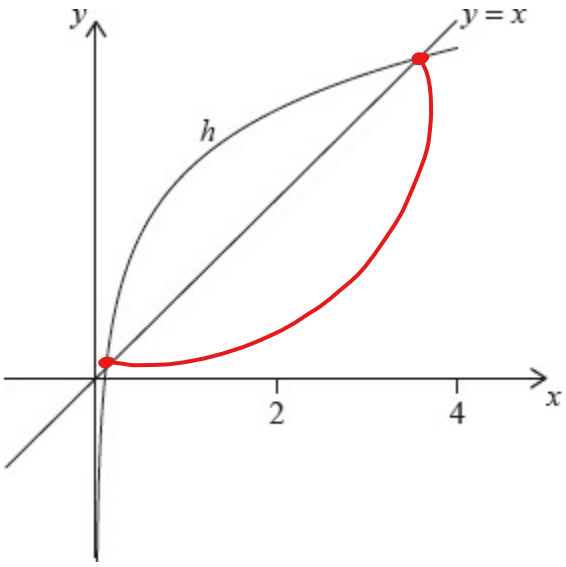
11c. *[1 mark]*

Write down the value of k.

$k = 3$

11d. *[2 marks]*

Let $h(x) = g(x) \times \cos(0.1x)$, for $0 < x < 4$. The following diagram shows the graph of h and the line $y = x$.



The graph of h intersects the graph of h^{-1} at two points. These points have x coordinates 0.111 and 3.31 correct to three significant figures.

Find $\int_{0.111}^{3.31} (h(x) - x)\,\mathrm{d}x$.

mr Ehrman's calculator
$y_1 = 3 + \ln\left(\frac{x}{2}\right)$
$y_2 = y_1 * \cos(.1x)$

calculation screen
$\int_{111}^{331} (y_2 - x)\,dx = 272$

11e. *[3 marks]*

Hence, find the area of the region enclosed by the graphs of h and h^{-1}.

(2724 .) x2 = 5448

545

11f. *[7 marks]*

Let d be the vertical distance from a point on the graph of h to the line $y = x$. There is a point $\mathrm{P}(a,\ b)$ on the graph of h where d is a maximum.

Find the coordinates of P, where $0.111 < a < 3.31$.

$d = h(x) - x$

$d' = h'(x) - 1$

$h'(x) - 1 = 0$

graph $h'(x)$

and $y = 1$ and intersect

In calculator

$y_3 = \frac{d}{dx}[y_2]\Big|_{x=x}$

$y_4 = 1$

calculate intersection

& unhighlight y_1 & y_2 *

x= 97368

h(97368) = 2.26938

P (974, 2.27)