AB Calculus Velocity and Other Rates of Change Homework Name: Key

1. A rock thrown vertically upward from the surface of the moon at a velocity of 24 m/sec (about 86 km/h) reaches a height of $s(t) = 24t - 0.8t^2$ meters in t seconds.

a) Find the rock's velocity and acceleration as functions of time. (The acceleration in this case is the acceleration of gravity on the moon). $s'(t) = 24 - 1.6t$ $s''(t) = -1.6$

b) How long did it take the rock to reach its highest point?

$24 - 1.6t = 0$ $-1.6t = -24$ $t = 15$ seconds

c) How high did the rock go?

$S(15) = 24(15) - .8(15)^2$ 180 meters

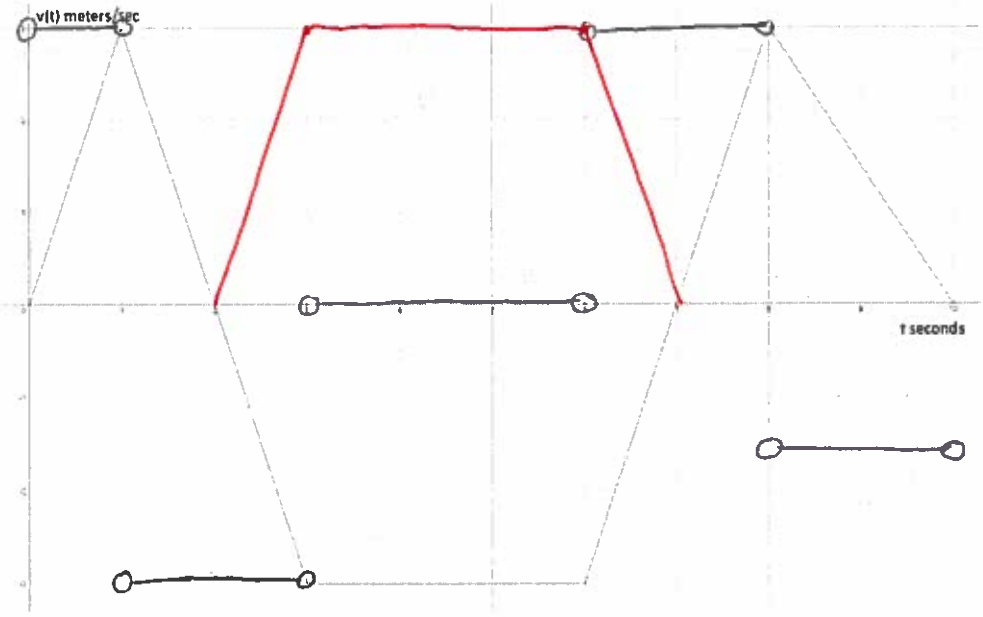
d) When did the rock reach half its maximum height?

$90 = 24t - .8t^2$ $8t^2 - 240t + 900 = 0$ $t \approx 4.393$ s, $t \approx 25.607$ s

e) How long was the rock aloft?

$0 = 24t - .8t^2$ $0 = .8t(30 - t)$ $t = 30$ s

2. The accompanying figure shows the velocity $v = \frac{ds}{dt} = f(t)$ meters/sec of a body moving along a coordinate line.



a) When does the body reverse direction? $t = 2s, 7s$

b) When (approximately) is the body moving at a constant speed? $t = (3, 6)$

c) Graph the body's speed for $0 \le t \le 10$ on the same graph in a different color. Red

d) Graph the acceleration, where defined, on the same grid in a different color. Black

3. A body's velocity at time t sec is $v(t) = 2t^3 - 9t^2 + 12t - 5$ m/sec. Find the body's speed each time the acceleration is 0.

$a(t) = 6t^2 - 18t + 12$

$= 6(t^2 - 3t + 2)$

$= 6(t-2)(t-1)$

$t = 1, 2$

$2(1)^3 - 9(1)^2 + 12(1) - 5$

$2 - 9 + 12 - 5$

$-7 + 7$

$= 0$ m/sec

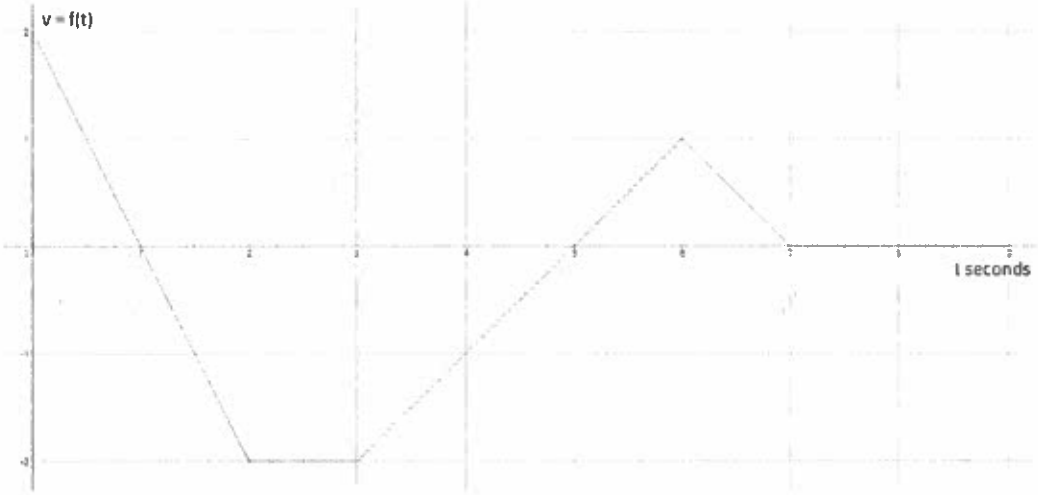
$2(2)^3 - 9(2)^2 + 12(2) - 5$

$16 - 36 + 24 - 5$

$-20 + 19$

$-1 \rightarrow 1$ m/sec

4. The accompanying figure shows the velocity $v = \frac{ds}{dt} = f(t)$ meters/sec of a body moving along a coordinate line.



a) When does the particle move forward? Move backward? Speed up? Slow down?

$(0,1) \cup (5,7)$ ← $\downarrow (1,5)$ $\downarrow (1,2) \cup (5,6)$ → $(0,1) \cup (3,5) \cup (6,7)$

b) When is the particle's acceleration positive? Negative? Zero?

$(3,6)$ ↙ ↓ $(0,2) \cup (6,7)$ → $(2,3) \cup (7,9)$

c) When does the particle move at its greatest speed?

$t = 0$ $t = (2,3)$

d) When does the particle stand still for more than an instant?

$(7,9)$

5. The number of gallons of water in a tank t minutes after the tank has started to drain is $Q(t) = 200(30 - t)^2$.

a) How fast is the water running out at the end of 10 minutes?

$200(900 - 60t + t^2)$

$180{,}000 - 12{,}000t + 200$

$Q'(t) = -12{,}000 + 400t$

$-12{,}000 + 400(10) = -8000$ g/min

b) What is the average rate at which the water flows out during the first 10 minutes? ↳ so 8000g/min

$\frac{Q'(10) - Q'(0)}{10 - 0} \rightarrow \frac{200(30-10)^2 - 200(30-0)^2}{10}$

$\frac{200(20)^2 - 200(30)^2}{10} \rightarrow 20^3 - 20(30)^2$

$8000 - 18000$

$= -10000 \rightarrow 10{,}000$ g/min