AP Calculus – Final Review Sheet

When you see the words ….		**This is what you think of doing**
1.	Find the zeros	Find roots. Set function = 0, factor or use quadratic equation if quadratic, graph to find zeros on calculator
2.	Show that $f(x)$ is even	Show that $f(-x) = f(x)$ symmetric to *y*-axis
3.	Show that $f(x)$ is odd	Show that $f(-x) = -f(x)$ OR $f(x) = -f(-x)$ symmetric around the origin
4.	Show that $\lim_{x\to a} f(x)$ exists	Show that $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$; exists and are equal
5.	Find $\lim_{x\to a} f(x)$, calculator allowed	Use TABLE [ASK], find y values for x-values close to a from left and right
6.	Find $\lim_{x\to a} f(x)$, no calculator	Substitute $x = a$ 1) limit is value if $\frac{b}{c}$, incl. $\frac{0}{c} = 0; c \neq 0$ 2) DNE for $\frac{b}{0}$ 3) $\frac{0}{0}$ DO MORE WORK! a) rationalize radicals b) simplify complex fractions c) factor/reduce d) known trig limits 1. $\lim_{x\to 0} \frac{\sin x}{x} = 1$ 2. $\lim_{x\to 0} \frac{1-\cos x}{x} = 0$ e) piece-wise fcn: check if RH = LH at break
7.	Find $\lim_{x\to\infty} f(x)$, calculator allowed	Use TABLE [ASK], find y values for large values of x, i.e. 999999999999
8.	Find $\lim_{x\to\infty} f(x)$, no calculator	Ratios of rates of changes 1) $\frac{fast}{slow} = DNE$ 2) $\frac{slow}{fast} = 0$ 3) $\frac{same}{same} = ratio$ of coefficients
9.	Find horizontal asymptotes of $f(x)$	Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$
10.	Find vertical asymptotes of $f(x)$	Find where $\lim_{x\to a^\pm} f(x) = \pm\infty$ 1) Factor/reduce $f(x)$ and set denominator = 0 2) $\ln x$ has VA at x = 0

11.	Find domain of $f(x)$	Assume domain is $(-\infty,\infty)$. Restrictable domains: denominators $\neq 0$, square roots of only non-negative numbers, log or ln of only positive numbers, real-world constraints
12.	Show that $f(x)$ is continuous	Show that 1) $\lim_{x\to a} f(x)$ exists ($\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$) 2) $f(a)$ exists 3) $\lim_{x\to a} f(x) = f(a)$
13.	Find the slope of the tangent line to $f(x)$ at x = a.	Find derivative $f'(a) = m$
14.	Find equation of the line tangent to $f(x)$ at (a,b)	$f'(a) = m$ and use $y - b = m(x-a)$ sometimes need to find $b = f(a)$
15.	Find equation of the line normal (perpendicular) to $f(x)$ at (a,b)	Same as above but $m = \dfrac{-1}{f'(a)}$
16.	Find the average rate of change of $f(x)$ on $[a,b]$	Find $\dfrac{f(b)-f(a)}{b-a}$
17.	Show that there exists a c in $[a,b]$ such that $f(c) = n$	Intermediate Value Theorem (IVT) Confirm that $f(x)$ is continuous on $[a,b]$, then show that $f(a) \leq n \leq f(b)$.
18.	Find the interval where $f(x)$ is increasing	Find $f'(x)$, set both numerator and denominator to zero to find critical points, make sign chart of $f'(x)$ and determine where $f'(x)$ is positive.
19.	Find interval where the slope of $f(x)$ is increasing	Find the derivative of $f'(x) = f''(x)$, set both numerator and denominator to zero to find critical points, make sign chart of $f''(x)$ and determine where $f''(x)$ is positive.
20.	Find instantaneous rate of change of $f(x)$ at a	Find $f'(a)$
21.	Given $s(t)$ (position function), find $v(t)$	Find $v(t) = s'(t)$
22.	Find $f'(x)$ by the limit definition *Frequently asked backwards*	$f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ or $f'(a) = \lim_{x\to a} \dfrac{f(x)-f(a)}{x-a}$
23.	Find the average velocity of a particle on $[a,b]$	Find $\dfrac{1}{b-a}\int_a^b v(t)\,dt$ OR $\dfrac{s(b)-s(a)}{b-a}$ depending on if you know $v(t)$ or $s(t)$
24.	Given $v(t)$, determine if a particle is speeding up at $t = k$	Find $v(k)$ and $a(k)$. If signs match, the particle is speeding up; if different signs, then the particle is slowing down.
25.	Given a graph of $f'(x)$, find where $f(x)$ is increasing	Determine where $f'(x)$ is positive (above the *x*-axis.)

26.	Given a table of x and $f(x)$ on selected values between a and b, estimate $f'(c)$ where c is between a and b.	Straddle c, using a value, k, greater than c and a value, h, less than c. so $f'(c) \approx \frac{f(k)-f(h)}{k-h}$
27.	Given a graph of $f'(x)$, find where $f(x)$ has a relative maximum.	Identify where $f'(x)=0$ crosses the x-axis from above to below OR where $f'(x)$ is discontinuous and jumps from above to below the x-axis.
28.	Given a graph of $f'(x)$, find where $f(x)$ is concave down.	Identify where $f'(x)$ is decreasing.
29.	Given a graph of $f'(x)$, find where $f(x)$ has point(s) of inflection.	Identify where $f'(x)$ changes from increasing to decreasing or vice versa.
30.	Show that a piecewise function is differentiable at the point a where the function rule splits	First, be sure that the function is continuous at $x = a$ by evaluating each function at x = a. Then take the derivative of each piece and show that $\lim_{x\to a^-} f'(x) = \lim_{x\to a+} f'(x)$
31.	Given a graph of $f(x)$ and $h(x) = f^{-1}(x)$, find $h^{'}(a)$	Find the point where a is the y-value on $f(x)$, sketch a tangent line and estimate $f'(b)$ at the point, then $h'(a) = \frac{1}{f'(b)}$
32.	Given the equation for $f(x)$ and $h(x) = f^{-1}(x)$, find $h^{'}(a)$	Understand that the point (a,b) is on $h(x)$ so the point (b,a) is on $f(x)$. So find b where $f(b)=a$ $h'(a) = \frac{1}{f'(b)}$
33.	Given the equation for $f(x)$, find its derivative algebraically.	1) know product/quotient/chain rules 2) know derivatives of basic functions a. Power Rule: polynomials, radicals, rationals b. $e^x; b^x$ c. $\ln x; \log_b x$ d. $\sin x; \cos x; \tan x$ e. $\arcsin x; \arccos x; \arctan x; \sin^{-1} x; etc$
34.	Given a relation of x and y, find $\frac{dy}{dx}$ algebraically.	Implicit Differentiation Find the derivative of each term, using product/quotient/chain appropriately, especially, chain rule: every derivative of y is multiplied by $\frac{dy}{dx}$; then group all $\frac{dy}{dx}$ terms on one side; factor out $\frac{dy}{dx}$ and solve.
35.	Find the derivative of $f(g(x))$	Chain Rule $f'(g(x)) \cdot g'(x)$

36.	Find the minimum value of a function on $[a,b]$	Solve $f'(x)=0$ or DNE, make a sign chart, find sign change from negative to positive for relative minimums and evaluate those candidates along with endpoints back into $f(x)$ and choose the smallest. NOTE: be careful to confirm that $f(x)$ exists for any x-values that make $f'(x)$ DNE.
37.	Find the minimum slope of a function on $[a,b]$	Solve $f''(x)=0$ or DNE, make a sign chart, find sign change from negative to positive for relative minimums and evaluate those candidates along with endpoints back into $f'(x)$ and choose the smallest. NOTE: be careful to confirm that $f(x)$ exists for any x-values that make $f''(x)$ DNE.
38.	Find critical values	Express $f'(x)$ as a fraction and solve for numerator and denominator each equal to zero.
39.	Find the absolute maximum of $f(x)$	Solve $f'(x)=0$ or DNE, make a sign chart, find sign change from positive to negative for relative maximums and evaluate those candidates into $f(x)$, also find $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$; choose the largest.
40.	Show that there exists a *c* in $[a,b]$ such that $f'(c)=0$	Rolle's Theorem Confirm that *f* is continuous and differentiable on the interval. Find *k* and *j* in $[a,b]$ such that $f(k)=f(j)$, then there is some *c* in $[k,j]$ such that $f'(c)=0$.
41.	Show that there exists a *c* in $[a,b]$ such that $f'(c)=m$	Mean Value Theorem Confirm that *f* is continuous and differentiable on the interval. Find *k* and *j* in $[a,b]$ such that $m=\dfrac{f(k)-f(j)}{k-j}$., then there is some *c* in $[k,j]$ such that $f'(c)=m$.
42.	Find range of $f(x)$ on $[a,b]$	Use max/min techniques to find values at relative max/mins. Also compare $f(a)$ and $f(b)$ (endpoints)
43.	Find range of $f(x)$ on $(-\infty,\infty)$	Use max/min techniques to find values at relative max/mins. Also compare $\lim\limits_{x\to\pm\infty} f(x)$.
44.	Find the locations of relative extrema of $f(x)$ given both $f'(x)$ and $f''(x)$. Particularly useful for relations of x and y where finding a change in sign would be difficult.	Second Derivative Test Find where $f'(x)=0$ OR DNE then check the value of $f''(x)$ there. If $f''(x)$ is positive, $f(x)$ has a relative minimum. If $f''(x)$ is negative, $f(x)$ has a relative maximum.

45.	Find inflection points of $f(x)$ algebraically.	Express $f''(x)$ as a fraction and set both numerator and denominator equal to zero. Make sign chart of $f''(x)$ to find where $f''(x)$ changes sign. (+ to – or – to +) NOTE: be careful to confirm that $f(x)$ exists for any x-values that make $f''(x)$ DNE.
46.	Show that the line $y = mx + b$ is tangent to $f(x)$ at (x_1, y_1)	Two relationships are required: same slope and point of intersection. Check that $m = f'(x_1)$ and that (x_1, y_1) is on both $f(x)$ and the tangent line.
47.	Find any horizontal tangent line(s) to $f(x)$ or a relation of *x* and *y*.	Write $\frac{dy}{dx}$ as a fraction. Set the numerator equal to zero. NOTE: be careful to confirm that any values are on the curve. Equation of tangent line is y = b. May have to find b.
48.	Find any vertical tangent line(s) to $f(x)$ or a relation of *x* and *y*.	Write $\frac{dy}{dx}$ as a fraction. Set the denominator equal to zero. NOTE: be careful to confirm that any values are on the curve. Equation of tangent line is x = a. May have to find a.
49.	Approximate the value of $f(0.1)$ by using the tangent line to *f* at $x = 0$	Find the equation of the tangent line to *f* using $y - y_1 = m(x - x_1)$ where $m = f'(0)$ and the point is $(0, f(0))$. Then plug in 0.1 into this line; be sure to use an approximate $(\approx)$ sign. Alternative linearization formula: $y = f'(a)(x-a) + f(a)$
50.	Find rates of change for volume problems.	Write the volume formula. Find $\frac{dV}{dt}$. Careful about product/ chain rules. Watch positive (increasing measure)/negative (decreasing measure) signs for rates.
51.	Find rates of change for Pythagorean Theorem problems.	$x^2 + y^2 = z^2$ $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$; can reduce 2's Watch positive (increasing distance)/negative (decreasing distance) signs for rates.
52.	Find the average value of $f(x)$ on $[a,b]$	Find $\frac{1}{b-a}\int_a^b f(x)\,dx$
53.	Find the average rate of change of $f(x)$ on $[a,b]$	$\frac{f(b) - f(a)}{b-a}$
54.	Given $v(t)$, find the total distance a particle travels on $[a,b]$	Find $\int_a^b \lvert v(t)\rvert dt$
55.	Given $v(t)$, find the change in position a particle travels on $[a,b]$	Find $\int_a^b v(t)\,dt$

56.	Given $v(t)$ and initial position of a particle, find the position at t = a.	Find $\int_0^a v(t)\,dt + s(0)$ Read carefully: starts at rest at the origin means $s(0)=0$ and $v(0)=0$
57.	$\frac{d}{dx}\int_a^x f(t)\,dt =$	$f(x)$
58.	$\frac{d}{dx}\int_a^{g(x)} f(t)\,dt$	$f(g(x))g'(x)$
59.	Find area using left Riemann sums	$A = base[x_0 + x_1 + x_2 + ... + x_{n-1}]$ Note: sketch a number line to visualize
60.	Find area using right Riemann sums	$A = base[x_1 + x_2 + x_3 + ... + x_n]$ Note: sketch a number line to visualize
61.	Find area using midpoint rectangles	Typically done with a table of values. Be sure to use only values that are given. If you are given 6 sets of points, you can only do 3 midpoint rectangles. Note: sketch a number line to visualize
62.	Find area using trapezoids	$A = \frac{base}{2}[x_0 + 2x_1 + 2x_2 + ... + 2x_{n-1} + x_n]$ This formula only works when the base (width) is the same. Also trapezoid area is the average of LH and RH. If different widths, you have to do individual trapezoids, $A = \frac{1}{2}h(b_1 + b_2)$
63.	Describe how you can tell if rectangle or trapezoid approximations over- or under-estimate area.	Overestimate area: LH for decreasing; RH for increasing; and trapezoids for concave up Underestimate area: LH for increasing; RH for decreasing and trapezoids for concave down DRAW A PICTURE with 2 shapes.
64.	Given $\int_a^b f(x)\,dx$, find $\int_a^b [f(x)+k]\,dx$	$\int_a^b [f(x)+k]\,dx = \int_a^b f(x)dx + \int_a^b kdx = \int_a^b f(x)dx + k(b-a)$
65.	Given $\frac{dy}{dx}$, draw a slope field	Use the given points and plug them into $\frac{dy}{dx}$, drawing little lines with the indicated slopes at the points.
66.	y is increasing proportionally to y	$\frac{dy}{dt} = ky$ translating to $y = Ae^{kt}$
67.	Solve the differential equation …	Separate the variables – *x* on one side, *y* on the other. The *dx* and *dy* must all be upstairs. Integrate each side, add C. Find C before solving for y,[unless $\ln y$, then solve for y first and find A]. When solving for y, choose + or – (not both), solution will be a continuous function passing through the initial value.
68.	Find the volume given a base bounded by $f(x)$ and $g(x)$ with $f(x) > g(x)$ and cross sections perpendicular to the *x*-axis are squares	The distance between the curves is the base of your square. So the volume is $\int_a^b (f(x)-g(x))^2\,dx$

69.	Given the value of $F(a)$ and $F'(x)=f(x)$, find $F(b)$	Usually, this problem contains an anti-derivative you cannot do. Utilize the fact that if $F(x)$ is the anti-derivative of *f*, then $\int_a^b f(x)dx = F(b)-F(a)$. So solve for $F(b)$ using the calculator to find the definite integral, $F(b)=\int_a^b f(x)dx + F(a)$
70.	Meaning of $\int_a^b f(t)\ dt$	The accumulation function: net (total if $f(x)$ is positive) amount of y-units for the function $f(x)$ beginning at x = a and ending at x = b.
71.	Given $v(t)$ and $s(0)$, find the greatest distance from the origin of a particle on $[a,b]$	Solve $v(t)=0$ OR DNE . Then integrate $v(t)$ adding $s(0)$ to find $s(t)$. Finally, compare *s*(each candidate) and s(each endpoint). Choose greatest distance (it might be negative!)
72.	Given a water tank with *g* gallons initially being filled at the rate of $F(t)$ gallons/min and emptied at the rate of $E(t)$ gallons/min on $[0,b]$, find a) the amount of water in the tank at *m* minutes	$g+\int_0^m (F(t)-E(t))dt$
73.	b) the rate the water amount is changing at *m*	$\frac{d}{dt}\int_0^m (F(t)-E(t))dt = F(m)-E(m)$
74.	c) the time when the water is at a minimum	Solve $F(t)-E(t)=0$ to find candidates, evaluate candidates and endpoints as $x=a$ in $g+\int_0^a (F(t)-E(t))dt$, choose the minimum value
75.	Find the area between $f(x)$ and $g(x)$ with $f(x) > g(x)$ on $[a,b]$	$A=\int_a^b [f(x)-g(x)]dx$
76.	Find the volume of the area between $f(x)$ and $g(x)$ with $f(x)>g(x)$, rotated about the *x*-axis.	$V=\pi\int_a^b \left[(f(x))^2-(g(x))^2\right]dx$
77.	Given $v(t)$ and $s(0)$, find $s(t)$	$s(t)=\int_0^t v(x)\ dx + s(0)$
78.	Find the line $x=c$ that divides the area under $f(x)$ on $[a,b]$ to two equal areas	$\frac{1}{2}\int_a^b f(x)dx = \int_a^c f(x)dx$ Note: this approach is usually easier to solve than $\int_a^c f(x)dx = \int_c^b f(x)dx$

79.	Find the volume given a base bounded by $f(x)$ and $g(x)$ with $f(x) > g(x)$ and cross sections perpendicular to the *x*-axis are semi-circles	The distance between the curves is the **diameter** of your circle. So the volume is $\frac{1}{2}\pi\int_a^b \left(\frac{f(x)-g(x)}{2}\right)^2 dx$